

thm_2Elist_2Elazy__list__case__compute
(TMQUuo1kUregktt5JP4b69VuJ2zHTXE6aF6)

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Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Elist_2Elist\ A0) \quad (1)$$

Let $c_2Elist_2ETL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ETL\ A_27a \in ((ty_2Elist_2Elist\ A_27a)^{(ty_2Elist_2Elist\ A_27a)}) \quad (2)$$

Let $c_2Elist_2EHD : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2EHD\ A_27a \in (A_27a^{(ty_2Elist_2Elist\ A_27a)}) \quad (3)$$

Let $c_2Elist_2ENULL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ENULL\ A_27a \in (2^{(ty_2Elist_2Elist\ A_27a)}) \quad (4)$$

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o\ (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap\ (ap\ (c_2Emin_2E_3D\ (2^2)))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x)$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ (ap\ (c_2Emin_2E_3D\ (2^{A_27a})))\ P))$

Definition 4 We define c_2Ebool_2EF to be $(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o\ (p \Rightarrow q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2.V2t)))\ t1)$

Definition 7 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if\ (\exists x \in A.p\ (ap\ P\ x))\ then\ (the\ (\lambda x.x \in A \wedge p\ x))$ of type $\iota \Rightarrow \iota$.

Definition 8 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.(ap\ (c_2Emin_2E_40\ A_27a)\ (V1t1 \wedge V2t2))))$

Let $c_2Elist_2Elist_CASE : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Elist_2Elist_CASE \\ & A_27a\ A_27b \in (((A_27b^{((A_27b^{(ty_2Elist_2Elist\ A_27a)})^{A_27a})})^{A_27b})^{(ty_2Elist_2Elist\ A_27a)}) \end{aligned} \quad (5)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \quad \forall V0b \in A_27b. (\forall V1f \in ((A_27b^{(ty_2Elist_2Elist\ A_27a)})^{A_27a}). \\ & \quad (\forall V2l \in (ty_2Elist_2Elist\ A_27a). ((ap\ (ap\ (ap\ (c_2Elist_2Elist_CASE \\ & \quad A_27a\ A_27b)\ V2l)\ V0b)\ V1f) = (ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27b) \\ & \quad (ap\ (c_2Elist_2ENULL\ A_27a)\ V2l))\ V0b)\ (ap\ (ap\ V1f\ (ap\ (c_2Elist_2EHD \\ & \quad A_27a)\ V2l))\ (ap\ (c_2Elist_2ETL\ A_27a)\ V2l)))))) \end{aligned} \quad (6)$$

Theorem 1

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \quad (c_2Elist_2Elist_CASE\ A_27a\ A_27b) = (\lambda V0l \in (ty_2Elist_2Elist \\ & \quad A_27a). (\lambda V1b \in A_27b. (\lambda V2f \in ((A_27b^{(ty_2Elist_2Elist\ A_27a)})^{A_27a}). \\ & \quad (ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27b)\ (ap\ (c_2Elist_2ENULL\ A_27a) \\ & \quad V0l))\ V1b)\ (ap\ (ap\ V2f\ (ap\ (c_2Elist_2EHD\ A_27a)\ V0l))\ (ap\ (c_2Elist_2ETL \\ & \quad A_27a)\ V0l)))))) \end{aligned}$$