

thm_2Ellist_2ELFILTER_EQ_NIL
(TMTHwT2cm6deA15E7351sHqKiWnD5bfbybN)

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Definition 1 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 2 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 5 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2$

Definition 6 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 7 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2$

Definition 8 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x))$ **then** (the $(\lambda x.x \in A \wedge p$ of type $\iota \Rightarrow \iota$).

Definition 9 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.(a$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{1}$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{2}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{3}$$

Definition 10 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 11 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (4)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (5)$$

Definition 12 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum. (ap\ c_2Enum_2EABS_num$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum)^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (6)$$

Definition 13 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum. (ap\ (ap\ c_2Earithmetic$

Definition 14 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum. V0x$.

Let $c_2Earithmetic_2E_2D : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2D \in ((ty_2Enum_2Enum)^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (7)$$

Let $ty_2Eoption_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. nonempty\ A0 \Rightarrow nonempty\ (ty_2Eoption_2Eoption\ A0) \quad (8)$$

Let $ty_2Ellist_2Ellist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. nonempty\ A0 \Rightarrow nonempty\ (ty_2Ellist_2Ellist\ A0) \quad (9)$$

Let $c_2Ellist_2Ellist_rep : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow c_2Ellist_2Ellist_rep\ A_27a \in ((ty_2Eoption_2Eoption\ A_27a)^{ty_2Enum_2Enum})^{(ty_2Ellist_2Ellist\ A_27a)} \quad (10)$$

Let $ty_2Eone_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eone_2Eone \quad (11)$$

Let $ty_2Esum_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. nonempty\ A0 \Rightarrow \forall A1. nonempty\ A1 \Rightarrow nonempty\ (ty_2Esum_2Esum\ A0\ A1) \quad (12)$$

Let $c_2Esum_2EABS_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow \forall A_27b. nonempty\ A_27b \Rightarrow c_2Esum_2EABS_sum\ A_27a\ A_27b \in ((ty_2Esum_2Esum\ A_27a\ A_27b)^{((2^{A_27b})^{A_27a})^2}) \quad (13)$$

Assume the following.

$$(\forall V0t \in 2.((p V0t) \vee (\neg(p V0t)))) \quad (20)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \end{aligned} \quad (21)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((\\ & (p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))) \end{aligned} \quad (22)$$

Assume the following.

$$\begin{aligned} & ((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge \\ & ((\neg False) \Leftrightarrow True)))) \end{aligned} \quad (23)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (24)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (25)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(\\ & p V0t)))))) \end{aligned} \quad (26)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0t1 \in A_27a.(\forall V1t2 \in \\ & A_27a.(((ap (ap (ap (c_2Ebool_2ECOND A_27a) c_2Ebool_2ET) V0t1) \\ & V1t2) = V0t1) \wedge ((ap (ap (ap (c_2Ebool_2ECOND A_27a) c_2Ebool_2EF) \\ & V0t1) V1t2) = V1t2)))) \end{aligned} \quad (27)$$

Assume the following.

$$\begin{aligned} & (\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow \\ & ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \end{aligned} \quad (28)$$

Assume the following.

$$\begin{aligned} & (\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Leftrightarrow (p V1t2)) \Leftrightarrow (((p \\ & V0t1) \Rightarrow (p V1t2)) \wedge ((p V1t2) \Rightarrow (p V0t1)))))) \end{aligned} \quad (29)$$

Assume the following.

$$2.(((\forall V0x \in 2.(\forall V1x_{.27} \in 2.(\forall V2y \in 2.(\forall V3y_{.27} \in 2.(((p V0x) \Leftrightarrow (p V1x_{.27})) \wedge ((p V1x_{.27}) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_{.27})))))) \Rightarrow ((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_{.27}) \Rightarrow (p V3y_{.27})))))) \Rightarrow (30)$$

Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow \forall A_{.27b}.nonempty A_{.27b} \Rightarrow \forall A_{.27c}.nonempty A_{.27c} \Rightarrow (\forall V0f \in (A_{.27b}^{A_{.27a}}).(\forall V1g \in (A_{.27a}^{A_{.27c}}).(\forall V2x \in A_{.27c}.((ap (ap (ap (c_{.2}Ecombin_{.2}Eo A_{.27c} A_{.27b} A_{.27a}) V0f) V1g) V2x) = (ap V0f (ap V1g V2x)))))) \Rightarrow (31)$$

Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow (\forall V0h \in A_{.27a}.(\forall V1t \in (ty_{.2}Ellist_{.2}Ellist A_{.27a}).(\neg((ap (ap (c_{.2}Ellist_{.2}ELCONS A_{.27a}) V0h) V1t) = (c_{.2}Ellist_{.2}ELNIL A_{.27a}))) \wedge (\neg((c_{.2}Ellist_{.2}ELNIL A_{.27a}) = (ap (ap (c_{.2}Ellist_{.2}ELCONS A_{.27a}) V0h) V1t)))))) \Rightarrow (32)$$

Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow (\forall V0P \in (2^{A_{.27a}}).(\forall V1Q \in (2^{(ty_{.2}Ellist_{.2}Ellist A_{.27a})}).(\forall V2h \in A_{.27a}.(\forall V3t \in (ty_{.2}Ellist_{.2}Ellist A_{.27a}).((p (ap V1Q (ap (ap (c_{.2}Ellist_{.2}ELCONS A_{.27a}) V2h) V3t))) \Rightarrow ((p (ap V0P V2h)) \wedge (p (ap V1Q V3t)))))) \Rightarrow (\forall V4ll \in (ty_{.2}Ellist_{.2}Ellist A_{.27a}).((p (ap V1Q V4ll)) \Rightarrow (p (ap (ap (c_{.2}Ellist_{.2}Eevery A_{.27a}) V0P) V4ll)))))) \Rightarrow (33)$$

Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow \forall A_{.27b}.nonempty A_{.27b} \Rightarrow ((\forall V0P \in (2^{A_{.27a}}).((ap (ap (c_{.2}Ellist_{.2}ELFILTER A_{.27a}) V0P) (c_{.2}Ellist_{.2}ELNIL A_{.27a})) = (c_{.2}Ellist_{.2}ELNIL A_{.27a}))) \wedge (\forall V1P \in (2^{A_{.27b}}).(\forall V2h \in A_{.27b}.(\forall V3t \in (ty_{.2}Ellist_{.2}Ellist A_{.27b}).((ap (ap (c_{.2}Ellist_{.2}ELFILTER A_{.27b}) V1P) (ap (ap (c_{.2}Ellist_{.2}ELCONS A_{.27b}) V2h) V3t)) = (ap (ap (ap (c_{.2}Ebool_{.2}ECOND (ty_{.2}Ellist_{.2}Ellist A_{.27b}) (ap V1P V2h)) (ap (ap (c_{.2}Ellist_{.2}ELCONS A_{.27b}) V2h) (ap (ap (c_{.2}Ellist_{.2}ELFILTER A_{.27b}) V1P) V3t))) (ap (ap (c_{.2}Ellist_{.2}ELFILTER A_{.27b}) V1P) V3t)))))) \Rightarrow (34)$$

Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow (\forall V0P \in (2^{A_{.27a}}).(\forall V1ll \in (ty_{.2}Ellist_{.2}Ellist A_{.27a}).((p (ap (ap (c_{.2}Ellist_{.2}Eevery A_{.27a}) (ap (ap (c_{.2}Ecombin_{.2}Eo A_{.27a} 2 2) c_{.2}Ebool_{.2}E_{.7E}) V0P)) V1ll)) \Rightarrow ((ap (ap (c_{.2}Ellist_{.2}ELFILTER A_{.27a}) V0P) V1ll) = (c_{.2}Ellist_{.2}ELNIL A_{.27a})))) \Rightarrow (35)$$

Theorem 1

$$\forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow (\forall V0P \in (2^{A_{27a}}). (\forall V1l \in (ty_2Ellist_2Ellist \ A_{27a}). ((ap \ (ap \ (c_2Ellist_2ELFILTER \ A_{27a}) \ V0P) \ V1l) = (c_2Ellist_2ELNIL \ A_{27a})) \Leftrightarrow (p \ (ap \ (ap \ (c_2Ellist_2Every \ A_{27a}) \ (ap \ (ap \ (c_2Ecombin_2Eo \ A_{27a} \ 2 \ 2) \ c_2Ebool_2E_7E) \ V0P)) \ V1l))))))$$