

thm_2Elist_2ELGENLIST__CHUNK__GENLIST (TMPJgjb6eRCNGk4CKGpCGJjS1jrbe9qrsE)

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Definition 1 We define `c_2Emin_2E_3D` to be $\lambda A. \lambda x \in A. \lambda y \in A. \text{inj_o } (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define `c_2Ebool_2ET` to be $(\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^2)) (\lambda V 0x \in 2.V 0x)) (\lambda V 1x \in 2.V 1x))$

Definition 3 We define `c_2Ebool_2E_21` to be $\lambda A. 27a : \iota. (\lambda V 0P \in (2^{A-27a}). (\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^{A-27a})) (\lambda V 0x \in 2.V 0x)) (\lambda V 1x \in 2.V 1x)))$

Definition 4 We define `c_2Ebool_2EF` to be $(\text{ap } (\text{c_2Ebool_2E_21 } 2) (\lambda V 0t \in 2.V 0t))$.

Definition 5 We define `c_2Ebool_2EBOUNDED` to be $(\lambda V 0v \in 2.c_2Ebool_2ET)$.

Definition 6 We define `c_2Ecombin_2EK` to be $\lambda A. 27a : \iota. \lambda A. 27b : \iota. (\lambda V 0x \in A. 27a. (\lambda V 1y \in A. 27b. V 0x))$

Definition 7 We define `c_2Ecombin_2ES` to be $\lambda A. 27a : \iota. \lambda A. 27b : \iota. \lambda A. 27c : \iota. (\lambda V 0f \in ((A. 27c^{A-27b})^{A-27a}))$

Definition 8 We define `c_2Ecombin_2EI` to be $\lambda A. 27a : \iota. (\text{ap } (\text{ap } (\text{c_2Ecombin_2ES } A. 27a (A. 27a^{A-27a})) A. 27a))$

Let `ty_2Elist_2Elist` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A 0. \text{nonempty } A 0 \Rightarrow \text{nonempty } (\text{ty_2Elist_2Elist } A 0) \quad (1)$$

Let `ty_2Enum_2Enum` : ι be given. Assume the following.

$$\text{nonempty } \text{ty_2Enum_2Enum} \quad (2)$$

Let `c_2Elist_2EGENLIST__AUX` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A. 27a. \text{nonempty } A. 27a \Rightarrow \text{c_2Elist_2EGENLIST_AUX } A. 27a \in (((\text{ty_2Elist_2Elist } A. 27a)^{(\text{ty_2Elist_2Elist } A. 27a)})^{\text{ty_2Enum_2Enum}})^{(A. 27a^{\text{ty_2Enum_2Enum}})} \quad (3)$$

Let `c_2Elist_2EGENLIST` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A. 27a. \text{nonempty } A. 27a \Rightarrow \text{c_2Elist_2EGENLIST } A. 27a \in (((\text{ty_2Elist_2Elist } A. 27a)^{(\text{ty_2Elist_2Elist } A. 27a)})^{\text{ty_2Enum_2Enum}})^{(A. 27a^{\text{ty_2Enum_2Enum}})} \quad (4)$$

Definition 9 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Let $ty_2Ellist_2Ellist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Ellist_2Ellist A0) \quad (5)$$

Let $c_2Ellist_2ELAPPEND : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Ellist_2ELAPPEND A_27a \in (((ty_2Ellist_2Ellist A_27a)^{(ty_2Ellist_2Ellist A_27a)})^{(ty_2Ellist_2Ellist A_27a)}) \quad (6)$$

Let $c_2Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2ECONS A_27a \in (((ty_2Elist_2Elist A_27a)^{(ty_2Elist_2Elist A_27a)})^{A_27a}) \quad (7)$$

Let $ty_2Eone_2Eone : \iota$ be given. Assume the following.

$$nonempty ty_2Eone_2Eone \quad (8)$$

Definition 10 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x))$ then (the $(\lambda x.x \in A \wedge p x)$ of type $\iota \Rightarrow \iota$).

