

thm_2Ellist_2ELFLATTEN_EQ_NIL
 (TMZG3jEaonkp2Q2v1rPdHeZVs7fgS4inb9z)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A. \lambda x \in A. \lambda y \in A. inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap (ap (c_2Emin_2E_3D (2^{A_27a})) (\lambda V1P \in 2.V1P)) (\lambda V2P \in 2.V2P)))$

Definition 4 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2. \lambda Q \in 2. inj_o (p \Rightarrow p \ Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2. (ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2EF))$

Definition 7 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2. (ap (c_2Ebool_2E_7E V2t) c_2Ebool_2EF))))))$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (1)$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (2)$$

Let $c_2Enum_2EAABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EAABS_num \in (ty_2Enum_2Enum^\omega) \quad (3)$$

Definition 8 We define c_2Enum_2E0 to be $(ap c_2Enum_2EAABS_num c_2Enum_2EZERO_REP)$.

Definition 9 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (4)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^\omega) \quad (5)$$

Definition 10 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (6)$$

Definition 11 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ (ap\ c_2Earithmeti$

Definition 12 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x.$

Let $ty_2Eoption_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Eoption_2Eoption\ A0) \quad (7)$$

Let $ty_2Ellist_2Ellist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Ellist_2Ellist\ A0) \quad (8)$$

Let $c_2Ellist_2Ellist_rep : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow c_2Ellist_2Ellist_rep\ A_27a \in \\ & (((ty_2Eoption_2Eoption\ A_27a)^{ty_2Enum_2Enum})^{(ty_2Ellist_2Ellist\ A_27a)}) \end{aligned} \quad (9)$$

Let $c_2Ellist_2Ellist_abs : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow c_2Ellist_2Ellist_abs\ A_27a \in \\ & ((ty_2Ellist_2Ellist\ A_27a)^{(ty_2Eoption_2Eoption\ A_27a)^{ty_2Enum_2Enum}}) \end{aligned} \quad (10)$$

Let $ty_2Eone_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eone_2Eone \quad (11)$$

Let $ty_2Esum_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Esum_2Esum \\ & \quad A0\ A1) \end{aligned} \quad (12)$$

Let $c_2Esum_2EABS_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Esum_2EABS_sum \\ & \quad A_27a\ A_27b \in ((ty_2Esum_2Esum\ A_27a\ A_27b)^{((2^{A_27b})^{A_27a})^2}) \end{aligned} \quad (13)$$

Definition 13 We define c_2Esum_2EINL to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0e \in A_27a.(ap\ (c_2Esum_2EABS$

Let $c_2Eoption_2Eoption_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow c_2Eoption_2Eoption_ABS\ A_27a \in \\ & ((ty_2Eoption_2Eoption\ A_27a)^{(ty_2Esum_2Esum\ A_27a\ ty_2Eone_2Eone)}) \end{aligned} \quad (14)$$

Definition 14 We define $c_2Eoption_2ESOME$ to be $\lambda A_27a : \iota. \lambda V0x \in A_27a. (ap (c_2Eoption_2Eoption_2ESOME) A_27a)$

Definition 15 We define $c_2Emin_2E_40$ to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p (ap P x)) \text{ then } (\lambda x. x \in A \wedge p x) \text{ else } \iota$

Definition 16 We define c_2Eone_2Eone to be $(ap (c_2Emin_2E_40 ty_2Eone_2Eone) (\lambda V0x \in ty_2Eone_2Eone. x = x))$

Definition 17 We define c_2Esum_2EINR to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0e \in A_27b. (ap (c_2Esum_2EABS A_27a) e)$

Definition 18 We define $c_2Eoption_2ENONE$ to be $\lambda A_27a : \iota. (ap (c_2Eoption_2Eoption_2ENONE) A_27a)$

Definition 19 We define $c_2Ellist_2ELHD$ to be $\lambda A_27a : \iota. \lambda V0ll \in (ty_2Ellist_2Ellist A_27a). (ap (ap (c_2Ellist_2ELHD) A_27a) V0ll)$

Let $c_2Eoption_2Eoption_CASE : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow \forall A_27b. nonempty A_27b \Rightarrow c_2Eoption_2Eoption_CASE A_27a A_27b \in (((A_27b^{(A_27b^{A_27a})^{A_27b}})^{(ty_2Eoption_2Eoption A_27a)})^{(ty_2Eoption_2Eoption A_27b)}) \quad (15)$$

Definition 20 We define $c_2Ellist_2ELTL$ to be $\lambda A_27a : \iota. \lambda V0ll \in (ty_2Ellist_2Ellist A_27a). (ap (ap (ap (c_2Ellist_2ELTL) A_27a) V0ll) A_27a)$

Let $c_2Eoption_2ETHE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow c_2Eoption_2ETHE A_27a \in (A_27a^{(ty_2Eoption_2Eoption A_27a)}) \quad (16)$$

Definition 21 We define $c_2Ellist_2ELNIL$ to be $\lambda A_27a : \iota. (ap (c_2Ellist_2Ellist_abs A_27a) (\lambda V0n \in ty_2Ellist A_27a. n = n))$

