thm_2Elist_2ESINGL__APPLY__MAP (TMG7B443g7JrzjQVNg5eL2fPdHBoxr37kmx)

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Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Let $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Elist\_2Elist\ A0) \tag{1}$$

Let $c\_2Elist\_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ENIL\ A\_27a \in (ty\_2Elist\_2Elist\ A\_27a) \tag{2}$$

Let $c\_2Elist\_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ECONS\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{A\_27a}) \tag{3}$$

Let $c\_2Elist\_2EMAP : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Elist\_2EMAP\ A\_27a\ A\_27b \in (((ty\_2Elist\_2Elist\ A\_27b)^{(ty\_2Elist\_2Elist\ A\_27a)})^{(A\_27b^{A\_27a})}) \tag{4}$$

Definition 3 *We define* c_2Ecombin_2EC *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.(\lambda V0f \in ((A\_27c^{A\_27b})^{A\_27c}$

Let $c\_2Elist\_2EFLAT : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2EFLAT\ A\_27a \in ((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ (ty\_2Elist\_2Elist\ A\_27a))}) \tag{5}$$

Definition 4 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 5 *We define* c_2Elist_2ELIST__BIND *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0l \in (ty\_2Elist\_2Elist\ A\_27b$

Definition 6 *We define* c_2Elist_2ELIST__APPLY *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0fs \in (ty\_2Elist\_2Elist\ (A$

Assume the following.

$$True \tag{6}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{7}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c.\ nonempty\ A\_27c \Rightarrow (\forall V0f \in ((A\_27c^{A\_27b})^{A\_27a}).(\forall V1x \in A\_27b.(\forall V2y \in A\_27a.((ap\ (ap\ (ap\ (\mathsf{c\_2Ecombin\_2EC}\ A\_27a\ A\_27b\ A\_27c)\ V0f)\ V1x)\ V2y) = (ap\ (ap\ V0f\ V2y)\ V1x))))) \tag{8}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0x \in A\_27b.(\forall V1f \in ((ty\_2Elist\_2Elist\ A\_27a)^{A\_27b}).((ap\ (ap\ (\mathsf{c\_2Elist\_2ELIST\_\_BIND}\ A\_27a\ A\_27b)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27b)\ V0x)\ (c\_2Elist\_2ENIL\ A\_27b)))\ V1f) = (ap\ V1f\ V0x)))) \tag{9}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0f \in (A\_27a^{A\_27b}).(\forall V1l \in (ty\_2Elist\_2Elist\ A\_27b).((ap\ (ap\ (\mathsf{c\_2Elist\_2ELIST\_\_APPLY}\ A\_27a\ A\_27b)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ (A\_27a^{A\_27b}))\ V0f)\ (c\_2Elist\_2ENIL\ (A\_27a^{A\_27b}))))\ V1l) = (ap\ (ap\ (c\_2Elist\_2EMAP\ A\_27b\ A\_27a)\ V0f)\ V1l))))$$