

thm_2Elist_2EOPT__MMAP__cong
(TMVAXkfxQ5gCMqAp1y9Jqvn2iPGkUJBJ1Hi)

October 26, 2020

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 4 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2EF$

Definition 7 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t$

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Elist_2Elist A0) \quad (1)$$

Let $c_2Elist_2ELIST_TO_SET : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2ELIST_TO_SET A_27a \in ((2^{A_27a})^{(ty_2Elist_2Elist A_27a)}) \quad (2)$$

Definition 8 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap V1f V0x)))$

Let $ty_2Eoption_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Eoption_2Eoption A0) \quad (3)$$

Let $c_2Eoption_2EOPTION_BIND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Eoption_2EOPTION_BIND A_27a A_27b \in (((ty_2Eoption_2Eoption A_27a)^{(ty_2Eoption_2Eoption A_27a)^{A_27b}})^{(ty_2Eoption_2Eoption A_27b)}) \quad (4)$$

Let $c_2Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ECONS\ A_27a \in (((ty_2Elist_2Elist\ A_27a)^{(ty_2Elist_2Elist\ A_27a)})^{A_27a}) \quad (5)$$

Let $ty_2Eone_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eone_2Eone \quad (6)$$

Definition 9 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2.$

Let $ty_2Esum_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Esum_2Esum\ A0\ A1) \quad (7)$$

Let $c_2Esum_2EABS_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Esum_2EABS_sum\ A_27a\ A_27b \in ((ty_2Esum_2Esum\ A_27a\ A_27b)^{((2^{A_27b})^{A_27a})^2}) \quad (8)$$

Definition 10 We define c_2Esum_2EINL to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0e \in A_27a.(ap\ (c_2Esum_2EABS_sum$

Let $c_2Eoption_2Eoption_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Eoption_2Eoption_ABS\ A_27a \in ((ty_2Eoption_2Eoption\ A_27a)^{(ty_2Esum_2Esum\ A_27a\ ty_2Eone_2Eone)}) \quad (9)$$

Definition 11 We define $c_2Eoption_2ESOME$ to be $\lambda A_27a : \iota.\lambda V0x \in A_27a.(ap\ (c_2Eoption_2Eoption_ABS$

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ENIL\ A_27a \in (ty_2Elist_2Elist\ A_27a) \quad (10)$$

Let $c_2Elist_2EOPT_MMAP : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Elist_2EOPT_MMAP\ A_27a\ A_27b \in (((ty_2Eoption_2Eoption\ (ty_2Elist_2Elist\ A_27a))^{(ty_2Elist_2Elist\ A_27b)})^{(ty_2Eoption_2Eoption_ABS\ A_27a)}) \quad (11)$$

Assume the following.

$$True \quad (12)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \quad (13)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (14)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0x \in A.27a.((V0x = V0x) \Leftrightarrow True)) \quad (15)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0x \in A.27a.(\forall V1y \in A.27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (16)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (17)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in (2^{A-27a}).(\forall V1Q \in (2^{A-27a}).((\forall V2x \in A.27a.((p (ap V0P V2x)) \wedge (p (ap V1Q V2x)))) \Leftrightarrow ((\forall V3x \in A.27a.(p (ap V0P V3x))) \wedge (\forall V4x \in A.27a.(p (ap V1Q V4x))))))) \quad (18)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p V0A) \Rightarrow (p V1B)) \Leftrightarrow ((\neg(p V0A)) \vee (p V1B)))) \quad (19)$$

Assume the following.