Definition 22 We define $c_2Ecombin_2Eo$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda A_27c : \iota. \lambda V0f \in (A_27b^{A_27c}). \lambda V1g \in (A_27c^{A_27b}). ap (c_2Ecombin_2Eo) A_27a$

Let $c_2Earithmetic_2E_2D : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2D \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (17)$$

Definition 23 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A_27a. (\lambda V2t2 \in A_27a. (V1t1 = t1 \wedge V2t2 = t2))) \vee (\lambda V1t1 \in A_27a. (\lambda V2t2 \in A_27a. (V1t1 = t1 \wedge V2t2 = t2))))$

Definition 24 We define $c_2Ellist_2ELCONS$ to be $\lambda A_27a : \iota. \lambda V0h \in A_27a. \lambda V1t \in (ty_2Ellist_2Ellist A_27a. ap (c_2Ellist_2ELCONS) A_27a) V0h$

Definition 25 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap V0P (ap (c_2Emin_2E_40) A_27a)))$

Definition 26 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c_2Ebool_2E_21) 2)))$

Definition 27 We define $c_2Ellist_2Eexists$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (\lambda V1a0 \in (ty_2Ellist_2Ellist A_27a). ap (c_2Ellist_2Eexists) A_27a) V0P))$

Definition 28 We define $c_2Ellist_2Eevery$ to be $\lambda A_27a : \iota. \lambda V0P \in (2^{A_27a}). (\lambda V1ll \in (ty_2Ellist_2Ellist A_27a). ap (c_2Ellist_2Eevery) A_27a) V0P))$

Let $c_2Ellist_2ELAPPEND : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow c_2Ellist_2ELAPPEND A_27a \in (((ty_2Ellist_2Ellist A_27a)^{(ty_2Ellist_2Ellist A_27a)})^{(ty_2Ellist_2Ellist A_27a)}) \quad (18)$$

Let $c_2Ellist_2ELFLATTEN : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow c_2Ellist_2ELFLATTEN A_27a \in (((ty_2Ellist_2Ellist A_27a)^{(ty_2Ellist_2Ellist (ty_2Ellist_2Ellist A_27a))})^{(ty_2Ellist_2Ellist (ty_2Ellist_2Ellist A_27a))}) \quad (19)$$

Assume the following.

$$True \quad (20)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (21)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p V0t))) \quad (22)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \quad (23)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t))))))) \quad (24)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t)) \Leftrightarrow (p V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True)))) \quad (25)$$

Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (26)$$

Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (27)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t))))))) \quad (28)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (29)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in 2. (\forall V1x_27 \in 2. (\forall V2y \in 2. (\forall V3y_27 \in \\ & 2. (((p V0x) \Leftrightarrow (p V1x_27)) \wedge ((p V1x_27) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_27)))))) \Rightarrow \\ & (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_27) \Rightarrow (p V3y_27)))))) \end{aligned} \quad (30)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in 2. \\ & (\forall V2x \in A_27a. (\forall V3x_27 \in A_27a. (\forall V4y \in A_27a. \\ & (\forall V5y_27 \in A_27a. (((p V0P) \Leftrightarrow (p V1Q)) \wedge ((p V1Q) \Rightarrow (V2x = V3x_27)) \wedge \\ & ((\neg(p V1Q)) \Rightarrow (V4y = V5y_27)))) \Rightarrow ((ap (ap (ap (c_2Ebool_2ECOND A_27a) \\ & V0P) V2x) V4y) = (ap (ap (ap (c_2Ebool_2ECOND A_27a) V1Q) V3x_27) \\ & V5y_27)))))))))) \end{aligned} \quad (31)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.\text{nonempty } A_27a \Rightarrow ((\forall V0t1 \in A_27a. (\forall V1t2 \in \\ & A_27a. ((ap (ap (ap (c_2Ebool_2ECOND A_27a) c_2Ebool_2ET) V0t1) \\ & V1t2) = V0t1))) \wedge (\forall V2t1 \in A_27a. (\forall V3t2 \in A_27a. ((ap \\ & (ap (ap (c_2Ebool_2ECOND A_27a) c_2Ebool_2EF) V2t1) V3t2) = V3t2)))))) \end{aligned} \quad (32)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0l1 \in (ty_2Ellist_2Ellist \\ & A_27a). (\forall V1l2 \in (ty_2Ellist_2Ellist A_27a). (((ap (ap (\\ & c_2Ellist_2ELAPPEND A_27a) V0l1) V1l2) = (c_2Ellist_2ELNIL A_27a)) \Leftrightarrow \\ & ((V0l1 = (c_2Ellist_2ELNIL A_27a)) \wedge (V1l2 = (c_2Ellist_2ELNIL \\ & A_27a)))))) \end{aligned} \quad (33)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). (\forall V1Q \in \\ & (2^{(ty_2Ellist_2Ellist A_27a)}). ((\forall V2h \in A_27a. (\forall V3t \in \\ & (ty_2Ellist_2Ellist A_27a). ((p (ap V1Q (ap (ap (c_2Ellist_2ELCONS \\ & A_27a) V2h) V3t))) \Rightarrow ((p (ap V0P V2h) \wedge (p (ap V1Q V3t))))))) \Rightarrow (\forall V4ll \in \\ & (ty_2Ellist_2Ellist A_27a). ((p (ap V1Q V4ll)) \Rightarrow (p (ap (ap (c_2Ellist_2EEvery \\ & A_27a) V0P) V4ll))))))) \end{aligned} \quad (34)$$

