

# thm\_2Elist\_2EoEL\_EQ\_EL (TMZcEiJbmk- wQc4dbyThHXbDwcipMEHe3ESs)

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**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A. \lambda x \in A. \lambda y \in A. inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2ET$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2))) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x)$

Let  $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty\_2Elist\_2Elist A0) \quad (1)$$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty ty\_2Enum\_2Enum \quad (2)$$

Let  $c\_2Elist\_2EEL : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Elist\_2EEL A\_27a \in ((A\_27a^{(ty\_2Elist\_2Elist A\_27a)})^{ty\_2Enum\_2Enum}) \quad (3)$$

Let  $c\_2Elist\_2ELENGTH : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Elist\_2ELENGTH A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Elist\_2Elist A\_27a)}) \quad (4)$$

**Definition 3** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota. (\lambda V0P \in (2^{A\_27a}). (ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a}))$

**Definition 4** We define  $c\_2Ebool\_2EF$  to be  $(ap (c\_2Ebool\_2E\_21 2)) (\lambda V0t \in 2.V0t)$ .

**Definition 5** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2. \lambda Q \in 2. inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 6** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2. (ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2EF$

**Definition 7** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c\_2Ebool\_2E\_21 2)) (\lambda V2t \in 2$

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \quad (5)$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \quad (6)$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \quad (7)$$

**Definition 8** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. (ap\ c\_2Enum\_2EABS\_num$

**Definition 9** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. P\ x) \text{ then } (the\ (\lambda x. x \in A \wedge P\ x))$  of type  $\iota \Rightarrow \iota$ .

**Definition 10** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota. (\lambda V0P \in (2^{A\_27a}). (ap\ V0P\ (ap\ (c\_2Emin\_2E\_40$

**Definition 11** We define  $c\_2Eprim\_rec\_2E\_3C$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. \lambda V1n \in ty\_2Enum\_2Enum$

Let  $ty\_2Eoption\_2Eoption : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0. nonempty\ A0 \Rightarrow nonempty\ (ty\_2Eoption\_2Eoption\ A0) \quad (8)$$

Let  $c\_2Elist\_2EoEL : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a. nonempty\ A\_27a \Rightarrow c\_2Elist\_2EoEL\ A\_27a \in (((ty\_2Eoption\_2Eoption\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{ty\_2Enum\_2Enum}) \quad (9)$$

Let  $ty\_2Eone\_2Eone : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Eone\_2Eone \quad (10)$$

**Definition 12** We define  $c\_2Eone\_2Eone$  to be  $(ap\ (c\_2Emin\_2E\_40\ ty\_2Eone\_2Eone))\ (\lambda V0x \in ty\_2Eone\_2Eone. 2)$

Let  $ty\_2Esum\_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0. nonempty\ A0 \Rightarrow \forall A1. nonempty\ A1 \Rightarrow nonempty\ (ty\_2Esum\_2Esum\ A0\ A1) \quad (11)$$

Let  $c\_2Esum\_2EABS\_sum : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a. nonempty\ A\_27a \Rightarrow \forall A\_27b. nonempty\ A\_27b \Rightarrow c\_2Esum\_2EABS\_sum\ A\_27a\ A\_27b \in ((ty\_2Esum\_2Esum\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})^2}) \quad (12)$$

**Definition 13** We define  $c\_2Esum\_2EINR$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0e \in A\_27b. (ap\ (c\_2Esum\_2EABS\_sum$



Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0xs \in (ty\_2Elist\_2Elist \\
& \quad A.27a).(\forall V1n \in ty\_2Enum\_2Enum.((ap\ (ap\ (c\_2Elist\_2EoEL \\
& \quad A.27a)\ V1n)\ V0xs) = (ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ (ty\_2Eoption\_2Eoption \\
& \quad A.27a))\ (ap\ (ap\ c\_2Eprim\_rec\_2E\_3C\ V1n)\ (ap\ (c\_2Elist\_2ELENGTH \\
& \quad A.27a)\ V0xs))))\ (ap\ (c\_2Eoption\_2ESOME\ A.27a)\ (ap\ (ap\ (c\_2Elist\_2EEL \\
& \quad A.27a)\ V1n)\ V0xs))))\ (c\_2Eoption\_2ENONE\ A.27a))))))
\end{aligned} \tag{22}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0P \in 2.(\forall V1x \in A.27a. \\
& \quad (\forall V2y \in A.27a.(((ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ (ty\_2Eoption\_2Eoption \\
& \quad A.27a))\ V0P)\ (ap\ (c\_2Eoption\_2ESOME\ A.27a)\ V1x))\ (c\_2Eoption\_2ENONE \\
& \quad A.27a)) = (c\_2Eoption\_2ENONE\ A.27a)) \Leftrightarrow (\neg(p\ V0P))) \wedge (((ap\ (ap\ ( \\
& \quad ap\ (c\_2Ebool\_2ECOND\ (ty\_2Eoption\_2Eoption\ A.27a))\ V0P)\ (c\_2Eoption\_2ENONE \\
& \quad A.27a))\ (ap\ (c\_2Eoption\_2ESOME\ A.27a)\ V1x)) = (c\_2Eoption\_2ENONE \\
& \quad A.27a)) \Leftrightarrow (p\ V0P))) \wedge (((ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ (ty\_2Eoption\_2Eoption \\
& \quad A.27a))\ V0P)\ (ap\ (c\_2Eoption\_2ESOME\ A.27a)\ V1x))\ (c\_2Eoption\_2ENONE \\
& \quad A.27a)) = (ap\ (c\_2Eoption\_2ESOME\ A.27a)\ V2y)) \Leftrightarrow ((p\ V0P) \wedge (V1x = V2y))) \wedge \\
& \quad (((ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ (ty\_2Eoption\_2Eoption\ A.27a)) \\
& \quad V0P)\ (c\_2Eoption\_2ENONE\ A.27a))\ (ap\ (c\_2Eoption\_2ESOME\ A.27a) \\
& \quad V1x)) = (ap\ (c\_2Eoption\_2ESOME\ A.27a)\ V2y)) \Leftrightarrow ((\neg(p\ V0P)) \wedge (V1x = \\
& \quad V2y))))))))))
\end{aligned} \tag{23}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \tag{24}$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \tag{25}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\
& \quad (((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False))))))
\end{aligned} \tag{26}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p\ V0A)) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\
& \quad ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False))))))
\end{aligned} \tag{27}$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p\ V0A)) \Rightarrow False) \Rightarrow (((p\ V0A) \Rightarrow False) \Rightarrow False))) \tag{28}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow ( \\
& (p \ V1q) \Leftrightarrow (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee ((p \ V1q) \vee (p \ V2r))) \wedge (((p \ V0p) \vee ((\neg( \\
& p \ V2r)) \vee (\neg(p \ V1q)))) \wedge (((p \ V1q) \vee ((\neg(p \ V2r)) \vee (\neg(p \ V0p)))) \wedge ((p \ V2r) \vee \\
& ((\neg(p \ V1q)) \vee (\neg(p \ V0p))))))))))
\end{aligned} \tag{29}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow ( \\
& (p \ V1q) \Rightarrow (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee (p \ V1q)) \wedge (((p \ V0p) \vee (\neg(p \ V2r))) \wedge (( \\
& \neg(p \ V1q)) \vee ((p \ V2r) \vee (\neg(p \ V0p))))))))))
\end{aligned} \tag{30}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \ V0p) \Rightarrow (p \ V1q))) \Rightarrow (p \ V0p))) \tag{31}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \ V0p) \Rightarrow (p \ V1q))) \Rightarrow (\neg(p \ V1q)))) \tag{32}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \ V0p) \vee (p \ V1q))) \Rightarrow (\neg(p \ V0p)))) \tag{33}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \ V0p) \vee (p \ V1q))) \Rightarrow (\neg(p \ V1q)))) \tag{34}$$

Assume the following.

$$(\forall V0p \in 2. ((\neg(\neg(p \ V0p))) \Rightarrow (p \ V0p))) \tag{35}$$

**Theorem 1**

$$\begin{aligned}
& \forall A\_27a. nonempty \ A\_27a \Rightarrow (\forall V0xs \in (ty\_2Elist\_2Elist \\
& A\_27a). (\forall V1n \in ty\_2Enum\_2Enum. (\forall V2y \in A\_27a. ((( \\
& ap \ (ap \ (c\_2Elist\_2EoEL \ A\_27a) \ V1n) \ V0xs) = (ap \ (c\_2Eoption\_2ESOME \\
& A\_27a) \ V2y)) \Leftrightarrow ((p \ (ap \ (ap \ c\_2Eprim\_rec\_2E\_3C \ V1n) \ (ap \ (c\_2Elist\_2ELENGTH \\
& A\_27a) \ V0xs))) \wedge (V2y = (ap \ (ap \ (c\_2Elist\_2EEL \ A\_27a) \ V1n) \ V0xs))))))
\end{aligned}$$