

thm_2Ellist_2ELDROP_EQ_LNIL (TMJix-aMKA5KnWVMDf34eFYbTigwXWAjw1eT)

October 26, 2020

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A. \lambda x \in A. \lambda y \in A. inj_o(x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_Ebool_2ET to be $(ap \ (ap \ (c_Emin_2E_3D \ (2^2)) \ (\lambda V0x \in 2.V0x)) \ (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_{\text{Ebool_2E_21}}$ to be $\lambda A.27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap\ (ap\ (ap\ (c_{\text{Emin_2E_3D}}\ (2^{A-27a})\ V)\ P)\ 0)\ A))$

Definition 4 We define c_2Ebool_2EF to be $(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 5 We define c_2 to be $\lambda P \in 2. \lambda Q \in 2. inj_o(p \rightarrow p \rightarrow Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap(c_2Ebool_2E_21 2))(\lambda V2t \in 2.$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (1)$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

nonempty *ty_2Enum_2Enum*

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{omega}) \quad (3)$$

Definition 7 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EAABS_num\ c_2Enum_2EZERO_REP)$.

Definition 8 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (4)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^\omega) \quad (5)$$

Definition 9 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num\ m)$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (6)$$

Definition 10 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ (ap\ c_2Earithmetic\ n))$

Definition 11 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let $c_2Earithmetic_2E_2D : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2D \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (7)$$

Let $ty_2Eoption_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Eoption_2Eoption\ A0) \quad (8)$$

Let $ty_2Ellist_2Ellist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Ellist_2Ellist\ A0) \quad (9)$$

Let $c_2Ellist_2Ellist_rep : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow c_2Ellist_2Ellist_rep\ A_27a \in \\ & (((ty_2Eoption_2Eoption\ A_27a)^{ty_2Enum_2Enum})^{(ty_2Ellist_2Ellist\ A_27a)}) \end{aligned} \quad (10)$$

Let $ty_2Eone_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eone_2Eone \quad (11)$$

Definition 12 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in$

Let $ty_2Esum_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Esum_2Esum \\ & \quad A0\ A1) \end{aligned} \quad (12)$$

Let $c_2Esum_2EABS_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Esum_2EABS_sum \\ & \quad A_27a\ A_27b \in ((ty_2Esum_2Esum\ A_27a\ A_27b)^{((2^{A_27b})^{A_27a})^2}) \end{aligned} \quad (13)$$

Definition 13 We define c_2Esum_2EINL to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0e \in A_27a.(ap\ (c_2Esum_2EABS\ e))$

Let $c_2Eoption_2Eoption_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow c_2Eoption_2Eoption_ABS\ A_27a \in \\ & ((ty_2Eoption_2Eoption\ A_27a)^{(ty_2Esum_2Esum\ A_27a\ ty_2Eone_2Eone)}) \end{aligned} \quad (14)$$

Definition 14 We define $c_2Eoption_2ESOME$ to be $\lambda A_27a : \iota. \lambda V0x \in A_27a. (ap (c_2Eoption_2Eoption_2ESOME A_27a) x))$

Definition 15 We define $c_2Emin_2E_40$ to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p (ap P x)) \text{ then } (\lambda x. x \in A \wedge P(x)) \text{ else } \perp$

Definition 16 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A_27a. (\lambda V2t2 \in A_27a. (ap (c_2Eoption_2Eoption_2ECOND A_27a) t1) t2))))$

Let $c_2Ellist_2Ellist_abs : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a. \text{nonempty } A_27a \Rightarrow c_2Ellist_2Ellist_abs A_27a \in \\ & ((ty_2Ellist_2Ellist A_27a)^{(ty_2Eoption_2Eoption A_27a)^{ty_2Enum_2Enum}}) \end{aligned} \quad (15)$$

Definition 17 We define $c_2Ellist_2ELCONS$ to be $\lambda A_27a : \iota. \lambda V0h \in A_27a. \lambda V1t \in (ty_2Ellist_2Ellist A_27a). (ap (c_2Ellist_2ELCONS A_27a) (h, t)))$

Definition 18 We define c_2Eone_2Eone to be $(ap (c_2Emin_2E_40 ty_2Eone_2Eone) (\lambda V0x \in ty_2Eone_2Eone. (ap (c_2Eoption_2Eoption_2Eone ty_2Eone_2Eone) x))))$

Definition 19 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2. (ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_7E)))$

Definition 20 We define c_2Esum_2EINR to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0e \in A_27b. (ap (c_2Esum_2EABS A_27a) e))$

Definition 21 We define $c_2Eoption_2ENONE$ to be $\lambda A_27a : \iota. (ap (c_2Eoption_2Eoption_2ENONE A_27a) \perp))$

