

# thm\_2Ellist\_2ELGENLIST\_\_EQ\_\_CONS (TMR- tYHKSKrMxrK4mS74rdTyrbQAPyEM2C5o)

October 26, 2020

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{1}$$

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \tag{2}$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \tag{3}$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \tag{4}$$

**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2ET$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a})) (\lambda V1x \in 2.V1x)) (\lambda V1y \in 2.V1y)))$

**Definition 4** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap c\_2Enum\_2EABS\_num (c\_2Enum\_2EREP\_num m))$

**Definition 5** We define  $c\_2Ebool\_2EBOUNDED$  to be  $(\lambda V0v \in 2.c\_2Ebool\_2ET)$ .

**Definition 6** We define  $c\_2Ecombin\_2ES$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.(\lambda V0f \in ((A\_27c^{A\_27b})^{A\_27a}))$

**Definition 7** We define  $c\_2Ecombin\_2EC$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.(\lambda V0f \in ((A\_27c^{A\_27b})^{A\_27a}))$

**Definition 8** We define  $c\_2Ecombin\_2EK$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0x \in A\_27a.(\lambda V1y \in A\_27b.V0x))$

**Definition 9** We define  $c\_2Ecombin\_2EI$  to be  $\lambda A\_27a : \iota.(ap (ap (c\_2Ecombin\_2ES A\_27a (A\_27a^{A\_27a})) (\lambda V1x \in 2.V1x)) (\lambda V1y \in 2.V1y))$

**Definition 10** We define  $c\_2Ecombin\_2Eo$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.(\lambda V0f \in (A\_27c^{A\_27b}).\lambda V1x \in 2.V1x)$

Let  $ty\_2Eone\_2Eone : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Eone\_2Eone \quad (5)$$

**Definition 11** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.$ **if**  $(\exists x \in A.p (ap\ P\ x))$  **then**  $(the\ (\lambda x.x \in A \wedge P\ x))$  of type  $\iota \Rightarrow \iota$ .

**Definition 12** We define  $c\_2Eone\_2Eone$  to be  $(ap\ (c\_2Emin\_2E\_40\ ty\_2Eone\_2Eone)\ (\lambda V0x \in ty\_2Eone\_2Eone\ x))$

**Definition 13** We define  $c\_2Ebool\_2EF$  to be  $(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V0t \in 2.V0t))$ .

**Definition 14** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o\ (p\ P \Rightarrow p\ Q)$  of type  $\iota$ .

**Definition 15** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap\ (ap\ c\_2Emin\_2E\_3D\_3D\_3E\ V0t)\ c\_2Ebool\_2EF))$

**Definition 16** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V2t \in 2.V2t))))$

Let  $ty\_2Esum\_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Esum\_2Esum\ A0\ A1) \quad (6)$$

Let  $c\_2Esum\_2EABS\_sum : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Esum\_2EABS\_sum\ A\_27a\ A\_27b \in ((ty\_2Esum\_2Esum\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})^2}) \quad (7)$$

**Definition 17** We define  $c\_2Esum\_2EINR$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0e \in A\_27b.(ap\ (c\_2Esum\_2EABS\_sum\ A\_27a\ A\_27b)\ V0e)$

Let  $ty\_2Eoption\_2Eoption : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Eoption\_2Eoption\ A0) \quad (8)$$

Let  $c\_2Eoption\_2Eoption\_abs : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Eoption\_2Eoption\_abs\ A\_27a \in ((ty\_2Eoption\_2Eoption\ A\_27a)^{(ty\_2Esum\_2Esum\ A\_27a\ ty\_2Eone\_2Eone)}) \quad (9)$$

**Definition 18** We define  $c\_2Eoption\_2ENONE$  to be  $\lambda A\_27a : \iota.(ap\ (c\_2Eoption\_2Eoption\_abs\ A\_27a)\ (ty\_2Eoption\_2Eoption\ A\_27a))$

Let  $ty\_2Ellist\_2Ellist : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Ellist\_2Ellist\ A0) \quad (10)$$

