

# thm\_2Elist\_2E nub\_\_set (TMLMdHsFCMdXU- gAeuyyuWRGHPwRZALWcn7Z)

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**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2E\_ET$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Ebool\_2E\_IN$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap V1f V0x)))$

**Definition 4** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 5** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a})))$

**Definition 6** We define  $c\_2Ebool\_2E\_5C\_2E\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V2t)))$

**Definition 7** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V2t)))$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty\_2Epair\_2Eprod A0 A1) \tag{1}$$

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epair\_2EABS\_prod A\_27a A\_27b \in ((ty\_2Epair\_2Eprod A\_27a A\_27b)^{(2^{A\_27b})^{A\_27a}}) \tag{2}$$

**Definition 8** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap (c\_2Ebool\_2E\_21 2) (ap (c\_2Emin\_2E\_3D (2^{A\_27a})))$

Let  $c\_2Epred\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epred\_set\_2EGSPEC A\_27a A\_27b \in ((2^{A\_27a})^{(ty\_2Epair\_2Eprod A\_27a 2)^{A\_27b}}) \tag{3}$$

**Definition 9** We define  $c\_2Epred\_set\_2EINSERT$  to be  $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.\lambda V1s \in (2^{A\_27a}).(ap (c\_2Ebool\_2E\_21 2) (ap (c\_2Emin\_2E\_3D (2^{A\_27a})))$



Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \quad (13)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (( \\ & (p\ V0t) \Rightarrow False) \Leftrightarrow (\neg(p\ V0t)))))) \end{aligned} \quad (14)$$

Assume the following.

$$\begin{aligned} & ((\forall V0t \in 2. ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge \\ & ((\neg False) \Leftrightarrow True))) \end{aligned} \quad (15)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (16)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg(p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg( \\ & p\ V0t)))))) \end{aligned} \quad (17)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t1 \in A\_27a. (\forall V1t2 \in \\ & A\_27a. (((ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ A\_27a)\ c\_2Ebool\_2ET)\ V0t1) \\ & V1t2) = V0t1) \wedge ((ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ A\_27a)\ c\_2Ebool\_2EF \\ & V0t1)\ V1t2) = V1t2)))))) \end{aligned} \quad (18)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\ & \forall V0h \in A\_27b. (\forall V1t \in (ty\_2Elist\_2Elist\ A\_27b). (( \\ & (ap\ (c\_2Elist\_2ELIST\_TO\_SET\ A\_27a)\ (c\_2Elist\_2ENIL\ A\_27a)) = \\ & (c\_2Epred\_set\_2EEMPTY\ A\_27a)) \wedge ((ap\ (c\_2Elist\_2ELIST\_TO\_SET \\ & A\_27b)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27b)\ V0h)\ V1t)) = (ap\ (ap\ (c\_2Epred\_set\_2EINSERT \\ & A\_27b)\ V0h)\ (ap\ (c\_2Elist\_2ELIST\_TO\_SET\ A\_27b)\ V1t)))))) \end{aligned} \quad (19)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{(ty\_2Elist\_2Elist\ A\_27a)}). \\ & (((p\ (ap\ V0P\ (c\_2Elist\_2ENIL\ A\_27a))) \wedge (\forall V1t \in (ty\_2Elist\_2Elist \\ & A\_27a). ((p\ (ap\ V0P\ V1t)) \Rightarrow (\forall V2h \in A\_27a. (p\ (ap\ V0P\ (ap\ (ap\ ( \\ & c\_2Elist\_2ECONS\ A\_27a)\ V2h)\ V1t)))))) \Rightarrow (\forall V3l \in (ty\_2Elist\_2Elist \\ & A\_27a). (p\ (ap\ V0P\ V3l)))))) \end{aligned} \quad (20)$$

Assume the following.

$$\begin{aligned} \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow & ((\forall V0x \in A_{.27a}.((p\ (ap\ (ap \\ & (c_{.2Ebool\_2EIN}\ A_{.27a})\ V0x)\ (ap\ (c_{.2Elist\_2ELIST\_TO\_SET}\ A_{.27a}) \\ & (c_{.2Elist\_2ENIL}\ A_{.27a})))) \Leftrightarrow False)) \wedge (\forall V1x \in A_{.27a}.(\forall V2h \in \\ & A_{.27a}.(\forall V3t \in (ty_{.2Elist\_2Elist}\ A_{.27a}).((p\ (ap\ (ap\ (c_{.2Ebool\_2EIN} \\ & A_{.27a})\ V1x)\ (ap\ (c_{.2Elist\_2ELIST\_TO\_SET}\ A_{.27a})\ (ap\ (ap\ (c_{.2Elist\_2ECONS} \\ & A_{.27a})\ V2h)\ V3t)))) \Leftrightarrow ((V1x = V2h) \vee (p\ (ap\ (ap\ (c_{.2Ebool\_2EIN}\ A_{.27a}) \\ & V1x)\ (ap\ (c_{.2Elist\_2ELIST\_TO\_SET}\ A_{.27a})\ V3t)))))))))) \end{aligned} \quad (21)$$

