

thm_2Elist_2ESUM__SNOC (TMVYkefVn- sxM9hHr3YSQGT9UNHayk64dcsZ)

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Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{1}$$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \tag{2}$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \tag{3}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{4}$$

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a}))$

Definition 4 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap c_2Enum_2EABS_num ($

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \tag{5}$$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{6}$$

Definition 5 We define c_2Enum_2E0 to be $(ap c_2Enum_2EABS_num c_2Enum_2EZERO_REP)$.

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Elist_2Elist\ A0) \quad (7)$$

Let $c_2Elist_2ESUM : \iota$ be given. Assume the following.

$$c_2Elist_2ESUM \in (ty_2Enum_2Enum^{(ty_2Elist_2Elist\ ty_2Enum_2Enum)}) \quad (8)$$

Definition 6 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o\ (p\ P \Rightarrow p\ Q)$ of type ι .

Let $c_2Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow c_2Elist_2ECONS\ A.27a \in (((ty_2Elist_2Elist\ A.27a)^{(ty_2Elist_2Elist\ A.27a)})_{A.27a}) \quad (9)$$

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow c_2Elist_2ENIL\ A.27a \in (ty_2Elist_2Elist\ A.27a) \quad (10)$$

Let $c_2Elist_2ESNOC : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow c_2Elist_2ESNOC\ A.27a \in (((ty_2Elist_2Elist\ A.27a)^{(ty_2Elist_2Elist\ A.27a)})_{A.27a}) \quad (11)$$

Definition 7 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2.$

Assume the following.

$$\begin{aligned} & ((\forall V0n \in ty_2Enum_2Enum.((ap\ (ap\ c_2Earithmetic_2E_2B \\ & \quad c_2Enum_2E0)\ V0n) = V0n)) \wedge (\forall V1m \in ty_2Enum_2Enum.(\forall V2n \in \\ & \quad ty_2Enum_2Enum.((ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Enum_2ESUC \\ & \quad V1m))\ V2n) = (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V1m) \\ & \quad V2n)))))) \end{aligned} \quad (12)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.((ap\ (ap\ c_2Earithmetic_2E_2B\ V0m)\ c_2Enum_2E0) = V0m)) \quad (13)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(\\ & \quad \forall V2p \in ty_2Enum_2Enum.((ap\ (ap\ c_2Earithmetic_2E_2B\ V0m) \\ & \quad (ap\ (ap\ c_2Earithmetic_2E_2B\ V1n)\ V2p)) = (ap\ (ap\ c_2Earithmetic_2E_2B \\ & \quad (ap\ (ap\ c_2Earithmetic_2E_2B\ V0m)\ V1n))\ V2p)))) \end{aligned} \quad (14)$$

Assume the following.

$$True \quad (15)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (16)$$

Assume the following.

$$\begin{aligned} & (((ap\ c_2Elist_2ESUM\ (c_2Elist_2ENIL\ ty_2Enum_2Enum)) = c_2Enum_2E0) \wedge \\ & (\forall V0h \in ty_2Enum_2Enum. (\forall V1t \in (ty_2Elist_2Elist \\ & ty_2Enum_2Enum). ((ap\ c_2Elist_2ESUM\ (ap\ (ap\ (c_2Elist_2ECONS \\ & ty_2Enum_2Enum)\ V0h)\ V1t)) = (ap\ (ap\ c_2Earithmetic_2E_2B\ V0h) \\ & (ap\ c_2Elist_2ESUM\ V1t))))))) \end{aligned} \quad (17)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{(ty_2Elist_2Elist\ A_27a)}). \\ & (((p\ (ap\ V0P\ (c_2Elist_2ENIL\ A_27a))) \wedge (\forall V1t \in (ty_2Elist_2Elist \\ & A_27a). ((p\ (ap\ V0P\ V1t)) \Rightarrow (\forall V2h \in A_27a. (p\ (ap\ V0P\ (ap\ (ap\ (\\ & c_2Elist_2ECONS\ A_27a)\ V2h)\ V1t))))))) \Rightarrow (\forall V3l \in (ty_2Elist_2Elist \\ & A_27a). (p\ (ap\ V0P\ V3l)))))) \end{aligned} \quad (18)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow ((\forall V0x \in A_27a. ((ap\ (ap\ (c_2Elist_2ESNOC \\ & A_27a)\ V0x)\ (c_2Elist_2ENIL\ A_27a)) = (ap\ (ap\ (c_2Elist_2ECONS \\ & A_27a)\ V0x)\ (c_2Elist_2ENIL\ A_27a)))) \wedge (\forall V1x \in A_27a. (\forall V2x_27 \in \\ & A_27a. (\forall V3l \in (ty_2Elist_2Elist\ A_27a). ((ap\ (ap\ (c_2Elist_2ESNOC \\ & A_27a)\ V1x)\ (ap\ (ap\ (c_2Elist_2ECONS\ A_27a)\ V2x_27)\ V3l)) = (ap\ (\\ & ap\ (c_2Elist_2ECONS\ A_27a)\ V2x_27)\ (ap\ (ap\ (c_2Elist_2ESNOC\ A_27a) \\ & V1x)\ V3l))))))) \end{aligned} \quad (19)$$

Theorem 1

$$\begin{aligned} & (\forall V0x \in ty_2Enum_2Enum. (\forall V1l \in (ty_2Elist_2Elist \\ & ty_2Enum_2Enum). ((ap\ c_2Elist_2ESUM\ (ap\ (ap\ (c_2Elist_2ESNOC \\ & ty_2Enum_2Enum)\ V0x)\ V1l)) = (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Elist_2ESUM \\ & V1l)\ V0x)))) \end{aligned}$$