

thm_2Ellist_2ELGENLIST__EQ__LNIL
(TMHwq9LMLQsdoDpz7tzTJBivpzyH2rSu67k)

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Definition 1 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 2 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a}))$

Definition 5 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 6 We define $c_2Ebool_2EBOUNDED$ to be $(\lambda V0v \in 2.c_2Ebool_2ET)$.

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{1}$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{2}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{3}$$

Definition 7 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 8 We define $c_2Earithmic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \tag{4}$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \tag{5}$$

Definition 9 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (6)$$

Definition 10 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ (ap\ c_2Earithmetic$

Definition 11 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let $c_2Earithmetic_2E_2D : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2D \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (7)$$

Let $ty_2Eoption_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Eoption_2Eoption\ A0) \quad (8)$$

Let $ty_2Ellist_2Ellist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Ellist_2Ellist\ A0) \quad (9)$$

Let $c_2Ellist_2Ellist_rep : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ellist_2Ellist_rep\ A_27a \in \\ (((ty_2Eoption_2Eoption\ A_27a)^{ty_2Enum_2Enum})^{(ty_2Ellist_2Ellist\ A_27a)}) \quad (10)$$

Let $ty_2Eone_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eone_2Eone \quad (11)$$

Definition 12 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in$

Let $ty_2Esum_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Esum_2Esum\ A0\ A1) \quad (12)$$

Let $c_2Esum_2EABS_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Esum_2EABS_sum\ A_27a\ A_27b \in ((ty_2Esum_2Esum\ A_27a\ A_27b)^{((2^{A_27b})^{A_27a})^2}) \quad (13)$$

Definition 13 We define c_2Esum_2EINL to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0e \in A_27a.(ap\ (c_2Esum_2EABS$

Let $c_2Eoption_2Eoption_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Eoption_2Eoption_ABS\ A_27a \in \\ ((ty_2Eoption_2Eoption\ A_27a)^{(ty_2Esum_2Esum\ A_27a\ ty_2Eone_2Eone)}) \quad (14)$$

Definition 14 We define $c_2Eoption_2ESOME$ to be $\lambda A_27a : \iota.\lambda V0x \in A_27a.(ap (c_2Eoption_2Eoption_2ESOME A_27a) x)$

Definition 15 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.\text{if } (\exists x \in A.p (ap P x)) \text{ then } (the (\lambda x.x \in A.\lambda P x))$ of type $\iota \Rightarrow \iota$.

Definition 16 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.(ap (c_2Ebool_2ECOND A_27a) t1) t2)))$

Let $c_2Ellist_2Ellist_abs : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Ellist_2Ellist_abs A_27a \in ((ty_2Ellist_2Ellist A_27a)^{(ty_2Eoption_2Eoption A_27a)^{ty_2Enum_2Enum}}) \quad (15)$$

Definition 17 We define $c_2Ellist_2ELCONS$ to be $\lambda A_27a : \iota.\lambda V0h \in A_27a.\lambda V1t \in (ty_2Ellist_2Ellist A_27a) (ap (c_2Ellist_2ELCONS A_27a) h) t$

Definition 18 We define c_2Eone_2Eone to be $(ap (c_2Emin_2E_40 ty_2Eone_2Eone) (\lambda V0x \in ty_2Eone_2Eone. x))$

Definition 19 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2ECOND A_27a) t)$

Definition 20 We define c_2Esum_2EINR to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0e \in A_27b.(ap (c_2Esum_2EABS A_27a) e)$

Definition 21 We define $c_2Eoption_2ENONE$ to be $\lambda A_27a : \iota.(ap (c_2Eoption_2Eoption_2ABS A_27a) (c_2Eoption_2ENONE A_27a))$

Definition 22 We define $c_2Ellist_2ELNIL$ to be $\lambda A_27a : \iota.(ap (c_2Ellist_2Ellist_abs A_27a) (\lambda V0n \in ty_2Ellist_2Ellist A_27a. n))$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \quad (16)$$

Definition 23 We define $c_2Ecombin_2EK$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0x \in A_27a.(\lambda V1y \in A_27b.V0x))$

