

# thm\_2Ellist\_2ELFINITE\_THM (TM- NeKDNnNapUEDEXHNnGkYGt6esw7SzYYb4)

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**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2E\_2T$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A-27a}))$

**Definition 4** We define  $c\_2Ebool\_2E\_2F$  to be  $(ap (c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 5** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 6** We define  $c\_2Ebool\_2E\_27E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2E\_2F$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \tag{1}$$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{2}$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \tag{3}$$

**Definition 7** We define  $c\_2Enum\_2E0$  to be  $(ap c\_2Enum\_2EABS\_num c\_2Enum\_2EZERO\_REP)$ .

**Definition 8** We define  $c\_2Earithmic\_2EZERO$  to be  $c\_2Enum\_2E0$ .

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \tag{4}$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \tag{5}$$

**Definition 9** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_num$

Let  $c\_2Earithmetic\_2E\_2B : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (6)$$

**Definition 10** We define  $c\_2Earithmetic\_2EBIT1$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

**Definition 11** We define  $c\_2Earithmetic\_2ENUMERAL$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.V0x$ .

Let  $c\_2Earithmetic\_2E\_2D : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (7)$$

Let  $ty\_2Eoption\_2Eoption : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Eoption\_2Eoption\ A0) \quad (8)$$

Let  $ty\_2Ellist\_2Ellist : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Ellist\_2Ellist\ A0) \quad (9)$$

Let  $c\_2Ellist\_2Ellist\_rep : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ellist\_2Ellist\_rep\ A\_27a \in \\ (((ty\_2Eoption\_2Eoption\ A\_27a)^{ty\_2Enum\_2Enum})^{(ty\_2Ellist\_2Ellist\ A\_27a)}) \quad (10)$$

Let  $ty\_2Eone\_2Eone : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Eone\_2Eone \quad (11)$$

**Definition 12** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V2t \in$

Let  $ty\_2Esum\_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Esum\_2Esum\ A0\ A1) \quad (12)$$

Let  $c\_2Esum\_2EABS\_sum : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Esum\_2EABS\_sum\ A\_27a\ A\_27b \in ((ty\_2Esum\_2Esum\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})^2}) \quad (13)$$

**Definition 13** We define  $c\_2Esum\_2EINL$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0e \in A\_27a.(ap\ (c\_2Esum\_2EABS$

Let  $c\_2Eoption\_2Eoption\_ABS : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Eoption\_2Eoption\_ABS\ A\_27a \in \\ ((ty\_2Eoption\_2Eoption\ A\_27a)^{(ty\_2Esum\_2Esum\ A\_27a\ ty\_2Eone\_2Eone)}) \quad (14)$$

**Definition 14** We define  $c\_2Eoption\_2ESOME$  to be  $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.(ap (c\_2Eoption\_2Eoption\_2ESOME A\_27a) x)$

**Definition 15** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.\text{if } (\exists x \in A.p (ap P x)) \text{ then } (the (\lambda x.x \in A.\lambda y.p (ap P y)))$  of type  $\iota \Rightarrow \iota$ .

**Definition 16** We define  $c\_2Ebool\_2ECOND$  to be  $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.(ap (c\_2Ebool\_2ECOND A\_27a) t1 t2))))$

Let  $c\_2Ellist\_2Ellist\_abs : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Ellist\_2Ellist\_abs A\_27a \in ((ty\_2Ellist\_2Ellist A\_27a)^{(ty\_2Eoption\_2Eoption A\_27a)^{ty\_2Enum\_2Enum}}) \quad (15)$$

**Definition 17** We define  $c\_2Ellist\_2ELCONS$  to be  $\lambda A\_27a : \iota.\lambda V0h \in A\_27a.\lambda V1t \in (ty\_2Ellist\_2Ellist A\_27a)$

**Definition 18** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap V0P (ap (c\_2Emin\_2E\_40 A\_27a) P))))$

**Definition 19** We define  $c\_2Eone\_2Eone$  to be  $(ap (c\_2Emin\_2E\_40 ty\_2Eone\_2Eone) (\lambda V0x \in ty\_2Eone\_2Eone))$

**Definition 20** We define  $c\_2Esum\_2EINR$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0e \in A\_27b.(ap (c\_2Esum\_2EABS A\_27a A\_27b) e)$

**Definition 21** We define  $c\_2Eoption\_2ENONE$  to be  $\lambda A\_27a : \iota.(ap (c\_2Eoption\_2Eoption\_2ABS A\_27a) (c\_2ENONE A\_27a))$

**Definition 22** We define  $c\_2Ellist\_2ELNIL$  to be  $\lambda A\_27a : \iota.(ap (c\_2Ellist\_2Ellist\_abs A\_27a) (\lambda V0n \in ty\_2Ellist\_2Ellist A\_27a))$

**Definition 23** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.(ap (c\_2Ebool\_2E\_5C\_2F A\_27a) t1 t2))))$

**Definition 24** We define  $c\_2Ellist\_2ELFINITE$  to be  $\lambda A\_27a : \iota.(\lambda V0a0 \in (ty\_2Ellist\_2Ellist A\_27a).(ap (c\_2Ellist\_2ELFINITE A\_27a) a0)))$

Assume the following.