Let  $c\_2Ellist\_2Ellist\_abs : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ellist\_2Ellist\_abs\ A\_27a \in ((ty\_2Ellist\_2Ellist\ A\_27a)^{(ty\_2Eoption\_2Eoption\ A\_27a)^{ty\_2Enum\_2Enum}}) \quad (11)$$

**Definition 19** We define  $c\_2Ellist\_2ELNIL$  to be  $\lambda A\_27a : \iota.(ap (c\_2Ellist\_2Ellist\_abs A\_27a) (\lambda V0n \in ty\_2Enum\_2EZERO\_REP : \iota))$ .  
Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \tag{12}$$

**Definition 20** We define  $c\_2Enum\_2E0$  to be  $(ap c\_2Enum\_2EABS\_num c\_2Enum\_2EZERO\_REP)$ .

**Definition 21** We define  $c\_2Earithmetic\_2EZERO$  to be  $c\_2Enum\_2E0$ .

Let  $c\_2Earithmetic\_2E\_2B : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{13}$$

**Definition 22** We define  $c\_2Earithmetic\_2EBIT1$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap (ap c\_2Earithmetic\_2E\_2B) V0n)$ .

**Definition 23** We define  $c\_2Earithmetic\_2ENUMERAL$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.V0x$ .

Let  $c\_2Earithmetic\_2E\_2D : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{14}$$

Let  $c\_2Ellist\_2Ellist\_rep : \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty A\_27a \Rightarrow c\_2Ellist\_2Ellist\_rep A\_27a \in \\ ((ty\_2Eoption\_2Eoption A\_27a)^{ty\_2Enum\_2Enum})^{(ty\_2Ellist\_2Ellist A\_27a)} \end{aligned} \tag{15}$$

**Definition 24** We define  $c\_2Esum\_2EINL$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0e \in A\_27a.(ap (c\_2Esum\_2EABS A\_27a) V0e)$ .

**Definition 25** We define  $c\_2Eoption\_2ESOME$  to be  $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.(ap (c\_2Eoption\_2Eoption A\_27a) V0x)$ .

**Definition 26** We define  $c\_2Ebool\_2ECOND$  to be  $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.(ap (c\_2Ebool\_2ECOND A\_27a) V1t1 V2t2))))$ .

**Definition 27** We define  $c\_2Ellist\_2ELCONS$  to be  $\lambda A\_27a : \iota.\lambda V0h \in A\_27a.\lambda V1t \in (ty\_2Ellist\_2Ellist A\_27a) V1t$ .

**Definition 28** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap V0P (ap (c\_2Emin\_2E\_40 A\_27a) V0P))))$ .

**Definition 29** We define  $c\_2Eprim\_rec\_2E\_3C$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum.V1n$ .

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty\_2Epair\_2Eprod A0 A1) \tag{16}$$

Let  $c\_2Epair\_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epair\_2ESND A\_27a A\_27b \in (A\_27b)^{(ty\_2Epair\_2Eprod A\_27a A\_27b)} \tag{17}$$

Let  $c\_2Epair\_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epair\_2EFST A\_27a A\_27b \in (A\_27a)^{(ty\_2Epair\_2Eprod A\_27a A\_27b)} \tag{18}$$

**Definition 30** We define  $c\_2Epair\_2EUNCURRY$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in ((A\_27c^{A\_27a})$   
 Let  $c\_2Eoption\_2EOPTION\_BIND : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Eoption\_2EOPTION\_BIND \\ & A\_27a\ A\_27b \in (((ty\_2Eoption\_2Eoption\ A\_27a)^{(ty\_2Eoption\_2Eoption\ A\_27a)^{A\_27b}})^{(ty\_2Eoption\_2Eoption\ A\_27a)^{A\_27b}}) \end{aligned} \quad (19)$$

Let  $c\_2Earithmetic\_2EFUNPOW : \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Earithmetic\_2EFUNPOW\ A\_27a \in \\ & (((A\_27a^{A\_27a})^{ty\_2Enum\_2Enum})^{(A\_27a)^{A\_27a}}) \end{aligned} \quad (20)$$

