

# thm\_2Elist\_2ENOT\_\_CONS\_\_NIL (TMSKcgpuB- NWbTisgCY3m6EY6HxAp4ZRuqtD)

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Let  $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Elist\_2Elist\ A0) \quad (1)$$

Let  $c\_2Elist\_2ECONS : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow c\_2Elist\_2ECONS\ A.27a \in (((ty\_2Elist\_2Elist\ A.27a)^{(ty\_2Elist\_2Elist\ A.27a)})^{A.27a}) \quad (2)$$

Let  $c\_2Elist\_2ENIL : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow c\_2Elist\_2ENIL\ A.27a \in (ty\_2Elist\_2Elist\ A.27a) \quad (3)$$

**Definition 1** We define  $c\_2Emin\_2E3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o\ (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2ET$  to be  $(ap\ (ap\ (c\_2Emin\_2E3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Ebool\_2E21$  to be  $\lambda A.27a : \iota.(\lambda V0P \in (2^{A.27a}).(ap\ (ap\ (c\_2Emin\_2E3D\ (2^{A.27a})))$

**Definition 4** We define  $c\_2Ebool\_2EF$  to be  $(ap\ (c\_2Ebool\_2E21\ 2)\ (\lambda V0t \in 2.V0t))$ .

**Definition 5** We define  $c\_2Emin\_2E3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o\ (p\ P \Rightarrow p\ Q)$  of type  $\iota$ .

**Definition 6** We define  $c\_2Ebool\_2E7E$  to be  $(\lambda V0t \in 2.(ap\ (ap\ c\_2Emin\_2E3D\_3D\_3E\ V0t)\ c\_2Ebool\_2EF))$

Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0x \in A.27a.(\forall V1y \in A.27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (4)$$

Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0a1 \in (ty\_2Elist\_2Elist\ A.27a).(\forall V1a0 \in A.27a.(\neg((c\_2Elist\_2ENIL\ A.27a) = (ap\ (ap\ (c\_2Elist\_2ECONS\ A.27a)\ V1a0)\ V0a1)))))) \quad (5)$$

**Theorem 1**

$$\forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow (\forall V_{0a1} \in (\text{ty\_2Elist\_2Elist } A_{27a}). (\forall V_{1a0} \in A_{27a}. (\neg((\text{ap } (\text{ap } (\text{c\_2Elist\_2ECONS } A_{27a}) V_{1a0}) V_{0a1}) = (\text{c\_2Elist\_2ENIL } A_{27a}))))))$$