thm_2Elist_2ENULL__LENGTH (TMGe6FLTAHXyC28v4bgBVD6mPEeKm5V5CVK)

October 26, 2020

Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o\ (x = y)$
of type $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\text{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \qquad (1)$$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \qquad (2)$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \qquad (3)$$

Definition 3 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Let $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Elist\_2Elist\ A0) \qquad (4)$$

Let $c\_2Elist\_2ELENGTH : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ELENGTH\ A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Elist\_2Elist\ A\_27a)})$$
$$(5)$$

Let $c\_2Elist\_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ENIL\ A\_27a \in (ty\_2Elist\_2Elist\ A\_27a)$$
$$(6)$$

Let $c\_2Elist\_2ENULL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ENULL\ A\_27a \in (2^{(ty\_2Elist\_2Elist\ A\_27a)})$$
$$(7)$$

Definition 4 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ ($c_2Emin_2E_3D $(2^{A\_27a}$

Assume the following.
$$True \tag{8}$$

Assume the following.
$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in \\ A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \tag{9}$$

Assume the following.
$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow \\ True)) \tag{10}$$

Assume the following.
$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0l \in (ty\_2Elist\_2Elist \\ A\_27a).(((ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V0l) = \mathsf{c\_2Enum\_2E0}) \Leftrightarrow (\\ V0l = (c\_2Elist\_2ENIL\ A\_27a)))) \tag{11}$$

Assume the following.
$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0l \in (ty\_2Elist\_2Elist \\ A\_27a).((p\ (ap\ (c\_2Elist\_2ENULL\ A\_27a)\ V0l)) \Leftrightarrow (V0l = (c\_2Elist\_2ENIL \\ A\_27a)))) \tag{12}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0l \in (ty\_2Elist\_2Elist \\ A\_27a).((p\ (ap\ (c\_2Elist\_2ENULL\ A\_27a)\ V0l)) \Leftrightarrow ((ap\ (c\_2Elist\_2ELENGTH \\ A\_27a)\ V0l) = \mathsf{c\_2Enum\_2E0})))$$