

# thm\_2Ellist\_2ELDROP\_\_EQ\_\_LNIL (TMJix- aMKA5KnWVMDf34eFYbTigwXWAjw1eT)

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**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2E\_2T$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A-27a}))$

**Definition 4** We define  $c\_2Ebool\_2E\_2F$  to be  $(ap (c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 5** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 6** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V2t))$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \tag{1}$$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{2}$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \tag{3}$$

**Definition 7** We define  $c\_2Enum\_2E0$  to be  $(ap\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2EZERO\_REP)$ .

**Definition 8** We define  $c\_2Earithmic\_2EZERO$  to be  $c\_2Enum\_2E0$ .

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \tag{4}$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \tag{5}$$

**Definition 9** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_num$

Let  $c\_2Earithmetic\_2E\_2B : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (6)$$

**Definition 10** We define  $c\_2Earithmetic\_2EBIT1$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

**Definition 11** We define  $c\_2Earithmetic\_2ENUMERAL$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.V0x$ .

Let  $c\_2Earithmetic\_2E\_2D : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (7)$$

Let  $ty\_2Eoption\_2Eoption : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Eoption\_2Eoption\ A0) \quad (8)$$

Let  $ty\_2Ellist\_2Ellist : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Ellist\_2Ellist\ A0) \quad (9)$$

Let  $c\_2Ellist\_2Ellist\_rep : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ellist\_2Ellist\_rep\ A\_27a \in ((ty\_2Eoption\_2Eoption\ A\_27a)^{ty\_2Enum\_2Enum})^{(ty\_2Ellist\_2Ellist\ A\_27a)} \quad (10)$$

Let  $ty\_2Eone\_2Eone : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Eone\_2Eone \quad (11)$$

**Definition 12** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V2t \in$

Let  $ty\_2Esum\_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Esum\_2Esum\ A0\ A1) \quad (12)$$

Let  $c\_2Esum\_2EABS\_sum : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Esum\_2EABS\_sum\ A\_27a\ A\_27b \in ((ty\_2Esum\_2Esum\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})^2}) \quad (13)$$

**Definition 13** We define  $c\_2Esum\_2EINL$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0e \in A\_27a.(ap\ (c\_2Esum\_2EABS$

Let  $c\_2Eoption\_2Eoption\_ABS : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Eoption\_2Eoption\_ABS\ A\_27a \in ((ty\_2Eoption\_2Eoption\ A\_27a)^{(ty\_2Esum\_2Esum\ A\_27a\ ty\_2Eone\_2Eone)}) \quad (14)$$



Let  $c\_2Eoption\_2EOPTION\_MAP : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Eoption\_2EOPTION\_MAP\ A\_27a\ A\_27b \in (((ty\_2Eoption\_2Eoption\ A\_27b)^{(ty\_2Eoption\_2Eoption\ A\_27a)})^{(A\_27b^{A\_27a}})) \quad (19)$$

Let  $c\_2Eoption\_2EOPTION\_BIND : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Eoption\_2EOPTION\_BIND\ A\_27a\ A\_27b \in (((ty\_2Eoption\_2Eoption\ A\_27a)^{(ty\_2Eoption\_2Eoption\ A\_27a)^{A\_27b}})^{(ty\_2Eoption\_2Eoption\ A\_27b)}) \quad (20)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow ((\forall V0f \in (A\_27a^{A\_27a}).(\forall V1x \in \\ & A\_27a.((ap\ (ap\ (ap\ (c\_2Earithmetic\_2EFUNPOW\ A\_27a)\ V0f)\ c\_2Enum\_2E0) \\ & V1x) = V1x))) \wedge (\forall V2f \in (A\_27a^{A\_27a}).(\forall V3n \in ty\_2Enum\_2Enum. \\ & (\forall V4x \in A\_27a.((ap\ (ap\ (ap\ (c\_2Earithmetic\_2EFUNPOW\ A\_27a) \\ & V2f)\ (ap\ c\_2Enum\_2ESUC\ V3n))\ V4x) = (ap\ (ap\ (ap\ (c\_2Earithmetic\_2EFUNPOW \\ & A\_27a)\ V2f)\ V3n)\ (ap\ V2f\ V4x))))))) \end{aligned} \quad (21)$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\neg(c\_2Enum\_2E0 = (ap\ c\_2Enum\_2ESUC\ V0n)))) \quad (22)$$