Definition 11 We define c_2Eone_2Eone to be $(ap (c_2Emin_2E_40 ty_2Eone_2Eone) (\lambda V0x \in ty_2Eone_2Eone.2))$

Definition 12 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_7E))$

Definition 13 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.2)))$

Let $ty_2Esum_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Esum_2Esum A0 A1) \quad (9)$$

Let $c_2Esum_2EABS_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Esum_2EABS_sum A_27a A_27b \in ((ty_2Esum_2Esum A_27a A_27b)^{((2^{A_27b})^{A_27a})^2}) \quad (10)$$

Definition 14 We define c_2Esum_2EINR to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0e \in A_27b.(ap (c_2Esum_2EABS_sum A_27a A_27b) V0e)$

Let $ty_2Eoption_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Eoption_2Eoption A0) \quad (11)$$

Let $c_2Eoption_2Eoption_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Eoption_2Eoption_ABS A_27a \in ((ty_2Eoption_2Eoption A_27a)^{(ty_2Esum_2Esum A_27a ty_2Eone_2Eone)}) \quad (12)$$

Definition 15 We define $c_2Eoption_2ENONE$ to be $\lambda A_27a : \iota.(ap (c_2Eoption_2Eoption_ABS A_27a) (c_2Eoption_2Eoption_ABS A_27a))$.
Let $c_2Ellist_2Ellist_abs : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Ellist_2Ellist_abs A_27a \in ((ty_2Ellist_2Ellist A_27a)^{(ty_2Eoption_2Eoption A_27a)^{ty_2Enum_2Enum}}) \quad (13)$$

Definition 16 We define $c_2Ellist_2ELNIL$ to be $\lambda A_27a : \iota.(ap (c_2Ellist_2Ellist_abs A_27a) (\lambda V0n \in ty_2Ellist_2Ellist A_27a. V0n))$.
Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2ENIL A_27a \in (ty_2Elist_2Elist A_27a) \quad (14)$$

Let $c_2Ellist_2EfromList : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Ellist_2EfromList A_27a \in ((ty_2Ellist_2Ellist A_27a)^{(ty_2Elist_2Elist A_27a)}) \quad (15)$$

Definition 17 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a. V2t2)))$.
Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \quad (16)$$

Definition 18 We define $c_2Ecombin_2Eo$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda A_27c : \iota. \lambda V0f \in (A_27b^{A_27c}). \lambda V1g \in (A_27c^{A_27b}). \lambda V2h \in (A_27a^{A_27c}). \lambda V3i \in (A_27a^{A_27b}). V2h (V1g (V0f i))$.
Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2ESND A_27a A_27b \in (A_27b^{(ty_2Epair_2Eprod A_27a A_27b)}) \quad (17)$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EFST A_27a A_27b \in (A_27a^{(ty_2Epair_2Eprod A_27a A_27b)}) \quad (18)$$

Definition 19 We define $c_2Epair_2EUNCURRY$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda A_27c : \iota. \lambda V0f \in ((A_27c^{A_27b})^{A_27a}). \lambda V1g \in (A_27b^{A_27c}). \lambda V2h \in (A_27a^{A_27c}). V2h (V1g (V0f))$.
Let $c_2Eoption_2EOPTION_BIND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Eoption_2EOPTION_BIND A_27a A_27b \in (((ty_2Eoption_2Eoption A_27a)^{(ty_2Eoption_2Eoption A_27a)^{A_27b}})^{(ty_2Eoption_2Eoption A_27a)^{A_27b}})^{(ty_2Eoption_2Eoption A_27a)^{A_27b}} \quad (19)$$

Let $c_2Earithmetic_2EFUNPOW : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Earithmetic_2EFUNPOW A_27a \in (((A_27a^{A_27a})^{ty_2Enum_2Enum})^{(A_27a^{A_27a})}) \quad (20)$$

Let $c_2Eoption_2EOPTION_MAP : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Eoption_2EOPTION_MAP\ A_27a\ A_27b \in (((ty_2Eoption_2Eoption\ A_27b)^{(ty_2Eoption_2Eoption\ A_27a)})^{(A_27b^{A_27a}})) \quad (21)$$

Definition 20 We define $c_2Ellist_2ELUNFOLD$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in ((ty_2Eoption_2Eoption$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (22)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (23)$$

