

thm_2Ellist_2ELAPPEND_fromList (TMcBkM- bXTF5cK8eVP5Y38Y95kn5GXx5vaQX)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 4 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Let $ty_2Ellist_2Ellist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Ellist_2Ellist A0) \quad (1)$$

Let $c_2Ellist_2EAPPEND : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Ellist_2EAPPEND A_27a \in (((ty_2Ellist_2Ellist A_27a)^{(ty_2Ellist_2Ellist A_27a)})^{(ty_2Ellist_2Ellist A_27a)}) \quad (2)$$

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Let $ty_2Ellist_2Ellist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Ellist_2Ellist A0) \quad (3)$$

Let $c_2Ellist_2ELAPPEND : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Ellist_2ELAPPEND A_27a \in (((ty_2Ellist_2Ellist A_27a)^{(ty_2Ellist_2Ellist A_27a)})^{(ty_2Ellist_2Ellist A_27a)}) \quad (4)$$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (5)$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (6)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{omega}) \quad (7)$$

Definition 6 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 7 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (omega^{ty_2Enum_2Enum}) \quad (8)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (omega^{omega}) \quad (9)$$

Definition 8 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num\ (ap\ c_2Enum_2EREP_num\ m))$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (10)$$

Definition 9 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ (ap\ c_2Earithmetic_2E_2B\ n)\ c_2Enum_2E0)$

Definition 10 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let $c_2Earithmetic_2E_2D : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2D \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (11)$$

Let $ty_2Eoption_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Eoption_2Eoption\ A0) \quad (12)$$

Let $c_2Ellist_2Ellist_rep : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ellist_2Ellist_rep\ A_27a \in (((ty_2Eoption_2Eoption\ A_27a)^{ty_2Enum_2Enum})^{(ty_2Ellist_2Ellist_rep\ A_27a)}) \quad (13)$$

Let $ty_2Eone_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eone_2Eone \quad (14)$$

Definition 11 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ t1)\ t2))$

Let $ty_2Esum_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Esum_2Esum\ A0\ A1) \quad (15)$$

Let $c_2Esum_2EABS_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Esum_2EABS_sum\ A_27a\ A_27b \in ((ty_2Esum_2Esum\ A_27a\ A_27b)^{((2^{A_27b})^{A_27a})^2}) \quad (16)$$

Definition 12 We define c_2Esum_2EINL to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0e \in A_27a. (ap\ (c_2Esum_2EABS_sum\ A_27a\ A_27b)\ V0e)$

Let $c_2Eoption_2Eoption_abs : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Eoption_2Eoption_abs\ A_27a \in ((ty_2Eoption_2Eoption\ A_27a)^{ty_2Esum_2Esum\ A_27a\ ty_2Eone_2Eone}) \quad (17)$$

Definition 13 We define $c_2Eoption_2ESOME$ to be $\lambda A_27a : \iota. \lambda V0x \in A_27a. (ap\ (c_2Eoption_2Eoption_abs\ A_27a)\ V0x)$

Definition 14 We define $c_2Emin_2E_40$ to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. P\ x) \text{ then } (the\ (\lambda x. x \in A \wedge P\ x)) \text{ of type } \iota \Rightarrow \iota.$

Definition 15 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A_27a. (\lambda V2t2 \in A_27a. (ap\ (c_2Emin_2E_40\ A_27a)\ (V1t1 \wedge V2t2))\ V0t)))$

Let $c_2Ellist_2Ellist_abs : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ellist_2Ellist_abs\ A_27a \in ((ty_2Ellist_2Ellist\ A_27a)^{ty_2Eenum_2Eenum}) \quad (18)$$

Definition 16 We define $c_2Ellist_2ELCONS$ to be $\lambda A_27a : \iota. \lambda V0h \in A_27a. \lambda V1t \in (ty_2Ellist_2Ellist\ A_27a). (ap\ (c_2Ellist_2Ellist_abs\ A_27a)\ V1t)$

Let $c_2Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ECONS\ A_27a \in (((ty_2Elist_2Elist\ A_27a)^{ty_2Elist_2Elist\ A_27a})^{A_27a}) \quad (19)$$

Definition 17 We define c_2Eone_2Eone to be $(ap\ (c_2Emin_2E_40\ ty_2Eone_2Eone)\ (\lambda V0x \in ty_2Eone_2Eone. V0x))$

Definition 18 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2. (ap\ (ap\ c_2Emin_2E_3D_3D_3E\ V0t)\ c_2Ebool_2ECOND\ A))$

Definition 19 We define c_2Esum_2EINR to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0e \in A_27b. (ap\ (c_2Esum_2EABS_sum\ A_27a\ A_27b)\ V0e)$

Definition 20 We define $c_2Eoption_2ENONE$ to be $\lambda A_27a : \iota. (ap\ (c_2Eoption_2Eoption_abs\ A_27a)\ (ty_2Eone_2Eone))$

Definition 21 We define $c_2Ellist_2ELNIL$ to be $\lambda A_27a : \iota. (ap\ (c_2Ellist_2Ellist_abs\ A_27a)\ (\lambda V0n \in ty_2Ellist_2Ellist\ A_27a. V0n))$

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ENIL\ A_27a \in (ty_2Elist_2Elist\ A_27a) \quad (20)$$