Assume the following.

$$True \quad (23)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (24)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \quad (25)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p\ V0t) \Leftrightarrow (p\ V0t))) \quad (26)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (27)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \quad (28)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a. (\forall V1y \in A\_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (29)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg(p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p\ V0t)))))) \quad (30)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}). (\forall V1a \in A\_27a. ((\exists V2x \in A\_27a. ((V2x = V1a) \wedge (p\ (ap\ V0P\ V2x)))) \Leftrightarrow (p\ (ap\ V0P\ V1a)))))) \quad (31)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0l \in (ty\_2Ellist\_2Ellist\ A\_27a). ((V0l = (c\_2Ellist\_2ELNIL\ A\_27a)) \vee (\exists V1h \in A\_27a. (\exists V2t \in (ty\_2Ellist\_2Ellist\ A\_27a). (V0l = (ap\ (ap\ (c\_2Ellist\_2ELCONS\ A\_27a)\ V1h)\ V2t)))))) \quad (32)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ((ap\ (c\_2Ellist\_2ELTL\ A\_27a)\ (c\_2Ellist\_2ELNIL\ A\_27a)) = (c\_2Eoption\_2ENONE\ (ty\_2Ellist\_2Ellist\ A\_27a))) \wedge (\forall V0h \in A\_27b. (\forall V1t \in (ty\_2Ellist\_2Ellist\ A\_27b). ((ap\ (c\_2Ellist\_2ELTL\ A\_27b)\ (ap\ (ap\ (c\_2Ellist\_2ELCONS\ A\_27b)\ V0h)\ V1t)) = (ap\ (c\_2Eoption\_2ESOME\ (ty\_2Ellist\_2Ellist\ A\_27b)\ V1t)))))) \quad (33)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ((ap\ (c\_2Ellist\_2ELLENGTH\ A\_27a)\ (c\_2Ellist\_2ELNIL\ A\_27a)) = (ap\ (c\_2Eoption\_2ESOME\ ty\_2Enum\_2Enum)\ c\_2Enum\_2E0)) \wedge (\forall V0h \in A\_27b. (\forall V1t \in (ty\_2Ellist\_2Ellist\ A\_27b). ((ap\ (c\_2Ellist\_2ELLENGTH\ A\_27b)\ (ap\ (ap\ (c\_2Ellist\_2ELCONS\ A\_27b)\ V0h)\ V1t)) = (ap\ (ap\ (c\_2Eoption\_2EOPTION\_MAP\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)\ c\_2Enum\_2ESUC)\ (ap\ (c\_2Ellist\_2ELLENGTH\ A\_27b)\ V1t)))))) \quad (34)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in (ty\_2Ellist\_2Ellist\ A\_27a). (((ap\ (c\_2Ellist\_2ELLENGTH\ A\_27a)\ V0x) = (ap\ (c\_2Eoption\_2ESOME\ ty\_2Enum\_2Enum)\ c\_2Enum\_2E0)) \Leftrightarrow (V0x = (c\_2Ellist\_2ELNIL\ A\_27a)))) \quad (35)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0n \in ty.2Enum.2Enum.( \\ & \quad \forall V1l \in (ty.2Ellist.2Ellist\ A.27a).((ap\ (ap\ (c.2Ellist.2ELDROP \\ A.27a)\ V0n)\ V1l) = (ap\ (ap\ (ap\ (c.2Earithmic.2EFUNPOW\ (ty.2Eoption.2Eoption \\ & \quad (ty.2Ellist.2Ellist\ A.27a)))\ (\lambda V2m \in (ty.2Eoption.2Eoption \\ & \quad (ty.2Ellist.2Ellist\ A.27a)).(ap\ (ap\ (c.2Eoption.2EOPTION\_BIND \\ & \quad (ty.2Ellist.2Ellist\ A.27a)\ (ty.2Ellist.2Ellist\ A.27a))\ V2m) \\ & \quad (c.2Ellist.2ELTL\ A.27a))))\ V0n)\ (ap\ (c.2Eoption.2ESOME\ (ty.2Ellist.2Ellist \\ & \quad A.27a)\ V1l)))))) \end{aligned} \tag{36}$$