Definition 21 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 22 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (24)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (25)$$

Definition 23 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (26)$$

Definition 24 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ (ap\ c_2Earithmetic$

Definition 25 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let $c_2Earithmetic_2E_2D : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2D \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (27)$$

Let $c_2Ellist_2Ellist_rep : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ellist_2Ellist_rep\ A_27a \in (((ty_2Eoption_2Eoption\ A_27a)^{(ty_2Enum_2Enum)})^{(ty_2Ellist_2Ellist\ A_27a)}) \quad (28)$$

Definition 26 We define c_2Esum_2EINL to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0e \in A_27a.(ap\ (c_2Esum_2EABS$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& \quad \forall V2p \in ty_2Enum_2Enum. ((ap (ap c_2Earithmetic_2E_2B V0m) \\
& (ap (ap c_2Earithmetic_2E_2B V1n) V2p)) = (ap (ap c_2Earithmetic_2E_2B \\
& \quad (ap (ap c_2Earithmetic_2E_2B V0m) V1n)) V2p))))))
\end{aligned} \tag{35}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. ((V0m = c_2Enum_2E0) \vee (\exists V1n \in \\
& \quad ty_2Enum_2Enum. (V0m = (ap c_2Enum_2ESUC V1n))))))
\end{aligned} \tag{36}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. ((ap c_2Enum_2ESUC V0m) = (ap (ap \\
& \quad c_2Earithmetic_2E_2B V0m) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\
& \quad c_2Earithmetic_2EZERO))))))
\end{aligned} \tag{37}$$

Assume the following.

$$\text{True} \tag{38}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p \\
& \quad V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2))))))
\end{aligned} \tag{39}$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p V0t))) \tag{40}$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in \\
\quad A_27a. (p V0t)) \Leftrightarrow (p V0t))) \tag{41}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2. (((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow \\
& (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge \\
& \quad (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t))))))
\end{aligned} \tag{42}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge \\
& \quad ((\neg False) \Leftrightarrow True))))
\end{aligned} \tag{43}$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow \\
\quad True)) \tag{44}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (45)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\forall V0f \in (A_27b^{A_27a}). (\forall V1g \in (A_27b^{A_27a}). ((V0f = V1g) \Leftrightarrow (\forall V2x \in A_27a. ((ap\ V0f\ V2x) = (ap\ V1g\ V2x)))))) \quad (46)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg(p\ V0t)))))) \quad (47)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (2^{A_27a}). ((\exists V2x \in A_27a. ((p\ V0P) \wedge (ap\ V1Q\ V2x))) \Leftrightarrow ((p\ V0P) \wedge (\exists V3x \in A_27a. (p\ (ap\ V1Q\ V3x)))))) \quad (48)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). (\forall V1Q \in 2. ((\forall V2x \in A_27a. ((p\ (ap\ V0P\ V2x)) \Rightarrow (p\ V1Q))) \Leftrightarrow ((\exists V3x \in A_27a. (p\ (ap\ V0P\ V3x)) \Rightarrow (p\ V1Q)))))) \quad (49)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p\ V0A) \vee (p\ V1B) \vee (p\ V2C)) \Leftrightarrow (((p\ V0A) \vee (p\ V1B)) \vee (p\ V2C)))))) \quad (50)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((p\ V0A) \vee (p\ V1B)) \Leftrightarrow ((p\ V1B) \vee (p\ V0A)))) \quad (51)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((p\ V0A) \Rightarrow (p\ V1B)) \Leftrightarrow ((\neg(p\ V0A)) \vee (p\ V1B)))) \quad (52)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \quad (53)$$

Assume the following.