Let  $c\_2Eoption\_2EOPTION\_MAP : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Eoption\_2EOPTION\_MAP \\ & A\_27a\ A\_27b \in (((ty\_2Eoption\_2Eoption\ A\_27b)^{(ty\_2Eoption\_2Eoption\ A\_27a)^{A\_27b}})^{(A\_27b)^{A\_27a}}) \end{aligned} \quad (21)$$

**Definition 31** We define  $c\_2Ellist\_2ELUNFOLD$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in ((ty\_2Eoption\_2Eoption$   
 Let  $c\_2Ellist\_2ELGENLIST : \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ellist\_2ELGENLIST\ A\_27a \in (( \\ & (ty\_2Ellist\_2Ellist\ A\_27a)^{(ty\_2Eoption\_2Eoption\ ty\_2Enum\_2Enum)})^{(A\_27a)^{ty\_2Enum\_2Enum}}) \end{aligned} \quad (22)$$

**Definition 32** We define  $c\_2Emarker\_2ECong$  to be  $\lambda V0x \in 2.V0x$ .

Let  $c\_2Eoption\_2Eoption\_CASE : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Eoption\_2Eoption\_CASE \\ & A\_27a\ A\_27b \in (((A\_27b)^{(A\_27b)^{A\_27a}})^{(A\_27b)^{(ty\_2Eoption\_2Eoption\ A\_27a)}}) \end{aligned} \quad (23)$$

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_prod \\ & A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{(2^{A\_27b})^{A\_27a}}) \end{aligned} \quad (24)$$

**Definition 33** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2$

**Definition 34** We define  $c\_2Epair\_2Epair\_CASE$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0p \in (ty\_2Epair\_$

**Definition 35** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V2t \in$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m) \\ & c\_2Enum\_2E0) = V0m)) \end{aligned} \quad (25)$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
& ((ap (ap c\_2Earithmetic\_2E\_2B c\_2Enum\_2E0) V0m) = V0m) \wedge (((ap ( \\
ap c\_2Earithmetic\_2E\_2B V0m) c\_2Enum\_2E0) = V0m) \wedge (((ap (ap c\_2Earithmetic\_2E\_2B \\
(ap c\_2Enum\_2ESUC V0m)) V1n) = (ap c\_2Enum\_2ESUC (ap (ap c\_2Earithmetic\_2E\_2B \\
V0m) V1n))) \wedge ((ap (ap c\_2Earithmetic\_2E\_2B V0m) (ap c\_2Enum\_2ESUC \\
V1n)) = (ap c\_2Enum\_2ESUC (ap (ap c\_2Earithmetic\_2E\_2B V0m) V1n))))))))) \\
& \tag{26}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
(ap (ap c\_2Earithmetic\_2E\_2B V0m) V1n) = (ap (ap c\_2Earithmetic\_2E\_2B \\
V1n) V0m)))) \\
& \tag{27}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
(ap (ap c\_2Earithmetic\_2E\_2B V0m) V1n) = (ap (ap c\_2Earithmetic\_2E\_2B \\
V1n) V0m)))) \\
& \tag{28}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
\forall V2p \in ty\_2Enum\_2Enum. ((ap (ap c\_2Earithmetic\_2E\_2B V0m) \\
(ap (ap c\_2Earithmetic\_2E\_2B V1n) V2p)) = (ap (ap c\_2Earithmetic\_2E\_2B \\
(ap (ap c\_2Earithmetic\_2E\_2B V0m) V1n)) V2p)))))) \\
& \tag{29}
\end{aligned}$$

Assume the following.

$$\text{True} \tag{30}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p \\
V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \\
& \tag{31}
\end{aligned}$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p V0t))) \tag{32}$$

Assume the following.

$$\forall A\_27a. nonempty A\_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in \\
A\_27a. (p V0t)) \Leftrightarrow (p V0t))) \tag{33}$$

Assume the following.