Assume the following.

$$\begin{aligned} \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow & (((ap\ (c_{.2Elist\_2Enub}\ A_{.27a})\ (c_{.2Elist\_2ENIL} \\ & A_{.27a})) = (c_{.2Elist\_2ENIL}\ A_{.27a})) \wedge (\forall V0x \in A_{.27a}.(\forall V1l \in \\ & (ty_{.2Elist\_2Elist}\ A_{.27a}).((ap\ (c_{.2Elist\_2Enub}\ A_{.27a})\ (ap\ (ap \\ & (c_{.2Elist\_2ECONS}\ A_{.27a})\ V0x)\ V1l)) = (ap\ (ap\ (ap\ (c_{.2Ebool\_2ECOND} \\ & (ty_{.2Elist\_2Elist}\ A_{.27a}))\ (ap\ (ap\ (c_{.2Ebool\_2EIN}\ A_{.27a})\ V0x)\ ( \\ & ap\ (c_{.2Elist\_2ELIST\_TO\_SET}\ A_{.27a})\ V1l)))\ (ap\ (c_{.2Elist\_2Enub} \\ & A_{.27a})\ V1l))\ (ap\ (ap\ (c_{.2Elist\_2ECONS}\ A_{.27a})\ V0x)\ (ap\ (c_{.2Elist\_2Enub} \\ & A_{.27a})\ V1l)))))) \end{aligned} \quad (22)$$

Assume the following.

$$\begin{aligned} \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow & (\forall V0s \in (2^{A_{.27a}}).(\forall V1t \in \\ & (2^{A_{.27a}}).((V0s = V1t) \Leftrightarrow (\forall V2x \in A_{.27a}.((p\ (ap\ (ap\ (c_{.2Ebool\_2EIN} \\ & A_{.27a})\ V2x)\ V0s)) \Leftrightarrow (p\ (ap\ (ap\ (c_{.2Ebool\_2EIN}\ A_{.27a})\ V2x)\ V1t)))))) \end{aligned} \quad (23)$$

Assume the following.

$$\begin{aligned} \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow & (\forall V0x \in A_{.27a}.(\neg(p\ (ap\ (ap \\ & (c_{.2Ebool\_2EIN}\ A_{.27a})\ V0x)\ (c_{.2Epred\_set\_2EEMPTY}\ A_{.27a})))))) \end{aligned} \quad (24)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \quad (25)$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \quad (26)$$

Assume the following.

$$\begin{aligned} (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\ ((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))))) \end{aligned} \quad (27)$$

Assume the following.

$$\begin{aligned} (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(\neg(p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\ ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))))) \end{aligned} \quad (28)$$

Assume the following.

$$(\forall V0A \in 2.((\neg(p V0A)) \Rightarrow False) \Rightarrow ((p V0A) \Rightarrow False) \Rightarrow False)) \quad (29)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow ( \\ & (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee (\neg( \\ & p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee \\ & ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \end{aligned} \quad (30)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow ( \\ & (p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee \\ & (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))) \end{aligned} \quad (31)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow ( \\ & (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge \\ & ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))) \end{aligned} \quad (32)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow ( \\ & (p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge (( \\ & \neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))) \end{aligned} \quad (33)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee \\ & (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \end{aligned} \quad (34)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p))) \quad (35)$$

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$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))) \quad (36)$$

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$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V0p)))) \quad (37)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V1q)))) \quad (38)$$

Assume the following.

$$(\forall V0p \in 2.((\neg(\neg(p V0p))) \Rightarrow (p V0p))) \quad (39)$$

**Theorem 1**

$$\forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow (\forall V0l \in (\text{ty\_2Elist\_2Elist } A_{27a}). ((\text{ap } (c\_2Elist\_2ELIST\_TO\_SET } A_{27a}) (\text{ap } (c\_2Elist\_2Enub } A_{27a}) V0l)) = (\text{ap } (c\_2Elist\_2ELIST\_TO\_SET } A_{27a}) V0l)))$$