$$(\forall V0x \in 2. (\forall V1x_27 \in 2. (\forall V2y \in 2. (\forall V3y_27 \in 2. (((((p\ V0x) \Leftrightarrow (p\ V1x_27)) \wedge ((p\ V1x_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y_27)))) \Rightarrow (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x_27) \Rightarrow (p\ V3y_27)))))) \quad (54)$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in 2. \\
& (\forall V2x \in A_27a. (\forall V3x_27 \in A_27a. (\forall V4y \in A_27a. \\
& (\forall V5y_27 \in A_27a. (((p\ V0P) \Leftrightarrow (p\ V1Q)) \wedge ((p\ V1Q) \Rightarrow (V2x = V3x_27)) \wedge \\
& ((\neg(p\ V1Q)) \Rightarrow (V4y = V5y_27)))))) \Rightarrow ((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a) \\
& V0P)\ V2x)\ V4y) = (ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ V1Q)\ V3x_27) \\
& V5y_27)))))))))
\end{aligned} \tag{55}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). (\forall V1a \in \\
& A_27a. ((\exists V2x \in A_27a. ((V2x = V1a) \wedge (p\ (ap\ V0P\ V2x)))) \Leftrightarrow (p\ (\\
& ap\ V0P\ V1a))))))
\end{aligned} \tag{56}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0f \in (2^{A_27a}). (\forall V1v \in \\
& A_27a. ((\forall V2x \in A_27a. ((V2x = V1v) \Rightarrow (p\ (ap\ V0f\ V2x)))) \Leftrightarrow (p\ (\\
& ap\ V0f\ V1v))))))
\end{aligned} \tag{57}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow ((\forall V0t1 \in A_27a. (\forall V1t2 \in \\
& A_27a. ((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ c_2Ebool_2ET)\ V0t1) \\
& V1t2) = V0t1))) \wedge (\forall V2t1 \in A_27a. (\forall V3t2 \in A_27a. ((ap \\
& (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ c_2Ebool_2EF)\ V2t1)\ V3t2) = V3t2))))))
\end{aligned} \tag{58}$$

Assume the following.

$$(\forall V0v \in 2. ((p\ (ap\ c_2Ebool_2EBOUNDED\ V0v)) \Leftrightarrow True)) \tag{59}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\
& nonempty\ A_27c \Rightarrow (\forall V0f \in (A_27b^{A_27a}). (\forall V1g \in (A_27a^{A_27c}). \\
& (\forall V2x \in A_27c. ((ap\ (ap\ (ap\ (c_2Ecombin_2Eo\ A_27c\ A_27b\ A_27a) \\
& V0f)\ V1g)\ V2x) = (ap\ V0f\ (ap\ V1g\ V2x))))))
\end{aligned} \tag{60}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\
& nonempty\ A_27c \Rightarrow \forall A_27d.nonempty\ A_27d \Rightarrow (\forall V0f \in (A_27b^{A_27a}). \\
& (\forall V1g \in (A_27a^{A_27c}). (\forall V2h \in (A_27c^{A_27d}). ((ap\ (\\
& ap\ (c_2Ecombin_2Eo\ A_27d\ A_27b\ A_27a)\ V0f)\ (ap\ (ap\ (c_2Ecombin_2Eo \\
& A_27d\ A_27a\ A_27c)\ V1g)\ V2h)) = (ap\ (ap\ (c_2Ecombin_2Eo\ A_27d\ A_27b \\
& A_27c)\ (ap\ (ap\ (c_2Ecombin_2Eo\ A_27c\ A_27b\ A_27a)\ V0f)\ V1g))\ V2h))))))
\end{aligned} \tag{61}$$

Assume the following.

$$\forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow (\forall V0x \in A_{.27a}.((ap\ (c_2Ecombin_2EI\ A_{.27a})\ V0x) = V0x)) \quad (62)$$

Assume the following.

$$\begin{aligned} & \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow \forall A_{.27b}.nonempty\ A_{.27b} \Rightarrow (\\ & \forall V0f \in (A_{.27b}^{A_{.27a}}).(((ap\ (ap\ (c_2Ecombin_2Eo\ A_{.27a}\ A_{.27b}) \\ & A_{.27b})\ (c_2Ecombin_2EI\ A_{.27b}))\ V0f) = V0f) \wedge ((ap\ (ap\ (c_2Ecombin_2Eo \\ & A_{.27a}\ A_{.27b}\ A_{.27a})\ V0f)\ (c_2Ecombin_2EI\ A_{.27a})) = V0f))) \end{aligned} \quad (63)$$

Assume the following.

