

thm\_2Elist\_2EMONO\_EXISTS  
(TMZnJ1Ep2JZ7dxNiXUh4P5wMXFUqZ4kQN9Y)

October 26, 2020

**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2E\_2T$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a}))$

**Definition 4** We define  $c\_2Ebool\_2E\_2F$  to be  $(ap (c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 5** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 6** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2E\_2F))$

**Definition 7** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V2t)))$

Let  $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty\_2Elist\_2Elist A0) \quad (1)$$

Let  $c\_2Elist\_2EEXISTS : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Elist\_2EEXISTS A\_27a \in ((2^{(ty\_2Elist\_2Elist A\_27a)})^{(2^{A\_27a})}) \quad (2)$$

Let  $c\_2Elist\_2ECONS : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Elist\_2ECONS A\_27a \in (((ty\_2Elist\_2Elist A\_27a)^{(ty\_2Elist\_2Elist A\_27a)})^{A\_27a}) \quad (3)$$

Let  $c\_2Elist\_2ENIL : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Elist\_2ENIL A\_27a \in (ty\_2Elist\_2Elist A\_27a) \quad (4)$$

**Definition 8** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V2t)))$

Assume the following.

$$True \quad (5)$$

Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in A\_27a.(p \ V0t)) \Leftrightarrow (p \ V0t))) \quad (6)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \wedge True) \Leftrightarrow \\ & (p \ V0t)) \wedge (((False \wedge (p \ V0t)) \Leftrightarrow False) \wedge (((p \ V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p \ V0t) \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)))))) \quad (7) \end{aligned}$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \vee (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \vee True) \Leftrightarrow True) \wedge \\ & (((False \vee (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee False) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee \\ & (p \ V0t)) \Leftrightarrow (p \ V0t)))))) \quad (8) \end{aligned}$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \Rightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (( \\ & (p \ V0t) \Rightarrow False) \Leftrightarrow (\neg(p \ V0t)))))) \quad (9) \end{aligned}$$

Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0x \in A\_27a. (\forall V1y \in A\_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (10)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \Leftrightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p \ V0t)) \wedge (((False \Leftrightarrow (p \ V0t)) \Leftrightarrow (\neg(p \ V0t))) \wedge (((p \ V0t) \Leftrightarrow False) \Leftrightarrow (\neg( \\ & p \ V0t)))))) \quad (11) \end{aligned}$$

Assume the following.

$$\begin{aligned} & (\forall V0P \in 2. (\forall V1Q \in 2. (\forall V2R \in 2. (((p \ V0P) \vee \\ & (p \ V1Q)) \Rightarrow (p \ V2R)) \Leftrightarrow (((p \ V0P) \Rightarrow (p \ V2R)) \wedge ((p \ V1Q) \Rightarrow (p \ V2R)))))) \quad (12) \end{aligned}$$

Assume the following.

$$\begin{aligned} & (\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p \ V0t1) \Rightarrow \\ & (p \ V1t2) \Rightarrow (p \ V2t3)) \Leftrightarrow (((p \ V0t1) \wedge (p \ V1t2)) \Rightarrow (p \ V2t3)))))) \quad (13) \end{aligned}$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in 2. (\forall V1x\_27 \in 2. (\forall V2y \in 2. (\forall V3y\_27 \in \\ & 2. (((p \ V0x) \Leftrightarrow (p \ V1x\_27)) \wedge ((p \ V1x\_27) \Rightarrow ((p \ V2y) \Leftrightarrow (p \ V3y\_27)))) \Rightarrow \\ & (((p \ V0x) \Rightarrow (p \ V2y)) \Leftrightarrow ((p \ V1x\_27) \Rightarrow (p \ V3y\_27)))))) \quad (14) \end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.\text{nonempty } A\_27a \Rightarrow ((\forall V0P \in (2^{A-27a}).((p (ap \\
& (ap (c\_2Elist\_2EEXISTS A\_27a) V0P) (c\_2Elist\_2ENIL A\_27a))) \Leftrightarrow \\
& \text{False})) \wedge (\forall V1P \in (2^{A-27a}).(\forall V2h \in A\_27a.(\forall V3t \in \\
& (ty\_2Elist\_2Elist A\_27a).((p (ap (ap (c\_2Elist\_2EEXISTS A\_27a) \\
& V1P) (ap (ap (c\_2Elist\_2ECONS A\_27a) V2h) V3t))) \Leftrightarrow ((p (ap V1P V2h)) \vee \\
& (p (ap (ap (c\_2Elist\_2EEXISTS A\_27a) V1P) V3t))))))))))
\end{aligned} \tag{15}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.\text{nonempty } A\_27a \Rightarrow (\forall V0P \in (2^{(ty\_2Elist\_2Elist A\_27a)}). \\
& (((p (ap V0P (c\_2Elist\_2ENIL A\_27a))) \wedge (\forall V1t \in (ty\_2Elist\_2Elist \\
& A\_27a).((p (ap V0P V1t)) \Rightarrow (\forall V2h \in A\_27a.(p (ap V0P (ap (ap ( \\
& c\_2Elist\_2ECONS A\_27a) V2h) V1t))))))) \Rightarrow (\forall V3l \in (ty\_2Elist\_2Elist \\
& A\_27a).(p (ap V0P V3l))))))
\end{aligned} \tag{16}$$

**Theorem 1**

$$\begin{aligned}
& \forall A\_27a.\text{nonempty } A\_27a \Rightarrow (\forall V0P \in (2^{A-27a}).(\forall V1Q \in \\
& (2^{A-27a}).(\forall V2l \in (ty\_2Elist\_2Elist A\_27a).((\forall V3x \in \\
& A\_27a.((p (ap V0P V3x)) \Rightarrow (p (ap V1Q V3x)))) \Rightarrow ((p (ap (ap (c\_2Elist\_2EEXISTS \\
& A\_27a) V0P) V2l)) \Rightarrow (p (ap (ap (c\_2Elist\_2EEXISTS A\_27a) V1Q) V2l))))))
\end{aligned}$$