Definition 24 We define $c_2Ecombin_2Eo$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in (A_27b^{A_27c}).\lambda V1x \in A_27a. f x$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2ESND A_27a A_27b \in (A_27b^{(ty_2Epair_2Eprod A_27a A_27b)}) \quad (17)$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EFST A_27a A_27b \in (A_27a^{(ty_2Epair_2Eprod A_27a A_27b)}) \quad (18)$$

Definition 25 We define $c_2Epair_2EUNCURRY$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in ((A_27c^{A_27b})^A_27a)$

Let $c_2Eoption_2EOPTION_BIND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Eoption_2EOPTION_BIND \\ & A_27a\ A_27b \in (((ty_2Eoption_2Eoption\ A_27a)^{(ty_2Eoption_2Eoption\ A_27a)^{A_27b}})^{(ty_2Eoption_2Eoption\ A_27a)}) \end{aligned} \quad (19)$$

Let $c_2Earithmetic_2EFUNPOW : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow c_2Earithmetic_2EFUNPOW\ A_27a \in \\ & (((A_27a^{A_27a})^{ty_2Enum_2Enum})^{(A_27a^{A_27a})}) \end{aligned} \quad (20)$$

Let $c_2Eoption_2EOPTION_MAP : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Eoption_2EOPTION_MAP \\ & A_27a\ A_27b \in (((ty_2Eoption_2Eoption\ A_27b)^{(ty_2Eoption_2Eoption\ A_27a)})^{(A_27b^{A_27a})}) \end{aligned} \quad (21)$$

Definition 26 We define $c_2Ellist_2ELUNFOLD$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in ((ty_2Eoption_2Eoption$

Let $c_2Ellist_2ELGENLIST : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow c_2Ellist_2ELGENLIST\ A_27a \in ((\\ & (ty_2Ellist_2Ellist\ A_27a)^{(ty_2Eoption_2Eoption\ ty_2Enum_2Enum)})^{(A_27a^{ty_2Enum_2Enum})}) \end{aligned} \quad (22)$$

Definition 27 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ (c_2Emin_2E_40$

Definition 28 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in$

Let $c_2Eoption_2Eoption_CASE : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Eoption_2Eoption_CASE \\ & A_27a\ A_27b \in (((A_27b^{(A_27b^{A_27a})})^{A_27b})^{(ty_2Eoption_2Eoption\ A_27a)}) \end{aligned} \quad (23)$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod \\ & A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \end{aligned} \quad (24)$$

Definition 29 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap\ (c_2$

Definition 30 We define $c_2Epair_2Epair_CASE$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0p \in (ty_2Epair_2E$

Definition 31 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum.((ap\ (ap\ c_2Earithmetic_2E_2B\ V0m) \\ & c_2Enum_2E0) = V0m)) \end{aligned} \quad (25)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (ap (ap c_2Earithmetic_2E_2B V0m) V1n) = (ap (ap c_2Earithmetic_2E_2B V1n) V0m)))) \quad (26)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum. ((V0m = c_2Enum_2E0) \vee (\exists V1n \in ty_2Enum_2Enum. (V0m = (ap c_2Enum_2ESUC V1n)))))) \quad (27)$$

Assume the following.

$$True \quad (28)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (29)$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p V0t))) \quad (30)$$

Assume the following.

$$((\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (31)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (32)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (33)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (34)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (35)$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in 2. \\
& (\forall V2x \in A_27a. (\forall V3x_27 \in A_27a. (\forall V4y \in A_27a. \\
& (\forall V5y_27 \in A_27a. (((p\ V0P) \Leftrightarrow (p\ V1Q)) \wedge ((p\ V1Q) \Rightarrow (V2x = V3x_27)) \wedge \\
& ((\neg(p\ V1Q)) \Rightarrow (V4y = V5y_27)))))) \Rightarrow ((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a) \\
& V0P)\ V2x)\ V4y) = (ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ V1Q)\ V3x_27) \\
& V5y_27)))))))))
\end{aligned} \tag{36}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow ((\forall V0t1 \in A_27a. (\forall V1t2 \in \\
& A_27a. ((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ c_2Ebool_2ET)\ V0t1) \\
& V1t2) = V0t1))) \wedge (\forall V2t1 \in A_27a. (\forall V3t2 \in A_27a. ((ap \\
& (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ c_2Ebool_2EF)\ V2t1)\ V3t2) = V3t2))))))
\end{aligned} \tag{37}$$