$$\begin{aligned} & \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow (\forall V0f \in (A_{.27a}^{ty_2Enum_2Enum}). \\ & (\forall V1n \in ty_2Enum_2Enum.(((ap\ (ap\ (c_2Elist_2EGENLIST\ A_{.27a}) \\ & V0f)\ (ap\ c_2Enum_2ESUC\ V1n)) = (ap\ (ap\ (c_2Elist_2ECONS\ A_{.27a})\ (\\ & ap\ V0f\ c_2Enum_2E0))\ (ap\ (ap\ (c_2Elist_2EGENLIST\ A_{.27a})\ (ap\ (ap \\ & (c_2Ecombin_2Eo\ ty_2Enum_2Enum\ A_{.27a}\ ty_2Enum_2Enum)\ V0f)\ c_2Enum_2ESUC)) \\ & V1n)))))) \end{aligned} \quad (64)$$

Assume the following.

$$\begin{aligned} & \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow (\forall V0f \in (A_{.27a}^{ty_2Enum_2Enum}). \\ & (\forall V1n \in ty_2Enum_2Enum.(((ap\ (ap\ (c_2Elist_2EGENLIST\ A_{.27a}) \\ & V0f)\ c_2Enum_2E0) = (c_2Elist_2ENIL\ A_{.27a})) \wedge ((ap\ (ap\ (c_2Elist_2EGENLIST \\ & A_{.27a})\ V0f)\ (ap\ c_2Earithmetic_2ENUMERAL\ V1n)) = (ap\ (ap\ (ap\ (c_2Elist_2EGENLIST_AUX \\ & A_{.27a})\ V0f)\ (ap\ c_2Earithmetic_2ENUMERAL\ V1n))\ (c_2Elist_2ENIL \\ & A_{.27a})))))) \end{aligned} \quad (65)$$

Assume the following.

$$\begin{aligned} & \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow (\forall V0l \in (ty_2Ellist_2Ellist \\ & A_{.27a}).((V0l = (c_2Ellist_2ELNIL\ A_{.27a})) \vee (\exists V1h \in A_{.27a}. \\ & (\exists V2t \in (ty_2Ellist_2Ellist\ A_{.27a}).(V0l = (ap\ (ap\ (c_2Ellist_2ELCONS \\ & A_{.27a})\ V1h)\ V2t)))))) \end{aligned} \quad (66)$$

Assume the following.

$$\begin{aligned} & \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow (\forall V0h \in A_{.27a}.(\forall V1t \in \\ & (ty_2Ellist_2Ellist\ A_{.27a}).((\neg((ap\ (ap\ (c_2Ellist_2ELCONS\ A_{.27a}) \\ & V0h)\ V1t) = (c_2Ellist_2ELNIL\ A_{.27a}))) \wedge (\neg((c_2Ellist_2ELNIL \\ & A_{.27a}) = (ap\ (ap\ (c_2Ellist_2ELCONS\ A_{.27a})\ V0h)\ V1t)))))) \end{aligned} \quad (67)$$

Assume the following.

