

# thm\_2Elist\_2ESET\_\_TO\_\_LIST\_\_EMPTY (TMRpsNHrsskohrTUoCycSDb6SiqqLyscEge)

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**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2E\_2ET$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a}))$

**Definition 4** We define  $c\_2Ebool\_2E\_2EF$  to be  $(ap (c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 5** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p \Rightarrow q)$  of type  $\iota$ .

**Definition 6** We define  $c\_2Ebool\_2E\_27E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2E\_2EF$

**Definition 7** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V2t$

Let  $c\_2Epred\_set\_2ECHOICE : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Epred\_set\_2ECHOICE A\_27a \in (A\_27a^{(2^{A\_27a})}) \quad (1)$$

**Definition 8** We define  $c\_2Epred\_set\_2EEMPTY$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.c\_2Ebool\_2E\_2EF)$ .

**Definition 9** We define  $c\_2Ebool\_2E\_2IN$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap V1f V0x)))$

**Definition 10** We define  $c\_2Ebool\_2E\_25C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V2t$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty\_2Epair\_2Eprod A0 A1) \quad (2)$$

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epair\_2EABS\_prod A\_27a A\_27b \in ((ty\_2Epair\_2Eprod A\_27a A\_27b)^{(2^{A\_27b})^{A\_27a}}) \quad (3)$$

**Definition 11** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap (c\_2Epred\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota)$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epred\_set\_2EGSPEC\ A\_27a\ A\_27b \in ((2^{A\_27a})^{(ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A\_27b}})$$
 (4)

**Definition 12** We define  $c\_2Epred\_set\_2EINSERT$  to be  $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.\lambda V1s \in (2^{A\_27a}).(ap (c\_2Epred\_set\_2EDELETE$

**Definition 13** We define  $c\_2Epred\_set\_2EDIFF$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap (c\_2Epred\_set\_2EDELETE$

**Definition 14** We define  $c\_2Epred\_set\_2EDELETE$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1x \in A\_27a.(ap (c\_2Epred\_set\_2EDELETE$

**Definition 15** We define  $c\_2Epred\_set\_2EREST$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).(ap (ap (c\_2Epred\_set\_2EDELETE$

Let  $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Elist\_2Elist\ A0)$$
 (5)

Let  $c\_2Elist\_2ECONS : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ECONS\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{A\_27a})$$
 (6)

Let  $c\_2Elist\_2ENIL : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ENIL\ A\_27a \in (ty\_2Elist\_2Elist\ A\_27a)$$
 (7)

**Definition 16** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.$ if  $(\exists x \in A.p (ap\ P\ x))$  then (the  $(\lambda x.x \in A \wedge p\ x)$  of type  $\iota \Rightarrow \iota$ .

**Definition 17** We define  $c\_2Ebool\_2ECOND$  to be  $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.(ap (c\_2Ebool\_2E\_21$

Let  $c\_2Ebool\_2EARB : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ebool\_2EARB\ A\_27a \in A\_27a$$
 (8)

**Definition 18** We define  $c\_2Epred\_set\_2EFINITE$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).(ap (c\_2Ebool\_2E\_21$

**Definition 19** We define  $c\_2Ecombin\_2EK$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0x \in A\_27a.(\lambda V1y \in A\_27b.V0x)$

**Definition 20** We define  $c\_2Ecombin\_2ES$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.(\lambda V0f \in ((A\_27c^{A\_27b})^{A\_27a})$

**Definition 21** We define  $c\_2Ecombin\_2EI$  to be  $\lambda A\_27a : \iota.(ap (ap (c\_2Ecombin\_2ES\ A\_27a\ (A\_27a^{A\_27a})$

**Definition 22** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap (c\_2Emin\_2E\_40$

**Definition 23** We define  $c\_2Erelation\_2EWF$  to be  $\lambda A\_27a : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).(ap (c\_2Ebool\_2E\_21$

**Definition 24** We define  $c\_2Erelation\_2ERESTRICT$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in (A\_27b^{A\_27a}).\lambda V1M$

**Definition 25** We define  $c\_2Erelation\_2ETC$  to be  $\lambda A\_27a : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda V1a \in A\_27a.\lambda V2b$