Assume the following.

$$(\forall V0v \in 2. ((p\ (ap\ c_2Ebool_2EBOUNDED\ V0v)) \Leftrightarrow True)) \tag{38}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0h \in A_27a. (\forall V1t \in \\
& (ty_2Ellist_2Ellist\ A_27a). ((\neg((ap\ (ap\ (c_2Ellist_2ELCONS\ A_27a) \\
& V0h)\ V1t) = (c_2Ellist_2ELNIL\ A_27a)))) \wedge (\neg((c_2Ellist_2ELNIL \\
& A_27a) = (ap\ (ap\ (c_2Ellist_2ELCONS\ A_27a)\ V0h)\ V1t))))))
\end{aligned} \tag{39}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \forall V0f \in ((ty_2Eoption_2Eoption\ (ty_2Epair_2Eprod\ A_27a \\
& A_27b))^{A_27a}). (\forall V1x \in A_27a. ((ap\ (ap\ (c_2Ellist_2ELUNFOLD \\
& A_27b\ A_27a)\ V0f)\ V1x) = (ap\ (ap\ (ap\ (c_2Eoption_2Eoption_CASE \\
& (ty_2Epair_2Eprod\ A_27a\ A_27b)\ (ty_2Ellist_2Ellist\ A_27b))\ (\\
& ap\ V0f\ V1x))\ (c_2Ellist_2ELNIL\ A_27b))\ (\lambda V2v \in (ty_2Epair_2Eprod \\
& A_27a\ A_27b). (ap\ (ap\ (c_2Epair_2Epair_CASE\ (ty_2Ellist_2Ellist \\
& A_27b)\ A_27a\ A_27b)\ V2v)\ (\lambda V3v1 \in A_27a. (\lambda V4v2 \in A_27b. (ap \\
& (ap\ (c_2Ellist_2ELCONS\ A_27b)\ V4v2)\ (ap\ (ap\ (c_2Ellist_2ELUNFOLD \\
& A_27b\ A_27a)\ V0f)\ V3v1))))))))))
\end{aligned} \tag{40}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.\text{nonempty } A_27a \Rightarrow ((\forall V0f \in (A_27a^{ty_2Enum_2Enum}). \\
& ((ap (ap (c_2Ellist_2ELGENLIST A_27a) V0f) (c_2Eoption_2ENONE \\
& ty_2Enum_2Enum))) = (ap (ap (c_2Ellist_2ELUNFOLD A_27a ty_2Enum_2Enum) \\
& (\lambda V1n \in ty_2Enum_2Enum.(ap (c_2Eoption_2ESOME (ty_2Epair_2Eprod \\
& ty_2Enum_2Enum A_27a)) (ap (ap (c_2Epair_2E_2C ty_2Enum_2Enum \\
& A_27a) (ap (ap c_2Earithmetic_2E_2B V1n) (ap c_2Earithmetic_2ENUMERAL \\
& (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))))) (ap V0f \\
& V1n)))))) c_2Enum_2E0))) \wedge (\forall V2f \in (A_27a^{ty_2Enum_2Enum}). \\
& (\forall V3lim \in ty_2Enum_2Enum.((ap (ap (c_2Ellist_2ELGENLIST \\
& A_27a) V2f) (ap (c_2Eoption_2ESOME ty_2Enum_2Enum) V3lim)) = (\\
ap (ap (c_2Ellist_2ELUNFOLD A_27a ty_2Enum_2Enum) (\lambda V4n \in ty_2Enum_2Enum. \\
& (ap (ap (ap (c_2Ebool_2ECOND (ty_2Eoption_2Eoption (ty_2Epair_2Eprod \\
& ty_2Enum_2Enum A_27a))) (ap (ap c_2Eprim_rec_2E_2C V4n) V3lim)) \\
& (ap (c_2Eoption_2ESOME (ty_2Epair_2Eprod ty_2Enum_2Enum A_27a)) \\
& (ap (ap (c_2Epair_2E_2C ty_2Enum_2Enum A_27a) (ap (ap c_2Earithmetic_2E_2B \\
& V4n) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\
& c_2Earithmetic_2EZERO)))))) (ap V2f V4n)))))) (c_2Eoption_2ENONE \\
& (ty_2Epair_2Eprod ty_2Enum_2Enum A_27a)))))) c_2Enum_2E0)))))) \\