Definition 22 We define $c_2Ellist_2ELNIL$ to be $\lambda A_27a : \iota. (ap (c_2Ellist_2Ellist_abs A_27a) (\lambda V0n \in ty_2Ellist_2Ellist A_27a. n)))$

Definition 23 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a})). (ap V0P (ap (c_2Emin_2E_40 ty_2Ebool_2E_3F) P)))$

Definition 24 We define $c_2Ellist_2Ellength_rel$ to be $\lambda A_27a : \iota. (\lambda V0a0 \in (ty_2Ellist_2Ellist A_27a)). (\lambda V0l \in (ty_2Ellist_2Ellist A_27a). (ap (c_2Ellist_2Ellength_rel A_27a) (a0, l))))$

Definition 25 We define $c_2Ellist_2ELFINITE$ to be $\lambda A_27a : \iota. (\lambda V0a0 \in (ty_2Ellist_2Ellist A_27a)). (ap (c_2Ellist_2ELFINITE A_27a) (a0, \perp))$

Definition 26 We define $c_2Ellist_2ELLENGTH$ to be $\lambda A_27a : \iota. \lambda V0ll \in (ty_2Ellist_2Ellist A_27a). (ap (c_2Ellist_2ELLENGTH A_27a) ll))$

Definition 27 We define $c_2Ellist_2ELHD$ to be $\lambda A_27a : \iota. \lambda V0ll \in (ty_2Ellist_2Ellist A_27a). (ap (ap (c_2Ellist_2ELHD A_27a) ll) \perp))$

Let $c_2Eoption_2Eoption_CASE : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a. \text{nonempty } A_27a \Rightarrow \forall A_27b. \text{nonempty } A_27b \Rightarrow c_2Eoption_2Eoption_CASE \\ & A_27a A_27b \in (((A_27b^{(A_27b^{A_27a})})^{A_27b})^{(ty_2Eoption_2Eoption A_27a)}) \end{aligned} \quad (16)$$

Definition 28 We define $c_2Ellist_2ELTL$ to be $\lambda A_27a : \iota. \lambda V0ll \in (ty_2Ellist_2Ellist A_27a). (ap (ap (c_2Ellist_2ELTL A_27a) ll) \perp))$

Let $c_2Earithmetic_2EFUNPOW : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a. \text{nonempty } A_27a \Rightarrow c_2Earithmetic_2EFUNPOW A_27a \in \\ & (((A_27a^{A_27a})^{ty_2Enum_2Enum})^{(A_27a^{A_27a})}) \end{aligned} \quad (17)$$

Let $c_2Ellist_2ELDROP : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a. \text{nonempty } A_27a \Rightarrow c_2Ellist_2ELDROP A_27a \in (((ty_2Eoption_2Eoption A_27a)^{(ty_2Ellist_2Ellist A_27a)})^{ty_2Enum_2Enum}) \end{aligned} \quad (18)$$

Let $c_2Eoption_2EOPTION_MAP : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a. nonempty A_27a \Rightarrow \forall A_27b. nonempty A_27b \Rightarrow c_2Eoption_2EOPTION_MAP \\ & A_27a A_27b \in (((ty_2Eoption_2Eoption A_27b)^{(ty_2Eoption_2Eoption A_27a)})^{(A_27b^{A_27a})}) \end{aligned} \quad (19)$$

Let $c_2Eoption_2EOPTION_BIND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a. nonempty A_27a \Rightarrow \forall A_27b. nonempty A_27b \Rightarrow c_2Eoption_2EOPTION_BIND \\ & A_27a A_27b \in (((ty_2Eoption_2Eoption A_27a)^{(ty_2Eoption_2Eoption A_27a)})^{(ty_2Eoption_2Eoption A_27b)})^{(ty_2Eoption_2Eoption A_27b)} \end{aligned} \quad (20)$$

Assume the following.

$$\begin{aligned} & \forall A_27a. nonempty A_27a \Rightarrow ((\forall V0f \in (A_27a^{A_27a}). (\forall V1x \in \\ & A_27a. ((ap (ap (ap (c_2Earithmetic_2EFUNPOW A_27a) V0f) c_2Enum_2E0) \\ & V1x) = V1x))) \wedge (\forall V2f \in (A_27a^{A_27a}). (\forall V3n \in ty_2Enum_2Enum. \\ & (\forall V4x \in A_27a. ((ap (ap (ap (c_2Earithmetic_2EFUNPOW A_27a) \\ & V2f) (ap c_2Enum_2ESUC V3n)) V4x) = (ap (ap (ap (c_2Earithmetic_2EFUNPOW \\ & A_27a) V2f) V3n) (ap V2f V4x))))))) \end{aligned} \quad (21)$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum. (\neg(c_2Enum_2E0 = (ap c_2Enum_2ESUC V0n)))) \quad (22)$$

Assume the following.