$$\begin{aligned} & \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow (\forall V0h1 \in A_{.27a}.(\forall V1t1 \in \\ & (ty_2Ellist_2Ellist\ A_{.27a}).(\forall V2h2 \in A_{.27a}.(\forall V3t2 \in \\ & (ty_2Ellist_2Ellist\ A_{.27a}).(((ap\ (ap\ (c_2Ellist_2ELCONS\ A_{.27a}) \\ & V0h1)\ V1t1) = (ap\ (ap\ (c_2Ellist_2ELCONS\ A_{.27a})\ V2h2)\ V3t2)) \Leftrightarrow ((\\ & V0h1 = V2h2) \wedge (V1t1 = V3t2)))))) \end{aligned} \quad (68)$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0f \in ((ty_2Eoption_2Eoption\ (ty_2Epair_2Eprod\ A_27a \\
& \quad A_27b))^{A_27a}).(\forall V1x \in A_27a.((ap\ (ap\ (c_2Ellist_2ELUNFOLD \\
& A_27b\ A_27a)\ V0f)\ V1x) = (ap\ (ap\ (ap\ (c_2Eoption_2Eoption_CASE \\
& \quad (ty_2Epair_2Eprod\ A_27a\ A_27b)\ (ty_2Ellist_2Ellist\ A_27b))\ (\\
& \quad ap\ V0f\ V1x))\ (c_2Ellist_2ELNIL\ A_27b))\ (\lambda V2v \in (ty_2Epair_2Eprod \\
& \quad A_27a\ A_27b). (ap\ (ap\ (c_2Epair_2Epair_CASE\ (ty_2Ellist_2Ellist \\
& \quad A_27b)\ A_27a\ A_27b)\ V2v)\ (\lambda V3v1 \in A_27a. (\lambda V4v2 \in A_27b. (ap \\
& \quad (ap\ (c_2Ellist_2ELCONS\ A_27b)\ V4v2)\ (ap\ (ap\ (c_2Ellist_2ELUNFOLD \\
& \quad A_27b\ A_27a)\ V0f)\ V3v1)))))))))) \\
& \hspace{15em} (69)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0l1 \in (ty_2Ellist_2Ellist \\
& \quad A_27a).(\forall V1l2 \in (ty_2Ellist_2Ellist\ A_27a).((V0l1 = \\
& V1l2) \Leftrightarrow (\exists V2R \in ((2^{(ty_2Ellist_2Ellist\ A_27a)})(ty_2Ellist_2Ellist\ A_27a)). \\
& \quad ((p\ (ap\ (ap\ V2R\ V0l1)\ V1l2)) \wedge (\forall V3l3 \in (ty_2Ellist_2Ellist \\
& \quad A_27a).(\forall V4l4 \in (ty_2Ellist_2Ellist\ A_27a).((p\ (ap\ (ap \\
& \quad V2R\ V3l3)\ V4l4)) \Rightarrow ((V3l3 = V4l4) \vee (\exists V5h \in A_27a. (\exists V6t1 \in \\
& \quad (ty_2Ellist_2Ellist\ A_27a).(\exists V7t2 \in (ty_2Ellist_2Ellist \\
& \quad A_27a).((V3l3 = (ap\ (ap\ (c_2Ellist_2ELCONS\ A_27a)\ V5h)\ V6t1)) \wedge \\
& \quad ((V4l4 = (ap\ (ap\ (c_2Ellist_2ELCONS\ A_27a)\ V5h)\ V7t2)) \wedge (p\ (ap\ (\\
& \quad ap\ V2R\ V6t1)\ V7t2)))))))))))))) \\
& \hspace{15em} (70)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow ((\forall V0x \in (ty_2Ellist_2Ellist \\
& \quad A_27a).((ap\ (ap\ (c_2Ellist_2ELAPPEND\ A_27a)\ (c_2Ellist_2ELNIL \\
& \quad A_27a))\ V0x) = V0x)) \wedge (\forall V1h \in A_27a. (\forall V2t \in (ty_2Ellist_2Ellist \\
& A_27a).(\forall V3x \in (ty_2Ellist_2Ellist\ A_27a).((ap\ (ap\ (c_2Ellist_2ELAPPEND \\
& \quad A_27a)\ (ap\ (ap\ (c_2Ellist_2ELCONS\ A_27a)\ V1h)\ V2t))\ V3x) = (ap\ (ap \\
& \quad (c_2Ellist_2ELCONS\ A_27a)\ V1h)\ (ap\ (ap\ (c_2Ellist_2ELAPPEND\ A_27a) \\