& \hspace{15em} (41)
\end{aligned}$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum. (\neg((ap c_2Enum_2ESUC V0n) = c_2Enum_2E0))) \quad (42)$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0opt \in (ty_2Eoption_2Eoption \\
& A_27a). ((V0opt = (c_2Eoption_2ENONE A_27a)) \vee (\exists V1x \in A_27a. \\
& (V0opt = (ap (c_2Eoption_2ESOME A_27a) V1x)))))) \quad (43)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.\text{nonempty } A_27a \Rightarrow \forall A_27b.\text{nonempty } A_27b \Rightarrow (\\
& (\forall V0v \in A_27b. (\forall V1f \in (A_27b^{A_27a}). ((ap (ap (ap (c_2Eoption_2Eoption_2CASE \\
& A_27a A_27b) (c_2Eoption_2ENONE A_27a) V0v) V1f) = V0v))) \wedge (\forall V2x \in \\
& A_27a. (\forall V3v \in A_27b. (\forall V4f \in (A_27b^{A_27a}). ((ap (ap \\
& (ap (c_2Eoption_2Eoption_2CASE A_27a A_27b) (ap (c_2Eoption_2ESOME \\
& A_27a) V2x)) V3v) V4f) = (ap V4f V2x))))))))) \quad (44)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in \\
& A_27a. (((ap (c_2Eoption_2ESOME A_27a) V0x) = (ap (c_2Eoption_2ESOME \\
& A_27a) V1y)) \Leftrightarrow (V0x = V1y)))) \quad (45)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0x \in A_27a. (\neg((c_2Eoption_2ENONE \\
& A_27a) = (ap (c_2Eoption_2ESOME A_27a) V0x)))) \quad (46)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\
& \quad nonempty\ A_27c \Rightarrow (\forall V0x \in A_27b. (\forall V1y \in A_27c. (\forall V2f \in \\
& \quad ((A_27a^{A_27c})^{A_27b}). ((ap\ (ap\ (c_2Epair_2Epair_CASE\ A_27a\ A_27b \\
& \quad A_27c)\ (ap\ (ap\ (c_2Epair_2E_2C\ A_27b\ A_27c)\ V0x)\ V1y))\ V2f) = (ap \\
& \quad (ap\ V2f\ V0x)\ V1y))))))
\end{aligned} \tag{47}$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum. (\neg (p\ (ap\ (ap\ c_2Eprim_rec_2E_3C\ V0n)\ c_2Enum_2E0)))) \tag{48}$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum. (p\ (ap\ (ap\ c_2Eprim_rec_2E_3C\ c_2Enum_2E0)\ (ap\ c_2Enum_2ESUC\ V0n)))) \tag{49}$$

Theorem 1

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0f \in (A_27a^{ty_2Enum_2Enum}). \\
& \quad (\forall V1n \in (ty_2Eoption_2Eoption\ ty_2Enum_2Enum). (((ap \\
& \quad (ap\ (c_2Ellist_2ELGENLIST\ A_27a)\ V0f)\ V1n) = (c_2Ellist_2ELNIL \\
& \quad A_27a)) \Leftrightarrow (V1n = (ap\ (c_2Eoption_2ESOME\ ty_2Enum_2Enum)\ c_2Enum_2E0)))) \wedge \\
& \quad (((c_2Ellist_2ELNIL\ A_27a) = (ap\ (ap\ (c_2Ellist_2ELGENLIST\ A_27a)\ \\
& \quad V0f)\ V1n)) \Leftrightarrow (V1n = (ap\ (c_2Eoption_2ESOME\ ty_2Enum_2Enum)\ c_2Enum_2E0))))))
\end{aligned}$$