$$True \quad (23)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p \\ & V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2))))) \quad (24)$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p V0t))) \quad (25)$$

Assume the following.

$$\begin{aligned} & \forall A_27a. nonempty A_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in \\ & A_27a. (p V0t) \Leftrightarrow (p V0t))) \end{aligned} \quad (26)$$

Assume the following.

$$((\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge \\ ((\neg False) \Leftrightarrow True))) \quad (27)$$

Assume the following.

$$\begin{aligned} & \forall A_27a. nonempty A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow \\ & True)) \end{aligned} \quad (28)$$

Assume the following.

$$\forall A_{27a}.nonempty A_{27a} \Rightarrow (\forall V0x \in A_{27a}.(\forall V1y \in A_{27a}.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (29)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t))))))) \quad (30)$$

Assume the following.

$$\forall A_{27a}.nonempty A_{27a} \Rightarrow (\forall V0P \in (2^{A_{27a}}).(\forall V1a \in A_{27a}.((\exists V2x \in A_{27a}.((V2x = V1a) \wedge (p (ap V0P V2x)))) \Leftrightarrow (p (ap V0P V1a))))) \quad (31)$$

Assume the following.

$$\begin{aligned} \forall A_{27a}.nonempty A_{27a} \Rightarrow & (\forall V0l \in (ty_2Ellist_2Ellist A_{27a}).((V0l = (c_2Ellist_2ELNIL A_{27a})) \vee (\exists V1h \in A_{27a}. \\ & (\exists V2t \in (ty_2Ellist_2Ellist A_{27a}).(V0l = (ap (ap (c_2Ellist_2ELCONS A_{27a}) V1h) V2t))))))) \end{aligned} \quad (32)$$

Assume the following.

$$\begin{aligned} \forall A_{27a}.nonempty A_{27a} \Rightarrow & \forall A_{27b}.nonempty A_{27b} \Rightarrow \\ & ((ap (c_2Ellist_2ELTL A_{27a}) (c_2Ellist_2ELNIL A_{27a})) = (c_2Eoption_2ENONE \\ & (ty_2Ellist_2Ellist A_{27a}))) \wedge (\forall V0h \in A_{27b}.(\forall V1t \in \\ & (ty_2Ellist_2Ellist A_{27b}).((ap (c_2Ellist_2ELTL A_{27b}) (ap \\ & (ap (c_2Ellist_2ELCONS A_{27b}) V0h) V1t)) = (ap (c_2Eoption_2ESOME \\ & (ty_2Ellist_2Ellist A_{27b})) V1t)))))) \end{aligned} \quad (33)$$

Assume the following.

$$\begin{aligned} \forall A_{27a}.nonempty A_{27a} \Rightarrow & \forall A_{27b}.nonempty A_{27b} \Rightarrow \\ & ((ap (c_2Ellist_2ELLENGTH A_{27a}) (c_2Ellist_2ELNIL A_{27a})) = \\ & (ap (c_2Eoption_2ESOME ty_2Enum_2Enum) c_2Enum_2E0)) \wedge (\forall V0h \in \\ & A_{27b}.(\forall V1t \in (ty_2Ellist_2Ellist A_{27b}).((ap (c_2Ellist_2ELLENGTH \\ & A_{27b}) (ap (ap (c_2Ellist_2ELCONS A_{27b}) V0h) V1t)) = (ap (ap (c_2Eoption_2EOPTION_MAP \\ & ty_2Enum_2Enum ty_2Enum_2Enum) c_2Enum_2ESUC) (ap (c_2Ellist_2ELLENGTH \\ & A_{27b}) V1t)))))) \end{aligned} \quad (34)$$

Assume the following.

$$\begin{aligned} \forall A_{27a}.nonempty A_{27a} \Rightarrow & (\forall V0x \in (ty_2Ellist_2Ellist A_{27a}).((ap (c_2Ellist_2ELLENGTH A_{27a}) V0x) = (ap (c_2Eoption_2ESOME \\ & ty_2Enum_2Enum) c_2Enum_2E0)) \Leftrightarrow (V0x = (c_2Ellist_2ELNIL A_{27a})))) \end{aligned} \quad (35)$$

Assume the following.

$$\begin{aligned} \forall A_{27a}.nonempty\ A_{27a} \Rightarrow & (\forall V0n \in ty_2Enum_2Enum. (\\ & \forall V1ll \in (ty_2Ellist_2Ellist\ A_{27a}). ((ap\ (ap\ (c_2Ellist_2ELDROP \\ & A_{27a})\ V0n)\ V1ll) = (ap\ (ap\ (ap\ (c_2Earithmetic_2EFUNPOW\ (ty_2Eoption_2Eoption \\ & A_{27a})\ V0n)\ V1ll))) \lambda V2m \in (ty_2Eoption_2Eoption \\ & (ty_2Ellist_2Ellist\ A_{27a})). (ap\ (ap\ (c_2Eoption_2EOPTION_BIND \\ & (ty_2Ellist_2Ellist\ A_{27a})\ (ty_2Ellist_2Ellist\ A_{27a}))\ V2m) \\ & (c_2Ellist_2ELTL\ A_{27a}))))\ V0n) (ap\ (c_2Eoption_2ESOME\ (ty_2Ellist_2Ellist \\ & A_{27a}))\ V1ll)))))) \end{aligned} \quad (36)$$

