

thm_2Elist_2EUNIQUE_FILTER (TMZHuU- JbDswRmGPETpCVojXL9NofKBW7BYn)

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Definition 1 We define `c_2Emin_2E_3D` to be $\lambda A. \lambda x \in A. \lambda y \in A. \text{inj_o } (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define `c_2Ebool_2E_2T` to be $(\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^2)) (\lambda V0x \in 2. V0x)) (\lambda V1x \in 2. V1x))$

Definition 3 We define `c_2Ebool_2E_21` to be $\lambda A. \lambda a : \iota. (\lambda V0P \in (2^{A-27a}). (\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^{A-27a}))))$

Definition 4 We define `c_2Ebool_2E_2F` to be $(\text{ap } (\text{c_2Ebool_2E_21 } 2) (\lambda V0t \in 2. V0t))$.

Definition 5 We define `c_2Emin_2E_3D_3D_3E` to be $\lambda P \in 2. \lambda Q \in 2. \text{inj_o } (p P \Rightarrow p Q)$ of type ι .

Definition 6 We define `c_2Ebool_2E_2F_5C` to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (\text{ap } (\text{c_2Ebool_2E_21 } 2) (\lambda V2t \in 2. V2t))))$

Definition 7 We define `c_2Emin_2E_40` to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p (\text{ap } P x)) \text{ then } (the (\lambda x. x \in A \wedge p x))$ of type $\iota \Rightarrow \iota$.

Definition 8 We define `c_2Ebool_2ECOND` to be $\lambda A. \lambda a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A. \lambda a. (\lambda V2t2 \in A. \lambda a. (\text{ap } (\text{c_2Emin_2E_40 } (2^{A-27a}))))))$

Let `ty_2Elist_2Elist` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. \text{nonempty } A0 \Rightarrow \text{nonempty } (\text{ty_2Elist_2Elist } A0) \quad (1)$$

Let `c_2Elist_2EFILTER` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A. \lambda a : \iota. \text{nonempty } A \Rightarrow \text{c_2Elist_2EFILTER } A \Rightarrow \text{c_2Elist_2EFILTER } (A \Rightarrow \text{c_2Elist_2EFILTER } A) \quad (2)$$

Let `c_2Elist_2ENULL` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A. \lambda a : \iota. \text{nonempty } A \Rightarrow \text{c_2Elist_2ENULL } A \Rightarrow \text{c_2Elist_2ENULL } (A \Rightarrow \text{c_2Elist_2ENULL } A) \quad (3)$$

Let `c_2Elist_2ELIST_TO_SET` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A. \lambda a : \iota. \text{nonempty } A \Rightarrow \text{c_2Elist_2ELIST_TO_SET } A \Rightarrow \text{c_2Elist_2ELIST_TO_SET } (A \Rightarrow \text{c_2Elist_2ELIST_TO_SET } A) \quad (4)$$

Definition 9 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a. (\lambda V1f \in (2^{A_27a}). (ap\ V1f\ V0x)))$

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ENIL\ A_27a \in (ty_2Elist_2Elist\ A_27a) \quad (5)$$

Let $c_2Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ECONS\ A_27a \in (((ty_2Elist_2Elist\ A_27a)(ty_2Elist_2Elist\ A_27a))A_27a) \quad (6)$$

Let $c_2Elist_2EAPPEND : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2EAPPEND\ A_27a \in (((ty_2Elist_2Elist\ A_27a)(ty_2Elist_2Elist\ A_27a))(ty_2Elist_2Elist\ A_27a)) \quad (7)$$

Definition 10 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap\ V0P\ (ap\ (c_2Emin_2E_40$

Definition 11 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2. (ap\ (ap\ c_2Emin_2E_3D_3D_3E\ V0t)\ c_2Ebool_2E$

Definition 12 We define $c_2Elist_2EUNIQUE$ to be $\lambda A_27a : \iota. \lambda V0e \in A_27a. \lambda V1L \in (ty_2Elist_2Elist\ A$

Definition 13 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in$

Assume the following.

$$True \quad (8)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (9)$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p\ V0t))) \quad (10)$$

Assume the following.

$$(\forall V0t \in 2. ((p\ V0t) \vee (\neg(p\ V0t)))) \quad (11)$$

Assume the following.