Let $c_2Ellist_2EfromList : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ellist_2EfromList\ A_27a \in ((ty_2Ellist_2Ellist\ A_27a)^{(ty_2Elist_2Elist\ A_27a)}) \quad (21)$$

Assume the following.

$$True \quad (22)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \end{aligned} \quad (23)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (24)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow ((\forall V0l \in (ty_2Elist_2Elist\ A_27a).((ap\ (ap\ (c_2Elist_2EAPPEND\ A_27a)\ (c_2Elist_2ENIL\ A_27a)) \\ & V0l) = V0l) \wedge (\forall V1l1 \in (ty_2Elist_2Elist\ A_27a).(\forall V2l2 \in \\ & (ty_2Elist_2Elist\ A_27a).(\forall V3h \in A_27a.((ap\ (ap\ (c_2Elist_2EAPPEND\ A_27a)\ (ap\ (ap\ (c_2Elist_2ECONS\ A_27a)\ V3h)\ V1l1))\ V2l2) = (ap\ (ap\ (c_2Elist_2ECONS\ A_27a)\ V3h)\ (ap\ (ap\ (c_2Elist_2EAPPEND\ A_27a)\ V1l1)\ V2l2))))))) \end{aligned} \quad (25)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{(ty_2Elist_2Elist\ A_27a)}). \\ & (((p\ (ap\ V0P\ (c_2Elist_2ENIL\ A_27a))) \wedge (\forall V1t \in (ty_2Elist_2Elist\ A_27a).((p\ (ap\ V0P\ V1t)) \Rightarrow (\forall V2h \in A_27a.(p\ (ap\ V0P\ (ap\ (ap\ (c_2Elist_2ECONS\ A_27a)\ V2h)\ V1t)))))) \Rightarrow (\forall V3l \in (ty_2Elist_2Elist\ A_27a).(p\ (ap\ V0P\ V3l)))))) \end{aligned} \quad (26)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0h1 \in A_27a.(\forall V1t1 \in \\ & (ty_2Ellist_2Ellist\ A_27a).(\forall V2h2 \in A_27a.(\forall V3t2 \in \\ & (ty_2Ellist_2Ellist\ A_27a).(((ap\ (ap\ (c_2Ellist_2ELCONS\ A_27a)\ V1t1) = (ap\ (ap\ (c_2Ellist_2ELCONS\ A_27a)\ V2h2)\ V3t2)) \Leftrightarrow ((\\ & V0h1 = V2h2) \wedge (V1t1 = V3t2)))))) \end{aligned} \quad (27)$$

Assume the following.

$$\begin{aligned} \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow & ((\forall V0x \in (ty_2Ellist_2Ellist \\ & A_{.27a}).((ap\ (ap\ (c_2Ellist_2ELAPPEND\ A_{.27a})\ (c_2Ellist_2ELNIL \\ & A_{.27a}))\ V0x) = V0x)) \wedge (\forall V1h \in A_{.27a}.(\forall V2t \in (ty_2Ellist_2Ellist \\ & A_{.27a}).(\forall V3x \in (ty_2Ellist_2Ellist\ A_{.27a}).((ap\ (ap\ (c_2Ellist_2ELAPPEND \\ & A_{.27a})\ (ap\ (ap\ (c_2Ellist_2ELCONS\ A_{.27a})\ V1h)\ V2t))\ V3x) = (ap\ (ap \\ & (c_2Ellist_2ELCONS\ A_{.27a})\ V1h)\ (ap\ (ap\ (c_2Ellist_2ELAPPEND\ A_{.27a}) \\ & V2t)\ V3x))))))))) \end{aligned} \quad (28)$$

Assume the following.

$$\begin{aligned} \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow & (((ap\ (c_2Ellist_2EfromList\ A_{.27a}) \\ & (c_2Ellist_2ENIL\ A_{.27a})) = (c_2Ellist_2ELNIL\ A_{.27a})) \wedge (\forall V0h \in \\ & A_{.27a}.(\forall V1t \in (ty_2Ellist_2Ellist\ A_{.27a}).((ap\ (c_2Ellist_2EfromList \\ & A_{.27a})\ (ap\ (ap\ (c_2Ellist_2ELCONS\ A_{.27a})\ V0h)\ V1t)) = (ap\ (ap\ (c_2Ellist_2ELCONS \\ & A_{.27a})\ V0h)\ (ap\ (c_2Ellist_2EfromList\ A_{.27a})\ V1t)))))) \end{aligned} \quad (29)$$

Theorem 1

$$\begin{aligned} \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow & (\forall V0l1 \in (ty_2Ellist_2Ellist \\ & A_{.27a}).(\forall V1l2 \in (ty_2Ellist_2Ellist\ A_{.27a}).((ap\ (ap\ (c_2Ellist_2ELAPPEND \\ & A_{.27a})\ (ap\ (c_2Ellist_2EfromList\ A_{.27a})\ V0l1))\ (ap\ (c_2Ellist_2EfromList \\ & A_{.27a})\ V1l2)) = (ap\ (c_2Ellist_2EfromList\ A_{.27a})\ (ap\ (ap\ (c_2Ellist_2ELAPPEND \\ & A_{.27a})\ V0l1)\ V1l2)))))) \end{aligned}$$