Assume the following.

$$\begin{aligned} (\forall V0P \in (2^{ty_2Enum_2Enum}). (((p\ (ap\ V0P\ c_2Enum_2E0)) \wedge \\ & (\forall V1n \in ty_2Enum_2Enum. ((p\ (ap\ V0P\ V1n)) \Rightarrow (p\ (ap\ V0P\ (ap\ c_2Enum_2ESUC \\ & V1n))))))) \Rightarrow (\forall V2n \in ty_2Enum_2Enum. (p\ (ap\ V0P\ V2n)))) \end{aligned} \quad (37)$$

Assume the following.

$$\begin{aligned} \forall A_{27a}.nonempty\ A_{27a} \Rightarrow & (\forall V0x \in A_{27a}. (\forall V1y \in \\ & A_{27a}. (((ap\ (c_2Eoption_2ESOME\ A_{27a})\ V0x) = (ap\ (c_2Eoption_2ESOME \\ & A_{27a})\ V1y)) \Leftrightarrow (V0x = V1y)))) \end{aligned} \quad (38)$$

Assume the following.

$$\forall A_{27a}.nonempty\ A_{27a} \Rightarrow (\forall V0x \in A_{27a}. (\neg((c_2Eoption_2ENONE \\ & A_{27a}) = (ap\ (c_2Eoption_2ESOME\ A_{27a})\ V0x)))) \quad (39)$$

Assume the following.

$$\begin{aligned} \forall A_{27a}.nonempty\ A_{27a} \Rightarrow & \forall A_{27b}.nonempty\ A_{27b} \Rightarrow (\\ & \forall V0f \in (A_{27b}^{A_{27a}}). (\forall V1x \in (ty_2Eoption_2Eoption \\ & A_{27a}). (\forall V2y \in A_{27b}. (((ap\ (ap\ (c_2Eoption_2EOPTION_MAP \\ & A_{27a}\ A_{27b})\ V0f)\ V1x) = (ap\ (c_2Eoption_2ESOME\ A_{27b})\ V2y)) \Leftrightarrow (\exists V3z \in \\ & A_{27a}. ((V1x = (ap\ (c_2Eoption_2ESOME\ A_{27a})\ V3z)) \wedge (V2y = (ap\ V0f \\ & V3z))))))) \end{aligned} \quad (40)$$

Assume the following.

$$\begin{aligned} \forall A_{27a}.nonempty\ A_{27a} \Rightarrow & \forall A_{27b}.nonempty\ A_{27b} \Rightarrow (\\ & (\forall V0f \in ((ty_2Eoption_2Eoption\ A_{27a})^{A_{27b}}). ((ap\ (ap\ (\\ & c_2Eoption_2EOPTION_BIND\ A_{27a}\ A_{27b})\ (c_2Eoption_2ENONE\ A_{27b})) \\ & V0f) = (c_2Eoption_2ENONE\ A_{27a}))) \wedge (\forall V1x \in A_{27b}. (\forall V2f \in \\ & ((ty_2Eoption_2Eoption\ A_{27a})^{A_{27b}}). ((ap\ (ap\ (c_2Eoption_2EOPTION_BIND \\ & A_{27a}\ A_{27b})\ (ap\ (c_2Eoption_2ESOME\ A_{27b})\ V1x))\ V2f) = (ap\ V2f\ V1x)))))) \end{aligned} \quad (41)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\ & ((ap\ c_2Enum_2ESUC\ V0m) = (ap\ c_2Enum_2ESUC\ V1n)) \Leftrightarrow (V0m = V1n)))) \quad (42)$$

Theorem 1
$$\begin{aligned} \forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0n \in ty_2Enum_2Enum. (\\ \forall V1ll \in (ty_2Ellist_2Ellist A_27a). (((ap (ap (c_2Ellist_2ELDROP \\ A_27a) V0n) V1ll) = (ap (c_2Eoption_2ESOME (ty_2Ellist_2Ellist \\ A_27a)) (c_2Ellist_2ELNIL A_27a))) \Leftrightarrow ((ap (c_2Ellist_2ELLENGTH \\ A_27a) V1ll) = (ap (c_2Eoption_2ESOME ty_2Enum_2Enum) V0n)))))) \end{aligned}$$