$$\forall A\_27a. nonempty A\_27a \Rightarrow (\forall V0t \in 2. ((\exists V1x \in \\
A\_27a. (p V0t)) \Leftrightarrow (p V0t))) \tag{34}$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \quad (35)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (36)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t)) \wedge ((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (37)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a.(V0x = V0x)) \quad (38)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \quad (39)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (40)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (41)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (2^{A\_27a}).((\exists V2x \in A\_27a.((p V0P) \wedge (p (ap V1Q V2x)))) \Leftrightarrow ((p V0P) \wedge (\exists V3x \in A\_27a.(p (ap V1Q V3x))))))) \quad (42)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).(\forall V1Q \in 2.((\forall V2x \in A\_27a.((p (ap V0P V2x)) \Rightarrow (p V1Q))) \Leftrightarrow ((\exists V3x \in A\_27a.(p (ap V0P V3x)) \Rightarrow (p V1Q)))))) \quad (43)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p V0A) \vee (p V1B) \vee (p V2C)) \Leftrightarrow (((p V0A) \vee (p V1B)) \vee (p V2C)))))) \quad (44)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p V0A) \vee (p V1B)) \Leftrightarrow ((p V1B) \vee (p V0A)))) \quad (45)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p V0A) \Rightarrow (p V1B)) \Leftrightarrow ((\neg(p V0A)) \vee (p V1B)))) \quad (46)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (47)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in 2.(\forall V1x_{27} \in 2.(\forall V2y \in 2.(\forall V3y_{27} \in \\ & 2.(((p V0x) \Leftrightarrow (p V1x_{27})) \wedge ((p V1x_{27}) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_{27})))))) \Rightarrow \\ & (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_{27}) \Rightarrow (p V3y_{27})))))) \end{aligned} \quad (48)$$

Assume the following.

$$\forall A_{27a}.nonempty A_{27a} \Rightarrow (\forall V0P \in (2^{A_{27a}}).(\forall V1a \in A_{27a}.((\exists V2x \in A_{27a}.((V2x = V1a) \wedge (p (ap V0P V2x)))) \Leftrightarrow (p (ap V0P V1a)))))) \quad (49)$$

Assume the following.

$$\forall A_{27a}.nonempty A_{27a} \Rightarrow (\forall V0f \in (2^{A_{27a}}).(\forall V1v \in A_{27a}.((\forall V2x \in A_{27a}.((V2x = V1v) \Rightarrow (p (ap V0f V2x)))) \Leftrightarrow (p (ap V0f V1v)))))) \quad (50)$$

Assume the following.

$$(\forall V0v \in 2.((p (ap c_2Ebool_2EBOUNDED V0v)) \Leftrightarrow True)) \quad (51)$$

Assume the following.

$$\begin{aligned} & \forall A_{27a}.nonempty A_{27a} \Rightarrow \forall A_{27b}.nonempty A_{27b} \Rightarrow \forall A_{27c}. \\ & nonempty A_{27c} \Rightarrow (\forall V0f \in (A_{27b}^{A_{27a}}).(\forall V1g \in (A_{27a}^{A_{27c}}). \\ & (\forall V2x \in A_{27c}.((ap (ap (ap (c_2Ecombin_2Eo A_{27c} A_{27b} A_{27a}) \\ & V0f) V1g) V2x) = (ap V0f (ap V1g V2x)))))) \end{aligned} \quad (52)$$

Assume the following.