$$(\forall V0P \in 2.(\forall V1Q \in 2.(\forall V2R \in 2.(((p V0P) \vee (p V1Q)) \Rightarrow (p V2R)) \Leftrightarrow (((p V0P) \Rightarrow (p V2R)) \wedge ((p V1Q) \Rightarrow (p V2R)))))) \quad (20)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (21)$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x_{.27} \in 2.(\forall V2y \in 2.(\forall V3y_{.27} \in 2.(((p V0x) \Leftrightarrow (p V1x_{.27})) \wedge ((p V1x_{.27}) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_{.27})))) \Rightarrow (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_{.27}) \Rightarrow (p V3y_{.27})))))) \quad (22)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0f \in (2^{A.27a}).(\forall V1v \in \\ A.27a.((\forall V2x \in A.27a.((V2x = V1v) \Rightarrow (p\ (ap\ V0f\ V2x)))) \Leftrightarrow (p\ (\\ ap\ V0f\ V1v)))))) \end{aligned} \quad (23)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0P \in (2^{(ty.2Elist.2Elist\ A.27a)}), \\ (((p\ (ap\ V0P\ (c.2Elist.2ENIL\ A.27a))) \wedge (\forall V1t \in (ty.2Elist.2Elist \\ A.27a).((p\ (ap\ V0P\ V1t)) \Rightarrow (\forall V2h \in A.27a.(p\ (ap\ V0P\ (ap\ (ap\ (\\ c.2Elist.2ECONS\ A.27a)\ V2h)\ V1t)))))) \Rightarrow (\forall V3l \in (ty.2Elist.2Elist \\ A.27a).(p\ (ap\ V0P\ V3l)))))) \end{aligned} \quad (24)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow ((\forall V0x \in A.27a.((p\ (ap\ (ap \\ (c.2Ebool.2EIN\ A.27a)\ V0x)\ (ap\ (c.2Elist.2ELIST_TO_SET\ A.27a) \\ (c.2Elist.2ENIL\ A.27a)))) \Leftrightarrow False)) \wedge (\forall V1x \in A.27a.(\forall V2h \in \\ A.27a.(\forall V3t \in (ty.2Elist.2Elist\ A.27a).((p\ (ap\ (ap\ (c.2Ebool.2EIN \\ A.27a)\ V1x)\ (ap\ (c.2Elist.2ELIST_TO_SET\ A.27a)\ (ap\ (ap\ (c.2Elist.2ECONS \\ A.27a)\ V2h)\ V3t)))) \Leftrightarrow ((V1x = V2h) \vee (p\ (ap\ (ap\ (c.2Ebool.2EIN\ A.27a) \\ V1x)\ (ap\ (c.2Elist.2ELIST_TO_SET\ A.27a)\ V3t)))))))))) \end{aligned} \quad (25)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\ (\forall V0f \in ((ty.2Eoption.2Eoption\ A.27a)^{A.27b}).((ap\ (ap\ (\\ c.2Elist.2EOPT_MMAP\ A.27a\ A.27b)\ V0f)\ (c.2Elist.2ENIL\ A.27b)) = \\ (ap\ (c.2Eoption.2ESOME\ (ty.2Elist.2Elist\ A.27a))\ (c.2Elist.2ENIL \\ A.27a)))) \wedge (\forall V1f \in ((ty.2Eoption.2Eoption\ A.27a)^{A.27b}). \\ (\forall V2h0 \in A.27b.(\forall V3t0 \in (ty.2Elist.2Elist\ A.27b). \\ ((ap\ (ap\ (c.2Elist.2EOPT_MMAP\ A.27a\ A.27b)\ V1f)\ (ap\ (ap\ (c.2Elist.2ECONS \\ A.27b)\ V2h0)\ V3t0)) = (ap\ (ap\ (c.2Eoption.2EOPTION_BIND\ (ty.2Elist.2Elist \\ A.27a)\ A.27a)\ (ap\ V1f\ V2h0))\ (\lambda V4h \in A.27a.(ap\ (ap\ (c.2Eoption.2EOPTION_BIND \\ (ty.2Elist.2Elist\ A.27a)\ (ty.2Elist.2Elist\ A.27a))\ (ap\ (ap\ (c.2Elist.2EOPT_MMAP \\ A.27a\ A.27b)\ V1f)\ V3t0))\ (\lambda V5t \in (ty.2Elist.2Elist\ A.27a).(\\ ap\ (c.2Eoption.2ESOME\ (ty.2Elist.2Elist\ A.27a))\ (ap\ (ap\ (c.2Elist.2ECONS \\ A.27a)\ V4h)\ V5t)))))))))) \end{aligned} \quad (26)$$

Theorem 1

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \quad \forall V0f1 \in ((ty_2Eoption_2Eoption\ A_27b)^{A_27a}). (\forall V1f2 \in \\ & \quad ((ty_2Eoption_2Eoption\ A_27b)^{A_27a}). (\forall V2x1 \in (ty_2Elist_2Elist \\ & \quad A_27a). (\forall V3x2 \in (ty_2Elist_2Elist\ A_27a). (((V2x1 = V3x2) \wedge \\ & \quad (\forall V4a \in A_27a. ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V4a)\ (ap\ (\\ & \quad c_2Elist_2ELIST_TO_SET\ A_27a)\ V3x2)))) \Rightarrow ((ap\ V0f1\ V4a) = (ap\ V1f2 \\ & \quad V4a)))))) \Rightarrow ((ap\ (ap\ (c_2Elist_2EOPT_MMAP\ A_27b\ A_27a)\ V0f1)\ V2x1) = \\ & \quad (ap\ (ap\ (c_2Elist_2EOPT_MMAP\ A_27b\ A_27a)\ V1f2)\ V3x2)))))) \end{aligned}$$