$$(\forall V0t \in 2. (((p\ V0t) \Rightarrow False) \Rightarrow (\neg(p\ V0t)))) \quad (12)$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(p\ V0t)) \Rightarrow ((p\ V0t) \Rightarrow False))) \quad (13)$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2. (((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow \\
& (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge \\
& (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \quad (14)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2. (((True \vee (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \vee True) \Leftrightarrow True) \wedge \\
& (((False \vee (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee False) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee \\
& (p V0t)) \Leftrightarrow (p V0t)))))) \quad (15)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2. (((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\
& True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((\\
& (p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (16)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge \\
& ((\neg False) \Leftrightarrow True)))) \quad (17)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0x \in A.27a. ((V0x = V0x) \Leftrightarrow \\
& True)) \quad (18)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0x \in A.27a. (\forall V1y \in \\
& A.27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (19)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2. (((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\
& (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(\\
& p V0t)))))) \quad (20)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0t1 \in A.27a. (\forall V1t2 \in \\
& A.27a. (((ap (ap (ap (c.2Ebool.2ECOND A.27a) c.2Ebool.2ET) V0t1) \\
& V1t2) = V0t1) \wedge ((ap (ap (ap (c.2Ebool.2ECOND A.27a) c.2Ebool.2EF \\
& V0t1) V1t2) = V1t2)))))) \quad (21)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0P \in (2^{A.27a}). ((\neg(\exists V1x \in \\
& A.27a. (p (ap V0P V1x)))) \Leftrightarrow (\forall V2x \in A.27a. (\neg(p (ap V0P V2x)))))) \quad (22)
\end{aligned}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p V0A) \vee (p V1B) \vee (p V2C))) \Leftrightarrow (((p V0A) \vee (p V1B)) \vee (p V2C)))))) \quad (23)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p V0A) \vee (p V1B)) \Leftrightarrow ((p V1B) \vee (p V0A)))))) \quad (24)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \wedge (p V1B))) \Leftrightarrow ((\neg(p V0A)) \vee (\neg(p V1B)))))) \wedge (((\neg(p V0A) \vee (p V1B)) \Leftrightarrow ((\neg(p V0A)) \wedge (\neg(p V1B))))))))) \quad (25)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow (p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (26)$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x_{.27} \in 2.(\forall V2y \in 2.(\forall V3y_{.27} \in 2.(((p V0x) \Leftrightarrow (p V1x_{.27})) \wedge ((p V1x_{.27}) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_{.27})))))) \Rightarrow (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_{.27}) \Rightarrow (p V3y_{.27})))))) \quad (27)$$

Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow (\forall V0P \in 2.(\forall V1Q \in 2.(\forall V2x \in A_{.27a}.(\forall V3x_{.27} \in A_{.27a}.(\forall V4y \in A_{.27a}.(\forall V5y_{.27} \in A_{.27a}.(((p V0P) \Leftrightarrow (p V1Q)) \wedge ((p V1Q) \Rightarrow (V2x = V3x_{.27})) \wedge ((\neg(p V1Q)) \Rightarrow (V4y = V5y_{.27})))))) \Rightarrow ((ap (ap (ap (c_{.2Ebool_2ECOND} A_{.27a}) V0P) V2x) V4y) = (ap (ap (ap (c_{.2Ebool_2ECOND} A_{.27a}) V1Q) V3x_{.27}) V5y_{.27}))))))))) \quad (28)$$

Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow (\forall V0P \in (2^{A_{.27a}}).(\forall V1a \in A_{.27a}.((\exists V2x \in A_{.27a}.((V2x = V1a) \wedge (p (ap V0P V2x)))) \Leftrightarrow (p (ap V0P V1a)))))) \quad (29)$$

Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow ((\forall V0l \in (ty_{.2Elist_2Elist} A_{.27a}).((ap (ap (c_{.2Elist_2EAPPEND} A_{.27a}) (c_{.2Elist_2ENIL} A_{.27a}) V0l) = V0l)) \wedge (\forall V1l1 \in (ty_{.2Elist_2Elist} A_{.27a}).(\forall V2l2 \in (ty_{.2Elist_2Elist} A_{.27a}).(\forall V3h \in A_{.27a}.((ap (ap (c_{.2Elist_2EAPPEND} A_{.27a}) (ap (ap (c_{.2Elist_2ECONS} A_{.27a}) V3h) V1l1)) V2l2) = (ap (ap (c_{.2Elist_2ECONS} A_{.27a}) V3h) (ap (ap (c_{.2Elist_2EAPPEND} A_{.27a}) V1l1) V2l2)))))))))) \quad (30)$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow ((\forall V0P \in (2^{A_27a}).((ap\ (\\
& ap\ (c_2Elist_2EFILTER\ A_27a)\ V0P)\ (c_2Elist_2ENIL\ A_27a)) = (c_2Elist_2ENIL \\
& A_27a))) \wedge (\forall V1P \in (2^{A_27a}).(\forall V2h \in A_27a.(\forall V3t \in \\
& (ty_2Elist_2Elist\ A_27a).((ap\ (ap\ (c_2Elist_2EFILTER\ A_27a) \\
& V1P)\ (ap\ (ap\ (c_2Elist_2ECONS\ A_27a)\ V2h)\ V3t)) = (ap\ (ap\ (ap\ (c_2Ebool_2ECOND \\
& (ty_2Elist_2Elist\ A_27a))\ (ap\ V1P\ V2h))\ (ap\ (ap\ (c_2Elist_2ECONS \\
& A_27a)\ V2h)\ (ap\ (ap\ (c_2Elist_2EFILTER\ A_27a)\ V1P)\ V3t))))\ (ap\ (ap \\
& (c_2Elist_2EFILTER\ A_27a)\ V1P)\ V3t))))))
\end{aligned} \tag{31}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0a0 \in A_27a.(\forall V1a1 \in \\
& (ty_2Elist_2Elist\ A_27a).(\forall V2a0_27 \in A_27a.(\forall V3a1_27 \in \\
& (ty_2Elist_2Elist\ A_27a).(((ap\ (ap\ (c_2Elist_2ECONS\ A_27a)\ V0a0) \\
& V1a1) = (ap\ (ap\ (c_2Elist_2ECONS\ A_27a)\ V2a0_27)\ V3a1_27)) \Leftrightarrow ((V0a0 = \\
& V2a0_27) \wedge (V1a1 = V3a1_27))))))
\end{aligned} \tag{32}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0a1 \in (ty_2Elist_2Elist \\
& A_27a).(\forall V1a0 \in A_27a.(\neg((ap\ (ap\ (c_2Elist_2ECONS\ A_27a) \\
& V1a0)\ V0a1) = (c_2Elist_2ENIL\ A_27a))))))
\end{aligned} \tag{33}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0l \in (ty_2Elist_2Elist \\
& A_27a).((ap\ (ap\ (c_2Elist_2EAPPEND\ A_27a)\ V0l)\ (c_2Elist_2ENIL \\
& A_27a)) = V0l))
\end{aligned} \tag{34}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0l1 \in (ty_2Elist_2Elist \\
& A_27a).(\forall V1l2 \in (ty_2Elist_2Elist\ A_27a).(\forall V2l3 \in \\
& (ty_2Elist_2Elist\ A_27a).((ap\ (ap\ (c_2Elist_2EAPPEND\ A_27a) \\
& V0l1)\ (ap\ (ap\ (c_2Elist_2EAPPEND\ A_27a)\ V1l2)\ V2l3)) = (ap\ (ap\ (c_2Elist_2EAPPEND \\
& A_27a)\ (ap\ (ap\ (c_2Elist_2EAPPEND\ A_27a)\ V0l1)\ V1l2))\ V2l3))))))
\end{aligned} \tag{35}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0l \in (ty_2Elist_2Elist \\
& A_27a).((p\ (ap\ (c_2Elist_2ENULL\ A_27a)\ V0l)) \Leftrightarrow (V0l = (c_2Elist_2ENIL \\
& A_27a))))
\end{aligned} \tag{36}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow ((\forall V0l1 \in (ty_2Elist_2Elist \\
& \quad A.27a).(\forall V1l2 \in (ty_2Elist_2Elist\ A.27a).(((c_2Elist_2ENIL \\
& \quad A.27a) = (ap\ (ap\ (c_2Elist_2EAPPEND\ A.27a)\ V0l1)\ V1l2)) \Leftrightarrow ((V0l1 = \\
& \quad (c_2Elist_2ENIL\ A.27a)) \wedge (V1l2 = (c_2Elist_2ENIL\ A.27a)))))) \wedge \\
& \quad (\forall V2l1 \in (ty_2Elist_2Elist\ A.27a).(\forall V3l2 \in (ty_2Elist_2Elist \\
& \quad A.27a).(((ap\ (ap\ (c_2Elist_2EAPPEND\ A.27a)\ V2l1)\ V3l2) = (c_2Elist_2ENIL \\
& \quad A.27a)) \Leftrightarrow ((V2l1 = (c_2Elist_2ENIL\ A.27a)) \wedge (V3l2 = (c_2Elist_2ENIL \\