& \quad V2t)\ V3x)))))) \\
& \hspace{15em} (71)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (((ap\ (c_2Ellist_2EfromList\ A_27a) \\
& \quad (c_2Ellist_2ENIL\ A_27a)) = (c_2Ellist_2ELNIL\ A_27a)) \wedge (\forall V0h \in \\
& A_27a. (\forall V1t \in (ty_2Ellist_2Ellist\ A_27a).((ap\ (c_2Ellist_2EfromList \\
& A_27a)\ (ap\ (ap\ (c_2Ellist_2ELCONS\ A_27a)\ V0h)\ V1t)) = (ap\ (ap\ (c_2Ellist_2ELCONS \\
& \quad A_27a)\ V0h)\ (ap\ (c_2Ellist_2EfromList\ A_27a)\ V1t)))))) \\
& \hspace{15em} (72)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow ((\forall V0f \in (A.27a^{ty.2Enum.2Enum}). \\
& ((ap (ap (c.2Ellist.2ELGENLIST\ A.27a)\ V0f)\ (c.2Eoption.2ENONE \\
& ty.2Enum.2Enum))) = (ap (ap (c.2Ellist.2ELUNFOLD\ A.27a\ ty.2Enum.2Enum) \\
& (\lambda V1n \in ty.2Enum.2Enum.(ap (c.2Eoption.2ESOME\ (ty.2Epair.2Eprod \\
& ty.2Enum.2Enum\ A.27a))\ (ap (ap (c.2Epair.2E.2C\ ty.2Enum.2Enum \\
& A.27a)\ (ap (ap\ c.2Earithmetic.2E.2B\ V1n)\ (ap\ c.2Earithmetic.2ENUMERAL \\
& (ap\ c.2Earithmetic.2EBIT1\ c.2Earithmetic.2EZERO))))\ (ap\ V0f \\
& V1n))))\ c.2Enum.2E0)) \wedge (\forall V2f \in (A.27a^{ty.2Enum.2Enum}). \\
& (\forall V3lim \in ty.2Enum.2Enum.((ap (ap (c.2Ellist.2ELGENLIST \\
& A.27a)\ V2f)\ (ap (c.2Eoption.2ESOME\ ty.2Enum.2Enum)\ V3lim)) = (\\
ap (ap (c.2Ellist.2ELUNFOLD\ A.27a\ ty.2Enum.2Enum)\ (\lambda V4n \in ty.2Enum.2Enum. \\
& (ap (ap (ap (c.2Ebool.2ECOND\ (ty.2Eoption.2Eoption\ (ty.2Epair.2Eprod \\
& ty.2Enum.2Enum\ A.27a)))\ (ap (ap\ c.2Eprim.2rec.2E.3C\ V4n)\ V3lim)) \\
& (ap (c.2Eoption.2ESOME\ (ty.2Epair.2Eprod\ ty.2Enum.2Enum\ A.27a))) \\
& (ap (ap (c.2Epair.2E.2C\ ty.2Enum.2Enum\ A.27a)\ (ap (ap\ c.2Earithmetic.2E.2B \\
& V4n)\ (ap\ c.2Earithmetic.2ENUMERAL\ (ap\ c.2Earithmetic.2EBIT1 \\
& c.2Earithmetic.2EZERO))))\ (ap\ V2f\ V4n))))\ (c.2Eoption.2ENONE \\
& (ty.2Epair.2Eprod\ ty.2Enum.2Enum\ A.27a))))\ c.2Enum.2E0)))))) \\
& \hspace{15em} (73)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0f \in (A.27a^{ty.2Enum.2Enum}). \\
& (\forall V1h \in A.27a.(\forall V2t \in (ty.2Ellist.2Ellist\ A.27a). \\
& (((ap (ap (c.2Ellist.2ELGENLIST\ A.27a)\ V0f)\ (c.2Eoption.2ENONE \\
& ty.2Enum.2Enum))) = (ap (ap (c.2Ellist.2ELCONS\ A.27a)\ V1h)\ V2t)) \Leftrightarrow \\
& ((V1h = (ap\ V0f\ c.2Enum.2E0)) \wedge (V2t = (ap (ap (c.2Ellist.2ELGENLIST \\
& A.27a)\ (ap (ap (c.2Ecombin.2Eo\ ty.2Enum.2Enum\ A.27a\ ty.2Enum.2Enum) \\
& V0f)\ (ap\ c.2Earithmetic.2E.2B\ (ap\ c.2Earithmetic.2ENUMERAL\ (\\
& ap\ c.2Earithmetic.2EBIT1\ c.2Earithmetic.2EZERO))))\ (c.2Eoption.2ENONE \\
& ty.2Enum.2Enum))))))))) \\
& \hspace{15em} (74)
\end{aligned}$$

