

thm\_2Elist\_2EoHD\_\_thm  
(TMHrPSZ7sLhbfgrFD75866E71BkYyuHsLeTD)

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**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2E\_21$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2))) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x)$

Let  $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty\_2Elist\_2Elist A0) \quad (1)$$

Let  $c\_2Elist\_2ECONS : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow c\_2Elist\_2ECONS A.27a \in (((ty\_2Elist\_2Elist A.27a)^{(ty\_2Elist\_2Elist A.27a)})^{A.27a}) \quad (2)$$

Let  $c\_2Elist\_2ENIL : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow c\_2Elist\_2ENIL A.27a \in (ty\_2Elist\_2Elist A.27a) \quad (3)$$

**Definition 3** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 4** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A.27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A-27a})) (\lambda V1t \in 2.V1t)) (\lambda V2t \in 2.V2t))$

**Definition 5** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2)) (\lambda V2t \in 2.V2t))$

Let  $ty\_2Eone\_2Eone : \iota$  be given. Assume the following.

$$nonempty ty\_2Eone\_2Eone \quad (4)$$

**Definition 6** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A \wedge p x))$  of type  $\iota \Rightarrow \iota$ .

**Definition 7** We define  $c\_2Eone\_2Eone$  to be  $(ap (c\_2Emin\_2E\_40 ty\_2Eone\_2Eone)) (\lambda V0x \in ty\_2Eone\_2Eone.V0x)$

**Definition 8** We define  $c\_Ebool\_2EF$  to be  $(ap (c\_Ebool\_2E\_21\ 2) (\lambda V0t \in 2.V0t))$ .

**Definition 9** We define  $c\_Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap (ap c\_Emin\_2E\_3D\_3D\_3E\ V0t) c\_Ebool\_2EF))$

Let  $ty\_2Esum\_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Esum\_2Esum\ A0\ A1) \quad (5)$$

Let  $c\_2Esum\_2EABS\_sum : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Esum\_2EABS\_sum\ A\_27a\ A\_27b \in ((ty\_2Esum\_2Esum\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})^2}) \quad (6)$$

**Definition 10** We define  $c\_2Esum\_2EINR$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0e \in A\_27b.(ap (c\_2Esum\_2EABS\_sum\ A\_27a\ A\_27b)\ V0e)$

Let  $ty\_2Eoption\_2Eoption : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Eoption\_2Eoption\ A0) \quad (7)$$

Let  $c\_2Eoption\_2Eoption\_ABS : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Eoption\_2Eoption\_ABS\ A\_27a \in ((ty\_2Eoption\_2Eoption\ A\_27a)^{(ty\_2Esum\_2Esum\ A\_27a\ ty\_2Eone\_2Eone)}) \quad (8)$$

**Definition 11** We define  $c\_2Eoption\_2ENONE$  to be  $\lambda A\_27a : \iota.(ap (c\_2Eoption\_2Eoption\_ABS\ A\_27a)\ V0e)$

Let  $c\_2Elist\_2Elist\_CASE : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Elist\_2Elist\_CASE\ A\_27a\ A\_27b \in (((A\_27b^{(A\_27b^{(ty\_2Elist\_2Elist\ A\_27a)})^{A\_27a}})^{A\_27b})^{(ty\_2Elist\_2Elist\ A\_27a)}) \quad (9)$$

**Definition 12** We define  $c\_2Esum\_2EINL$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0e \in A\_27a.(ap (c\_2Esum\_2EABS\_sum\ A\_27a\ A\_27b)\ V0e)$

**Definition 13** We define  $c\_2Eoption\_2ESOME$  to be  $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.(ap (c\_2Eoption\_2Eoption\_ABS\ A\_27a)\ V0x)$

**Definition 14** We define  $c\_2Elist\_2EoHD$  to be  $\lambda A\_27a : \iota.\lambda V0l \in (ty\_2Elist\_2Elist\ A\_27a).(ap (ap (ap (c\_2Elist\_2Elist\_CASE\ A\_27a\ A\_27a)\ V0l)\ V0l)\ V0l)$

Assume the following.

$$True \quad (10)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \quad (11)$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\
& (\forall V0v \in A\_27b. (\forall V1f \in ((A\_27b^{(ty\_2Elist\_2Elist\ A\_27a)})^{A\_27a}). \\
& ((ap\ (ap\ (ap\ (c\_2Elist\_2Elist\_CASE\ A\_27a\ A\_27b)\ (c\_2Elist\_2ENIL \\
& A\_27a))\ V0v)\ V1f) = V0v))) \wedge (\forall V2a0 \in A\_27a. (\forall V3a1 \in \\
& (ty\_2Elist\_2Elist\ A\_27a). (\forall V4v \in A\_27b. (\forall V5f \in ( \\
& (A\_27b^{(ty\_2Elist\_2Elist\ A\_27a)})^{A\_27a}). ((ap\ (ap\ (ap\ (c\_2Elist\_2Elist\_CASE \\
& A\_27a\ A\_27b)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V2a0)\ V3a1))\ V4v)\ V5f) = \\
& (ap\ (ap\ V5f\ V2a0)\ V3a1))))))
\end{aligned} \tag{12}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a. (\forall V1y \in \\
& A\_27a. (((ap\ (c\_2Eoption\_2ESOME\ A\_27a)\ V0x) = (ap\ (c\_2Eoption\_2ESOME \\
& A\_27a)\ V1y)) \Leftrightarrow (V0x = V1y))))
\end{aligned} \tag{13}$$

**Theorem 1**

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\
& \forall V0h \in A\_27b. (\forall V1t \in (ty\_2Elist\_2Elist\ A\_27b). (( \\
& (ap\ (c\_2Elist\_2EoHD\ A\_27a)\ (c\_2Elist\_2ENIL\ A\_27a)) = (c\_2Eoption\_2ENONE \\
& A\_27a)) \wedge ((ap\ (c\_2Elist\_2EoHD\ A\_27b)\ (ap\ (ap\ (c\_2Elist\_2ECONS \\
& A\_27b)\ V0h)\ V1t)) = (ap\ (c\_2Eoption\_2ESOME\ A\_27b)\ V0h))))
\end{aligned}$$