Assume the following.

$$\begin{aligned}
& \forall A_{27a}. nonempty A_{27a} \Rightarrow (\forall V0ll \in (ty_2Ellist_2Ellist \\
& (ty_2Ellist_2Ellist A_{27a})).((ap (c_2Ellist_2ELFLATTEN A_{27a}) \\
& V0ll) = (ap (ap (ap (c_2Ebool_2ECOND (ty_2Ellist_2Ellist A_{27a})) \\
& (ap (ap (c_2Ellist_2Eevery (ty_2Ellist_2Ellist A_{27a})) (ap (c_2Emin_2E_3D \\
& (ty_2Ellist_2Ellist A_{27a})) (c_2Ellist_2ELNIL A_{27a}))) V0ll)) \\
& (c_2Ellist_2ELNIL A_{27a})) (ap (ap (ap (c_2Ebool_2ECOND (ty_2Ellist_2Ellist \\
& A_{27a})) (ap (ap (c_2Emin_2E_3D (ty_2Ellist_2Ellist A_{27a})) (ap \\
& (c_2Eoption_2ETHE (ty_2Ellist_2Ellist A_{27a})) (ap (c_2Ellist_2ELHD \\
& (ty_2Ellist_2Ellist A_{27a})) V0ll)))) (c_2Ellist_2ELNIL A_{27a})) \\
& (ap (c_2Ellist_2ELFLATTEN A_{27a}) (ap (c_2Eoption_2ETHE (ty_2Ellist_2Ellist \\
& (ty_2Ellist_2Ellist A_{27a}))) (ap (c_2Ellist_2ELTL (ty_2Ellist_2Ellist \\
& A_{27a})) V0ll)))) (ap (ap (c_2Ellist_2ELCONS A_{27a}) (ap (c_2Eoption_2ETHE \\
& A_{27a}) (ap (c_2Ellist_2ELHD A_{27a}) (ap (c_2Eoption_2ETHE (ty_2Ellist_2Ellist \\
& A_{27a})) (ap (c_2Ellist_2ELHD (ty_2Ellist_2Ellist A_{27a})) V0ll)))) \\
& (ap (c_2Ellist_2ELFLATTEN A_{27a}) (ap (ap (c_2Ellist_2ELCONS (\\
& ty_2Ellist_2Ellist A_{27a})) (ap (c_2Eoption_2ETHE (ty_2Ellist_2Ellist \\
& A_{27a})) (ap (c_2Ellist_2ELTL A_{27a}) (ap (c_2Eoption_2ETHE (ty_2Ellist_2Ellist \\
& A_{27a})) (ap (c_2Ellist_2ELHD (ty_2Ellist_2Ellist A_{27a})) V0ll)))) \\
& (ap (c_2Eoption_2ETHE (ty_2Ellist_2Ellist (ty_2Ellist_2Ellist \\
& A_{27a}))) (ap (c_2Ellist_2ELTL (ty_2Ellist_2Ellist A_{27a})) V0ll))))))) \\
& (35)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_{27a}. nonempty A_{27a} \Rightarrow (\forall V0h \in (ty_2Ellist_2Ellist \\
& A_{27a}).(\forall V1t \in (ty_2Ellist_2Ellist (ty_2Ellist_2Ellist \\
& A_{27a})).((ap (c_2Ellist_2ELFLATTEN A_{27a}) (ap (ap (c_2Ellist_2ELCONS \\
& (ty_2Ellist_2Ellist A_{27a})) V0h) V1t)) = (ap (ap (c_2Ellist_2ELAPPEND \\
& A_{27a}) V0h) (ap (c_2Ellist_2ELFLATTEN A_{27a}) V1t))))) \\
& (36)
\end{aligned}$$

Theorem 1

$$\begin{aligned}
& \forall A_{27a}. nonempty A_{27a} \Rightarrow (\forall V0ll \in (ty_2Ellist_2Ellist \\
& (ty_2Ellist_2Ellist A_{27a})).(((ap (c_2Ellist_2ELFLATTEN A_{27a}) \\
& V0ll) = (c_2Ellist_2ELNIL A_{27a})) \Leftrightarrow (p (ap (ap (c_2Ellist_2Eevery \\
& (ty_2Ellist_2Ellist A_{27a})) (ap (c_2Emin_2E_3D (ty_2Ellist_2Ellist \\
& A_{27a})) (c_2Ellist_2ELNIL A_{27a}))) V0ll)))) \\
&
\end{aligned}$$