Assume the following.

$$\begin{aligned} & (\forall V0P \in (2^{ty.2Enum.2Enum}).(((p\ (ap\ V0P\ c.2Enum.2E0)) \wedge \\ & (\forall V1n \in ty.2Enum.2Enum.((p\ (ap\ V0P\ V1n)) \Rightarrow (p\ (ap\ V0P\ (ap\ c.2Enum.2ESUC \\ & \quad V1n)))))) \Rightarrow (\forall V2n \in ty.2Enum.2Enum.(p\ (ap\ V0P\ V2n)))))) \end{aligned} \tag{37}$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0x \in A.27a.(\forall V1y \in \\ & A.27a.(((ap\ (c.2Eoption.2ESOME\ A.27a)\ V0x) = (ap\ (c.2Eoption.2ESOME \\ & \quad A.27a)\ V1y)) \Leftrightarrow (V0x = V1y)))))) \end{aligned} \tag{38}$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0x \in A.27a.(\neg((c.2Eoption.2ENONE \\ & \quad A.27a) = (ap\ (c.2Eoption.2ESOME\ A.27a)\ V0x)))))) \end{aligned} \tag{39}$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow ( \\ & \quad \forall V0f \in (A.27b^{A.27a}).(\forall V1x \in (ty.2Eoption.2Eoption \\ & \quad A.27a).(\forall V2y \in A.27b.(((ap\ (ap\ (c.2Eoption.2EOPTION\_MAP \\ & \quad A.27a\ A.27b)\ V0f)\ V1x) = (ap\ (c.2Eoption.2ESOME\ A.27b)\ V2y)) \Leftrightarrow (\exists V3z \in \\ & \quad A.27a.((V1x = (ap\ (c.2Eoption.2ESOME\ A.27a)\ V3z)) \wedge (V2y = (ap\ V0f \\ & \quad \quad V3z)))))))))) \end{aligned} \tag{40}$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow ( \\ & \quad (\forall V0f \in ((ty.2Eoption.2Eoption\ A.27a)^{A.27b}).((ap\ (ap\ ( \\ & \quad c.2Eoption.2EOPTION\_BIND\ A.27a\ A.27b)\ (c.2Eoption.2ENONE\ A.27b)) \\ & \quad V0f) = (c.2Eoption.2ENONE\ A.27a))) \wedge (\forall V1x \in A.27b.(\forall V2f \in \\ & \quad ((ty.2Eoption.2Eoption\ A.27a)^{A.27b}).((ap\ (ap\ (c.2Eoption.2EOPTION\_BIND \\ & \quad A.27a\ A.27b)\ (ap\ (c.2Eoption.2ESOME\ A.27b)\ V1x))\ V2f) = (ap\ V2f\ V1x)))))) \end{aligned} \tag{41}$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty.2Enum.2Enum.(\forall V1n \in ty.2Enum.2Enum.( \\ & ((ap\ c.2Enum.2ESUC\ V0m) = (ap\ c.2Enum.2ESUC\ V1n)) \Leftrightarrow (V0m = V1n)))) \end{aligned} \tag{42}$$

**Theorem 1**

$$\forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow (\forall V0n \in ty\_2Enum\_2Enum. (\forall V1ll \in (ty\_2Ellist\_2Ellist\ A_{.27a}). (((ap\ (ap\ (c\_2Ellist\_2ELDROPA_{.27a})\ V0n)\ V1ll) = (ap\ (c\_2Eoption\_2ESOME\ (ty\_2Ellist\_2Ellist\ A_{.27a})\ (c\_2Ellist\_2ELNIL\ A_{.27a}))) \Leftrightarrow ((ap\ (c\_2Ellist\_2ELLENGTH\ A_{.27a})\ V1ll) = (ap\ (c\_2Eoption\_2ESOME\ ty\_2Enum\_2Enum)\ V0n))))))$$