Assume the following.

$$(\forall V0n \in ty.2Enum.2Enum.(\neg((ap\ c.2Enum.2ESUC\ V0n) = c.2Enum.2E0))) \quad (75)$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0opt \in (ty.2Eoption.2Eoption \\
& A.27a).((V0opt = (c.2Eoption.2ENONE\ A.27a)) \vee (\exists V1x \in A.27a. \\
& (V0opt = (ap (c.2Eoption.2ESOME\ A.27a)\ V1x)))))) \quad (76)
\end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\ & (\forall V0v \in A.27b. (\forall V1f \in (A.27b^{A.27a}). ((ap\ (ap\ (ap\ (c.2Eoption.2Eoption_CASE \\ & A.27a\ A.27b)\ (c.2Eoption.2ENONE\ A.27a))\ V0v)\ V1f) = V0v))) \wedge (\forall V2x \in \\ & A.27a. (\forall V3v \in A.27b. (\forall V4f \in (A.27b^{A.27a}). ((ap\ (ap \\ & (ap\ (c.2Eoption.2Eoption_CASE\ A.27a\ A.27b)\ (ap\ (c.2Eoption.2ESOME \\ & A.27a)\ V2x))\ V3v)\ V4f) = (ap\ V4f\ V2x)))))) \end{aligned} \quad (77)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0x \in A.27a. (\forall V1y \in \\ & A.27a. (((ap\ (c.2Eoption.2ESOME\ A.27a)\ V0x) = (ap\ (c.2Eoption.2ESOME \\ & A.27a)\ V1y)) \Leftrightarrow (V0x = V1y)))) \end{aligned} \quad (78)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0x \in A.27a. (\neg((c.2Eoption.2ENONE \\ & A.27a) = (ap\ (c.2Eoption.2ESOME\ A.27a)\ V0x)))) \end{aligned} \quad (79)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow \forall A.27c. \\ & nonempty\ A.27c \Rightarrow (\forall V0x \in A.27b. (\forall V1y \in A.27c. (\forall V2f \in \\ & ((A.27a^{A.27c})^{A.27b}). ((ap\ (ap\ (c.2Epair.2Epair_CASE\ A.27a\ A.27b \\ & A.27c)\ (ap\ (ap\ (c.2Epair.2E.2C\ A.27b\ A.27c)\ V0x)\ V1y))\ V2f) = (ap \\ & (ap\ V2f\ V0x)\ V1y)))))) \end{aligned} \quad (80)$$

Assume the following.

$$\begin{aligned} & (\forall V0n \in ty.2Enum.2Enum. (\neg(p\ (ap\ (ap\ c.2Eprim_rec.2E.3C \\ & V0n)\ c.2Enum.2E0)))) \end{aligned} \quad (81)$$

Assume the following.

$$\begin{aligned} & (\forall V0n \in ty.2Enum.2Enum. (p\ (ap\ (ap\ c.2Eprim_rec.2E.3C\ c.2Enum.2E0) \\ & (ap\ c.2Enum.2ESUC\ V0n)))) \end{aligned} \quad (82)$$

Theorem 1

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0f \in (A.27a^{ty.2Enum.2Enum}). \\ & (\forall V1n \in ty.2Enum.2Enum. ((ap\ (ap\ (c.2Ellist.2ELGENLIST \\ & A.27a)\ V0f)\ (c.2Eoption.2ENONE\ ty.2Enum.2Enum)) = (ap\ (ap\ (c.2Ellist.2ELAPPEND \\ & A.27a)\ (ap\ (c.2Ellist.2EfromList\ A.27a)\ (ap\ (ap\ (c.2Elist.2EGENLIST \\ & A.27a)\ V0f)\ V1n)))\ (ap\ (ap\ (c.2Ellist.2ELGENLIST\ A.27a)\ (ap\ (ap \\ & (c.2Ecombin.2Eo\ ty.2Enum.2Enum\ A.27a\ ty.2Enum.2Enum)\ V0f)\ (ap \\ & c.2Earithmetic.2E.2B\ V1n)))\ (c.2Eoption.2ENONE\ ty.2Enum.2Enum)))))) \end{aligned}$$