**Definition 26** We define  $c\_2Erelation\_2Eapprox$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda V1M$

**Definition 27** We define  $c\_2Erelation\_2Ethe\_fun$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda V1M$

**Definition 28** We define  $c\_2Erelation\_2EWFREC$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda V1M$

**Definition 29** We define  $c\_2Elist\_2ESET\_TO\_LIST$  to be  $\lambda A\_27a : \iota.(ap (ap (c\_2Erelation\_2EWFREC (2^{A\_27a})$

Assume the following.

$$True \tag{9}$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{10}$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{11}$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow \neg(p V0t)) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow \neg( \\ & p V0t)))))) \end{aligned} \tag{12}$$

Assume the following.

$$\begin{aligned} & (\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow \\ & ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \end{aligned} \tag{13}$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in 2. \\ & (\forall V2x \in A\_27a.(\forall V3x\_27 \in A\_27a.(\forall V4y \in A\_27a. \\ & (\forall V5y\_27 \in A\_27a.(((p V0P) \Leftrightarrow (p V1Q)) \wedge (((p V1Q) \Rightarrow (V2x = V3x\_27)) \wedge \\ & ((\neg(p V1Q)) \Rightarrow (V4y = V5y\_27)))))) \Rightarrow ((ap (ap (ap (c\_2Ebool\_2ECOND A\_27a) \\ & V0P) V2x) V4y) = (ap (ap (ap (c\_2Ebool\_2ECOND A\_27a) V1Q) V3x\_27) \\ & V5y\_27))))))))) \end{aligned} \tag{14}$$

Assume the following.

$$\begin{aligned} \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow & ((\forall V0t1 \in A_{.27a}.(\forall V1t2 \in \\ & A_{.27a}.((ap\ (ap\ (ap\ (c_{.2Ebool\_2ECOND}\ A_{.27a})\ c_{.2Ebool\_2ET})\ V0t1) \\ & V1t2) = V0t1))) \wedge (\forall V2t1 \in A_{.27a}.(\forall V3t2 \in A_{.27a}.((ap \\ & (ap\ (ap\ (c_{.2Ebool\_2ECOND}\ A_{.27a})\ c_{.2Ebool\_2EF})\ V2t1)\ V3t2) = V3t2)))))) \end{aligned} \quad (15)$$

Assume the following.

$$\begin{aligned} \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow & (\forall V0s \in (2^{A_{.27a}}).((p\ (ap \\ & (c_{.2Epred\_set\_2EFINITE}\ A_{.27a})\ V0s)) \Rightarrow ((ap\ (c_{.2Elist\_2ESET\_TO\_LIST} \\ & A_{.27a})\ V0s) = (ap\ (ap\ (ap\ (c_{.2Ebool\_2ECOND}\ (ty_{.2Elist\_2Elist}\ A_{.27a})) \\ & (ap\ (ap\ (c_{.2Emin\_2E\_3D}\ (2^{A_{.27a}}))\ V0s)\ (c_{.2Epred\_set\_2EEMPTY} \\ & A_{.27a})))\ (c_{.2Elist\_2ENIL}\ A_{.27a}))\ (ap\ (ap\ (c_{.2Elist\_2ECONS}\ A_{.27a}) \\ & (ap\ (c_{.2Epred\_set\_2ECHOICE}\ A_{.27a})\ V0s))\ (ap\ (c_{.2Elist\_2ESET\_TO\_LIST} \\ & A_{.27a})\ (ap\ (c_{.2Epred\_set\_2EREST}\ A_{.27a})\ V0s)))))) \end{aligned} \quad (16)$$

Assume the following.

$$\forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow (p\ (ap\ (c_{.2Epred\_set\_2EFINITE}\ A_{.27a})\ (c_{.2Epred\_set\_2EEMPTY}\ A_{.27a}))) \quad (17)$$

**Theorem 1**

$$\forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow ((ap\ (c_{.2Elist\_2ESET\_TO\_LIST}\ A_{.27a})\ (c_{.2Epred\_set\_2EEMPTY}\ A_{.27a})) = (c_{.2Elist\_2ENIL}\ A_{.27a}))$$