$$True \quad (16)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (17)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p V0t))) \quad (18)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0t \in 2.((\exists V1x \in A\_27a.(p V0t)) \Leftrightarrow (p V0t))) \quad (19)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \quad (20)$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \vee (p \vee V0t)) \Leftrightarrow True) \wedge (((p \vee V0t) \vee True) \Leftrightarrow True) \wedge \\
& (((False \vee (p \vee V0t)) \Leftrightarrow (p \vee V0t)) \wedge (((p \vee V0t) \vee False) \Leftrightarrow (p \vee V0t)) \wedge (((p \vee V0t) \vee \\
& (p \vee V0t)) \Leftrightarrow (p \vee V0t))))))
\end{aligned} \tag{21}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Rightarrow (p \vee V0t)) \Leftrightarrow (p \vee V0t)) \wedge (((p \vee V0t) \Rightarrow True) \Leftrightarrow \\
& True) \wedge (((False \Rightarrow (p \vee V0t)) \Leftrightarrow True) \wedge (((p \vee V0t) \Rightarrow (p \vee V0t)) \Leftrightarrow True) \wedge (( \\
& (p \vee V0t) \Rightarrow False) \Leftrightarrow \neg(p \vee V0t))))))
\end{aligned} \tag{22}$$

Assume the following.

$$\forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0x \in A.27a.((V0x = V0x) \Leftrightarrow True)) \tag{23}$$

Assume the following.

$$\forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0x \in A.27a.(\forall V1y \in A.27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{24}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Leftrightarrow (p \vee V0t)) \Leftrightarrow (p \vee V0t)) \wedge (((p \vee V0t) \Leftrightarrow True) \Leftrightarrow \\
& (p \vee V0t)) \wedge (((False \Leftrightarrow (p \vee V0t)) \Leftrightarrow \neg(p \vee V0t)) \wedge (((p \vee V0t) \Leftrightarrow False) \Leftrightarrow \neg( \\
& p \vee V0t))))))
\end{aligned} \tag{25}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in 2.(\forall V1y \in 2.(\forall V2z \in 2.(\forall V3w \in \\
& 2.((((p \vee V0x) \Rightarrow (p \vee V1y)) \wedge ((p \vee V2z) \Rightarrow (p \vee V3w))) \Rightarrow (((p \vee V0x) \wedge (p \vee V2z)) \Rightarrow \\
& ((p \vee V1y) \wedge (p \vee V3w)))))))
\end{aligned} \tag{26}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in 2.(\forall V1y \in 2.(\forall V2z \in 2.(\forall V3w \in \\
& 2.((((p \vee V0x) \Rightarrow (p \vee V1y)) \wedge ((p \vee V2z) \Rightarrow (p \vee V3w))) \Rightarrow (((p \vee V0x) \vee (p \vee V2z)) \Rightarrow \\
& ((p \vee V1y) \vee (p \vee V3w)))))))
\end{aligned} \tag{27}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).(\forall V1Q \in \\
& (2^{A.27a}).((\forall V2x \in A.27a.((p \ (ap \ V0P \ V2x)) \Rightarrow (p \ (ap \ V1Q \ V2x)))))) \Rightarrow \\
& ((\exists V3x \in A.27a.(p \ (ap \ V0P \ V3x))) \Rightarrow (\exists V4x \in A.27a.(p \ ( \\
& ap \ V1Q \ V4x))))))
\end{aligned} \tag{28}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).(\forall V1a \in \\
& A.27a.((\exists V2x \in A.27a.((V2x = V1a) \wedge (p \ (ap \ V0P \ V2x)))))) \Leftrightarrow (p \ ( \\
& ap \ V0P \ V1a))))
\end{aligned} \tag{29}$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0h \in A\_27a. (\forall V1t \in \\ (ty\_2Ellist\_2Ellist\ A\_27a). (\neg((ap\ (ap\ (c\_2Ellist\_2ELCONS\ A\_27a) \\ V0h)\ V1t) = (c\_2Ellist\_2ELNIL\ A\_27a)))) \wedge (\neg((c\_2Ellist\_2ELNIL \\ A\_27a) = (ap\ (ap\ (c\_2Ellist\_2ELCONS\ A\_27a)\ V0h)\ V1t)))))) \end{aligned} \quad (30)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0h1 \in A\_27a. (\forall V1t1 \in \\ (ty\_2Ellist\_2Ellist\ A\_27a). (\forall V2h2 \in A\_27a. (\forall V3t2 \in \\ (ty\_2Ellist\_2Ellist\ A\_27a). (((ap\ (ap\ (c\_2Ellist\_2ELCONS\ A\_27a) \\ V0h1)\ V1t1) = (ap\ (ap\ (c\_2Ellist\_2ELCONS\ A\_27a)\ V2h2)\ V3t2)) \Leftrightarrow (( \\ V0h1 = V2h2) \wedge (V1t1 = V3t2))))))) \end{aligned} \quad (31)$$

**Theorem 1**

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\ ((p\ (ap\ (c\_2Ellist\_2ELFINITE\ A\_27a)\ (c\_2Ellist\_2ELNIL\ A\_27a))) \Leftrightarrow \\ True) \wedge (\forall V0h \in A\_27b. (\forall V1t \in (ty\_2Ellist\_2Ellist \\ A\_27b). ((p\ (ap\ (c\_2Ellist\_2ELFINITE\ A\_27b)\ (ap\ (ap\ (c\_2Ellist\_2ELCONS \\ A\_27b)\ V0h)\ V1t))) \Leftrightarrow (p\ (ap\ (c\_2Ellist\_2ELFINITE\ A\_27b)\ V1t)))))) \end{aligned}$$