& \quad A.27a)))))))
\end{aligned} \tag{37}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0l1 \in (ty_2Elist_2Elist \\
& \quad A.27a).(\forall V1l2 \in (ty_2Elist_2Elist\ A.27a).(\forall V2e \in \\
& \quad A.27a.(((ap\ (ap\ (c_2Elist_2EAPPEND\ A.27a)\ V0l1)\ V1l2) = (ap\ (ap \\
& \quad (c_2Elist_2ECONS\ A.27a)\ V2e)\ (c_2Elist_2ENIL\ A.27a))) \Leftrightarrow (((V0l1 = \\
& \quad (ap\ (ap\ (c_2Elist_2ECONS\ A.27a)\ V2e)\ (c_2Elist_2ENIL\ A.27a))) \wedge \\
& \quad (V1l2 = (c_2Elist_2ENIL\ A.27a))) \vee ((V0l1 = (c_2Elist_2ENIL\ A.27a)) \wedge \\
& \quad (V1l2 = (ap\ (ap\ (c_2Elist_2ECONS\ A.27a)\ V2e)\ (c_2Elist_2ENIL\ A.27a))))))))))
\end{aligned} \tag{38}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow ((\forall V0l1 \in (ty_2Elist_2Elist \\
& \quad A.27a).(\forall V1l2 \in (ty_2Elist_2Elist\ A.27a).(\forall V2l3 \in \\
& \quad (ty_2Elist_2Elist\ A.27a).(((ap\ (ap\ (c_2Elist_2EAPPEND\ A.27a) \\
& \quad V0l1)\ V1l2) = (ap\ (ap\ (c_2Elist_2EAPPEND\ A.27a)\ V0l1)\ V2l3)) \Leftrightarrow (V1l2 = \\
& \quad V2l3)))))) \wedge (\forall V3l1 \in (ty_2Elist_2Elist\ A.27a).(\forall V4l2 \in \\
& \quad (ty_2Elist_2Elist\ A.27a).(\forall V5l3 \in (ty_2Elist_2Elist\ A.27a). \\
& \quad (((ap\ (ap\ (c_2Elist_2EAPPEND\ A.27a)\ V4l2)\ V3l1) = (ap\ (ap\ (c_2Elist_2EAPPEND \\
& \quad A.27a)\ V5l3)\ V3l1)) \Leftrightarrow (V4l2 = V5l3))))))
\end{aligned} \tag{39}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0x \in A.27a.(\forall V1l \in \\
& \quad (ty_2Elist_2Elist\ A.27a).((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A.27a)\ V0x) \\
& \quad (ap\ (c_2Elist_2ELIST_TO_SET\ A.27a)\ V1l))) \Leftrightarrow (\exists V2l1 \in (\\
& \quad ty_2Elist_2Elist\ A.27a).(\exists V3l2 \in (ty_2Elist_2Elist\ A.27a). \\
& \quad (V1l = (ap\ (ap\ (c_2Elist_2EAPPEND\ A.27a)\ V2l1)\ (ap\ (ap\ (c_2Elist_2ECONS \\
& \quad A.27a)\ V0x)\ V3l2))))))
\end{aligned} \tag{40}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).(\forall V1l \in \\
& \quad (ty_2Elist_2Elist\ A.27a).((\neg((ap\ (ap\ (c_2Elist_2EFILTER\ A.27a) \\
& \quad V0P)\ V1l) = (c_2Elist_2ENIL\ A.27a))) \Leftrightarrow (\exists V2x \in A.27a.((p\ (\\
& \quad ap\ (ap\ (c_2Ebool_2EIN\ A.27a)\ V2x)\ (ap\ (c_2Elist_2ELIST_TO_SET \\
& \quad A.27a)\ V1l))) \wedge (p\ (ap\ V0P\ V2x))))))
\end{aligned} \tag{41}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). (\forall V1L \in \\ & (ty_2Elist_2Elist\ A_27a). (\forall V2M \in (ty_2Elist_2Elist\ A_27a). \\ & ((ap\ (ap\ (c_2Elist_2EFILTER\ A_27a)\ V0P)\ (ap\ (ap\ (c_2Elist_2EAPPEND \\ & A_27a)\ V1L)\ V2M))) = (ap\ (ap\ (c_2Elist_2EAPPEND\ A_27a)\ (ap\ (ap\ (c_2Elist_2EFILTER \\ & A_27a)\ V0P)\ V1L))\ (ap\ (ap\ (c_2Elist_2EFILTER\ A_27a)\ V0P)\ V2M)))))) \\ & \hspace{15em} (42) \end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). (\forall V1ls \in \\ & (ty_2Elist_2Elist\ A_27a). ((p\ (ap\ (c_2Elist_2ENULL\ A_27a)\ (ap \\ & (ap\ (c_2Elist_2EFILTER\ A_27a)\ V0P)\ V1ls))) \Leftrightarrow (\forall V2x \in A_27a. \\ & ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V2x)\ (ap\ (c_2Elist_2ELIST_TO_SET \\ & A_27a)\ V1ls))) \Rightarrow (\neg(p\ (ap\ V0P\ V2x)))))) \\ & \hspace{15em} (43) \end{aligned}$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \quad (44)$$