$$\forall A_{27a}.nonempty A_{27a} \Rightarrow (\forall V0x \in A_{27a}.((ap (c_2Ecombin_2EI A_{27a}) V0x) = V0x)) \quad (53)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\ & \forall V0f \in (A\_27b^{A\_27a}).(((ap\ (ap\ (c\_2Ecombin\_2Eo\ A\_27a\ A\_27b \\ & A\_27b)\ (c\_2Ecombin\_2EI\ A\_27b))\ V0f) = V0f) \wedge ((ap\ (ap\ (c\_2Ecombin\_2Eo \\ & A\_27a\ A\_27b\ A\_27a)\ V0f)\ (c\_2Ecombin\_2EI\ A\_27a)) = V0f))) \end{aligned} \quad (54)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0l \in (ty\_2Ellist\_2Ellist \\ & A\_27a).(V0l = (c\_2Ellist\_2ELNIL\ A\_27a)) \vee (\exists V1h \in A\_27a. \\ & (\exists V2t \in (ty\_2Ellist\_2Ellist\ A\_27a).(V0l = (ap\ (ap\ (c\_2Ellist\_2ELCONS \\ & A\_27a)\ V1h)\ V2t)))))) \end{aligned} \quad (55)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0h \in A\_27a.(\forall V1t \in \\ & (ty\_2Ellist\_2Ellist\ A\_27a).(\neg((ap\ (ap\ (c\_2Ellist\_2ELCONS\ A\_27a) \\ & V0h)\ V1t) = (c\_2Ellist\_2ELNIL\ A\_27a))) \wedge (\neg((c\_2Ellist\_2ELNIL \\ & A\_27a) = (ap\ (ap\ (c\_2Ellist\_2ELCONS\ A\_27a)\ V0h)\ V1t)))))) \end{aligned} \quad (56)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0h1 \in A\_27a.(\forall V1t1 \in \\ & (ty\_2Ellist\_2Ellist\ A\_27a).(\forall V2h2 \in A\_27a.(\forall V3t2 \in \\ & (ty\_2Ellist\_2Ellist\ A\_27a).(((ap\ (ap\ (c\_2Ellist\_2ELCONS\ A\_27a) \\ & V0h1)\ V1t1) = (ap\ (ap\ (c\_2Ellist\_2ELCONS\ A\_27a)\ V2h2)\ V3t2)) \Leftrightarrow (( \\ & V0h1 = V2h2) \wedge (V1t1 = V3t2)))))) \end{aligned} \quad (57)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\ & \forall V0f \in ((ty\_2Eoption\_2Eoption\ (ty\_2Epair\_2Eprod\ A\_27a \\ & A\_27b))^{A\_27a}).(\forall V1x \in A\_27a.((ap\ (ap\ (c\_2Ellist\_2ELUNFOLD \\ & A\_27b\ A\_27a)\ V0f)\ V1x) = (ap\ (ap\ (ap\ (c\_2Eoption\_2Eoption\_CASE \\ & (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)\ (ty\_2Ellist\_2Ellist\ A\_27b))\ ( \\ & ap\ V0f\ V1x))\ (c\_2Ellist\_2ELNIL\ A\_27b))\ (\lambda V2v \in (ty\_2Epair\_2Eprod \\ & A\_27a\ A\_27b).(ap\ (ap\ (c\_2Epair\_2Epair\_CASE\ (ty\_2Ellist\_2Ellist \\ & A\_27b)\ A\_27a\ A\_27b)\ V2v)\ (\lambda V3v1 \in A\_27a.(\lambda V4v2 \in A\_27b.(ap \\ & (ap\ (c\_2Ellist\_2ELCONS\ A\_27b)\ V4v2)\ (ap\ (ap\ (c\_2Ellist\_2ELUNFOLD \\ & A\_27b\ A\_27a)\ V0f)\ V3v1)))))))))) \end{aligned} \quad (58)$$



Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0ll1 \in (ty\_2Ellist\_2Ellist \\
& \quad A.27a).(\forall V1ll2 \in (ty\_2Ellist\_2Ellist\ A.27a).((V0ll1 = \\
& \quad V1ll2) \Leftrightarrow (\exists V2R \in ((2^{(ty\_2Ellist\_2Ellist\ A.27a)})(ty\_2Ellist\_2Ellist\ A.27a)). \\
& \quad ((p\ (ap\ (ap\ V2R\ V0ll1)\ V1ll2)) \wedge (\forall V3ll3 \in (ty\_2Ellist\_2Ellist \\
& \quad A.27a).(\forall V4ll4 \in (ty\_2Ellist\_2Ellist\ A.27a).((p\ (ap\ (ap \\
& \quad V2R\ V3ll3)\ V4ll4)) \Rightarrow ((V3ll3 = V4ll4) \vee (\exists V5h \in A.27a.(\exists V6t1 \in \\
& \quad (ty\_2Ellist\_2Ellist\ A.27a).(\exists V7t2 \in (ty\_2Ellist\_2Ellist \\
& \quad A.27a).((V3ll3 = (ap\ (ap\ (c\_2Ellist\_2ELCONS\ A.27a)\ V5h)\ V6t1))) \wedge \\
& \quad ((V4ll4 = (ap\ (ap\ (c\_2Ellist\_2ELCONS\ A.27a)\ V5h)\ V7t2)) \wedge (p\ (ap\ ( \\
& \quad \quad ap\ V2R\ V6t1)\ V7t2)))))))))))))))))
\end{aligned} \tag{59}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow ((\forall V0f \in (A.27a^{ty\_2Enum\_2Enum}). \\
& \quad ((ap\ (ap\ (c\_2Ellist\_2ELGENLIST\ A.27a)\ V0f)\ (c\_2Eoption\_2ENONE \\
& \quad ty\_2Enum\_2Enum)) = (ap\ (ap\ (c\_2Ellist\_2ELUNFOLD\ A.27a\ ty\_2Enum\_2Enum) \\
& \quad (\lambda V1n \in ty\_2Enum\_2Enum.(ap\ (c\_2Eoption\_2ESOME\ (ty\_2Epair\_2Eprod \\
& \quad ty\_2Enum\_2Enum\ A.27a))\ (ap\ (ap\ (c\_2Epair\_2E\_2C\ ty\_2Enum\_2Enum \\
& \quad A.27a)\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V1n)\ (ap\ c\_2Earithmetic\_2ENUMERAL \\
& \quad (ap\ c\_2Earithmetic\_2EBIT1\ c\_2Earithmetic\_2EZERO))))\ (ap\ V0f \\
& \quad V1n))))\ c\_2Enum\_2E0))) \wedge (\forall V2f \in (A.27a^{ty\_2Enum\_2Enum}). \\
& \quad (\forall V3lim \in ty\_2Enum\_2Enum.((ap\ (ap\ (c\_2Ellist\_2ELGENLIST \\
& \quad A.27a)\ V2f)\ (ap\ (c\_2Eoption\_2ESOME\ ty\_2Enum\_2Enum)\ V3lim)) = ( \\
& \quad ap\ (ap\ (c\_2Ellist\_2ELUNFOLD\ A.27a\ ty\_2Enum\_2Enum)\ (\lambda V4n \in ty\_2Enum\_2Enum. \\
& \quad (ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ (ty\_2Eoption\_2Eoption\ (ty\_2Epair\_2Eprod \\
& \quad ty\_2Enum\_2Enum\ A.27a)))\ (ap\ (ap\ c\_2Eprim\_rec\_2E\_3C\ V4n)\ V3lim)) \\
& \quad (ap\ (c\_2Eoption\_2ESOME\ (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ A.27a)) \\
& \quad (ap\ (ap\ (c\_2Epair\_2E\_2C\ ty\_2Enum\_2Enum\ A.27a)\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
& \quad V4n)\ (ap\ c\_2Earithmetic\_2ENUMERAL\ (ap\ c\_2Earithmetic\_2EBIT1 \\
& \quad c\_2Earithmetic\_2EZERO))))\ (ap\ V2f\ V4n))))\ (c\_2Eoption\_2ENONE \\
& \quad (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ A.27a))))\ c\_2Enum\_2E0))))))
\end{aligned} \tag{60}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow ( \\
& \quad (\forall V0v \in A.27b.(\forall V1f \in (A.27b^{A.27a}).((ap\ (ap\ (ap\ (c\_2Eoption\_2Eoption\_CASE \\
& \quad A.27a\ A.27b)\ (c\_2Eoption\_2ENONE\ A.27a))\ V0v)\ V1f) = V0v))) \wedge (\forall V2x \in \\
& \quad A.27a.(\forall V3v \in A.27b.(\forall V4f \in (A.27b^{A.27a}).((ap\ (ap \\
& \quad (ap\ (c\_2Eoption\_2Eoption\_CASE\ A.27a\ A.27b)\ (ap\ (c\_2Eoption\_2ESOME \\
& \quad A.27a)\ V2x))\ V3v)\ V4f) = (ap\ V4f\ V2x))))))
\end{aligned} \tag{61}$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow \forall A.27c. \\ & \quad nonempty\ A.27c \Rightarrow (\forall V0x \in A.27b. (\forall V1y \in A.27c. (\forall V2f \in \\ & \quad ((A.27a^{A.27c})^{A.27b}). ((ap\ (ap\ (c.2Epair.2Epair\_CASE\ A.27a\ A.27b \\ & \quad A.27c)\ (ap\ (ap\ (c.2Epair.2E.2C\ A.27b\ A.27c)\ V0x)\ V1y))\ V2f) = (ap \\ & \quad (ap\ V2f\ V0x)\ V1y)))))) \end{aligned} \quad (62)$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \quad (63)$$

Assume the following.

$$(\forall V0A \in 2. ((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \quad (64)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\ & \quad ((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))))) \end{aligned} \quad (65)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(\neg(p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\ & \quad ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))))) \end{aligned} \quad (66)$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p\ V0A)) \Rightarrow False) \Rightarrow (((p\ V0A) \Rightarrow False) \Rightarrow False))) \quad (67)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p\ V0p) \Leftrightarrow ( \\ & \quad (p\ V1q) \Leftrightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((p\ V1q) \vee (p\ V2r))) \wedge (((p\ V0p) \vee ((\neg \\ & \quad p\ V2r) \vee (\neg(p\ V1q)))) \wedge ((p\ V1q) \vee ((\neg(p\ V2r)) \vee (\neg(p\ V0p)))) \wedge ((p\ V2r) \vee \\ & \quad ((\neg(p\ V1q)) \vee (\neg(p\ V0p)))))))))) \end{aligned} \quad (68)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p\ V0p) \Leftrightarrow ( \\ & \quad (p\ V1q) \wedge (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((\neg(p\ V1q)) \vee (\neg(p\ V2r)))) \wedge (((p\ V1q) \vee \\ & \quad (\neg(p\ V0p))) \wedge ((p\ V2r) \vee (\neg(p\ V0p)))))))))) \end{aligned} \quad (69)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p\ V0p) \Leftrightarrow ( \\ & \quad (p\ V1q) \vee (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee (\neg(p\ V1q))) \wedge (((p\ V0p) \vee (\neg(p\ V2r))) \wedge \\ & \quad ((p\ V1q) \vee ((p\ V2r) \vee (\neg(p\ V0p)))))))))) \end{aligned} \quad (70)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \Rightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee \neg(p V2r))) \wedge (\neg(p V1q) \vee ((p V2r) \vee \neg(p V0p)))))))) \quad (71)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow \neg(p V1q)) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (\neg(p V1q) \vee \neg(p V0p)))))) \quad (72)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p)))) \quad (73)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow \neg(p V1q)))) \quad (74)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \vee (p V1q))) \Rightarrow \neg(p V0p)))) \quad (75)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \vee (p V1q))) \Rightarrow \neg(p V1q)))) \quad (76)$$

Assume the following.

$$(\forall V0p \in 2. ((\neg(\neg(p V0p))) \Rightarrow (p V0p))) \quad (77)$$

### Theorem 1

$$\begin{aligned} & \forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0f \in (A.27a^{ty.2Enum.2Enum}). \\ & \quad (\forall V1h \in A.27a. (\forall V2t \in (ty.2Ellist.2Ellist \ A.27a). \\ & \quad ((ap (ap (c.2Ellist.2ELGENLIST \ A.27a) \ V0f) (c.2Eoption.2ENONE \\ & \quad ty.2Enum.2Enum)) = (ap (ap (c.2Ellist.2ELCONS \ A.27a) \ V1h) \ V2t)) \Leftrightarrow \\ & \quad ((V1h = (ap \ V0f \ c.2Enum.2E0)) \wedge (V2t = (ap (ap (c.2Ellist.2ELGENLIST \\ & \quad A.27a) (ap (ap (c.2Ecombin.2Eo \ ty.2Enum.2Enum \ A.27a \ ty.2Enum.2Enum) \\ & \quad V0f) (ap \ c.2Earithmetic.2E.2B (ap \ c.2Earithmetic.2ENUMERAL ( \\ & \quad ap \ c.2Earithmetic.2EBIT1 \ c.2Earithmetic.2EZERO)))))) (c.2Eoption.2ENONE \\ & \quad ty.2Enum.2Enum)))))) \end{aligned}$$