Assume the following.

$$(\forall V0A \in 2. ((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \quad (45)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\ & (((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))) \\ & \hspace{15em} (46) \end{aligned}$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(p\ V0A)) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\ & ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))) \\ & \hspace{15em} (47) \end{aligned}$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p\ V0A)) \Rightarrow False) \Rightarrow (((p\ V0A) \Rightarrow False) \Rightarrow False))) \quad (48)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p\ V0p) \Leftrightarrow (\\ & (p\ V1q) \Leftrightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((p\ V1q) \vee (p\ V2r))) \wedge (((p\ V0p) \vee ((\neg \\ & p\ V2r)) \vee (\neg(p\ V1q)))) \wedge (((p\ V1q) \vee ((\neg(p\ V2r)) \vee (\neg(p\ V0p)))) \wedge ((p\ V2r) \vee \\ & ((\neg(p\ V1q)) \vee (\neg(p\ V0p)))))))))) \\ & \hspace{15em} (49) \end{aligned}$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p\ V0p) \Leftrightarrow (\\ & (p\ V1q) \vee (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee (\neg(p\ V1q))) \wedge (((p\ V0p) \vee (\neg(p\ V2r))) \wedge \\ & ((p\ V1q) \vee ((p\ V2r) \vee (\neg(p\ V0p)))))))))) \\ & \hspace{15em} (50) \end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee \neg(p V2r))) \wedge (\\
& \neg(p V1q)) \vee ((p V2r) \vee \neg(p V0p)))))))))) \quad (51)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow \neg(p V1q))) \Leftrightarrow (((p V0p) \vee \\
& (p V1q)) \wedge (\neg(p V1q)) \vee \neg(p V0p)))))) \quad (52)
\end{aligned}$$

Theorem 1

$$\begin{aligned}
& \forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow (\forall V0e \in A_{27a}. (\forall V1L \in \\
& (ty_2Elist_2Elist A_{27a}). ((p (ap (ap (c_2Elist_2EUNIQUE A_{27a}) \\
& V0e) V1L)) \Leftrightarrow ((ap (ap (c_2Elist_2EFILTER A_{27a}) (ap (c_2Emin_2E_3D \\
& A_{27a}) V0e)) V1L) = (ap (ap (c_2Elist_2ECONS A_{27a}) V0e) (c_2Elist_2ENIL \\
& A_{27a}))))))
\end{aligned}$$