

thm\_2Ellist\_2ELHDTL\_\_CONS\_\_THM  
 (TMWxNZHD-  
 CXe11RUayMVqgWy4rFeEUNcgkmh)

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**Definition 1** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p \ P \Rightarrow p \ Q)$  of type  $\iota$ .

**Definition 2** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 3** We define  $c\_2Ebool\_2ET$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 4** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a})) (\lambda V1x \in 2.V1x)) (\lambda V2x \in 2.V2x)))$

**Definition 5** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V2t))))$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \quad (1)$$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty \ ty\_2Enum\_2Enum \quad (2)$$

Let  $c\_2Enum\_2EAABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EAABS\_num \in (ty\_2Enum\_2Enum)^{\omega} \quad (3)$$

**Definition 6** We define  $c\_2Enum\_2E0$  to be  $(ap c\_2Enum\_2EAABS\_num c\_2Enum\_2EZERO\_REP)$ .

Let  $ty\_2Eoption\_2Eoption : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty\_2Eoption\_2Eoption A0) \quad (4)$$

Let  $ty\_2Ellist\_2Ellist : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty\_2Ellist\_2Ellist A0) \quad (5)$$

Let  $c\_2Ellist\_2Ellist\_rep : \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty A\_27a \Rightarrow c\_2Ellist\_2Ellist\_rep A\_27a \in \\ & ((ty\_2Eoption\_2Eoption A\_27a)^{ty\_2Enum\_2Enum})^{(ty\_2Ellist\_2Ellist A\_27a)} \end{aligned} \quad (6)$$

**Definition 7** We define  $c\_2Ellist\_2ELHD$  to be  $\lambda A\_27a : \iota. \lambda V0l \in (ty\_2Ellist\_2Ellist\ A\_27a).(ap\ (ap\ (c\_2E$

**Definition 8** We define  $c\_2Earithmetic\_2EZERO$  to be  $c\_2Enum\_2E0$ .

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \quad (7)$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \quad (8)$$

**Definition 9** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_num\ m)$

Let  $c\_2Earithmetic\_2E\_2B : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (9)$$

**Definition 10** We define  $c\_2Earithmetic\_2EBIT1$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmeti$

**Definition 11** We define  $c\_2Earithmetic\_2ENUMERAL$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.V0x$ .

Let  $c\_2Earithmetic\_2E\_2D : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (10)$$

Let  $ty\_2Eone\_2Eone : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Eone\_2Eone \quad (11)$$

Let  $ty\_2Esum\_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A0.nonempty\ A0 \Rightarrow & \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Esum\_2Esum \\ & A0\ A1) \end{aligned} \quad (12)$$

Let  $c\_2Esum\_2EABS\_sum : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow & \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Esum\_2EABS\_sum \\ & A\_27a\ A\_27b \in ((ty\_2Esum\_2Esum\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})^2}) \end{aligned} \quad (13)$$

**Definition 12** We define  $c\_2Esum\_2EINL$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0e \in A\_27a.(ap\ (c\_2Esum\_2EABS$

Let  $c\_2Eoption\_2Eoption\_ABS : \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow & c\_2Eoption\_2Eoption\_ABS\ A\_27a \in \\ & ((ty\_2Eoption\_2Eoption\ A\_27a)^{(ty\_2Esum\_2Esum\ A\_27a\ ty\_2Eone\_2Eone)}) \end{aligned} \quad (14)$$

**Definition 13** We define  $c\_2Eoption\_2ESOME$  to be  $\lambda A\_27a : \iota. \lambda V0x \in A\_27a.(ap\ (c\_2Eoption\_2Eoption\_$

**Definition 14** We define  $c\_2Ebool\_2EF$  to be  $(ap (c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 15** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.\text{if } (\exists x \in A.p (ap P x)) \text{ then } (\text{the } (\lambda x.x \in A \wedge p x)) \text{ of type } \iota \Rightarrow \iota$ .

**Definition 16** We define  $c\_2Ebool\_2ECOND$  to be  $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.(ap (c\_2Ebool\_2EF V0t) V1t1) V2t2)))$ .

Let  $c\_2Ellist\_2Ellist\_abs : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.\text{nonempty } A\_27a \Rightarrow c\_2Ellist\_2Ellist\_abs A\_27a \in ((ty\_2Ellist\_2Ellist A\_27a)^{(ty\_2Eoption\_2Eoption A\_27a)^{ty\_2Eenum\_2Eenum}}) \quad (15)$$

**Definition 17** We define  $c\_2Ellist\_2ELCONS$  to be  $\lambda A\_27a : \iota.\lambda V0h \in A\_27a.\lambda V1t \in (ty\_2Ellist\_2Ellist A\_27a).(\lambda V2t \in 2.(ap (c\_2Ellist\_2ELCONS V0h V1t) V2t))$ .

**Definition 18** We define  $c\_2Eone\_2Eone$  to be  $(ap (c\_2Emin\_2E\_40 ty\_2Eone\_2Eone) (\lambda V0x \in ty\_2Eone\_2Eone.(\lambda V1x \in 2.(ap (c\_2Eone\_2Eone V0x) V1x))))$ .

**Definition 19** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2EF)))$ .

**Definition 20** We define  $c\_2Esum\_2EINR$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0e \in A\_27b.(ap (c\_2Esum\_2EABS A\_27a V0e) V0e))$ .

**Definition 21** We define  $c\_2Eoption\_2ENONE$  to be  $\lambda A\_27a : \iota.(ap (c\_2Eoption\_2Eoption\_ABS A\_27a) V0a))$ .

Let  $c\_2Eoption\_2Eoption\_CASE : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.\text{nonempty } A\_27a \Rightarrow \forall A\_27b.\text{nonempty } A\_27b \Rightarrow c\_2Eoption\_2Eoption\_CASE A\_27a A\_27b \in (((A\_27b^{(A\_27b^{A\_27a})})^{A\_27b})^{(ty\_2Eoption\_2Eoption A\_27a)}) \quad (16)$$

**Definition 22** We define  $c\_2Ellist\_2ELTL$  to be  $\lambda A\_27a : \iota.\lambda V0ll \in (ty\_2Ellist\_2Ellist A\_27a).(ap (ap (ap (c\_2Ellist\_2ELTL V0ll) V0ll)))$ .

Assume the following.

$$True \quad (17)$$

Assume the following.

$$\forall A\_27a.\text{nonempty } A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \quad (18)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.\text{nonempty } A\_27a \Rightarrow & (\forall V0h \in A\_27a.(\forall V1t \in \\ & (ty\_2Ellist\_2Ellist A\_27a).((ap (c\_2Ellist\_2ELHD A\_27a) (ap \\ & (ap (c\_2Ellist\_2ELCONS A\_27a) V0h) V1t)) = (ap (c\_2Eoption\_2ESOME \\ & A\_27a) V0h)))) \end{aligned} \quad (19)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.\text{nonempty } A\_27a \Rightarrow & (\forall V0h \in A\_27a.(\forall V1t \in \\ & (ty\_2Ellist\_2Ellist A\_27a).((ap (c\_2Ellist\_2ELTL A\_27a) (ap \\ & (ap (c\_2Ellist\_2ELCONS A\_27a) V0h) V1t)) = (ap (c\_2Eoption\_2ESOME \\ & (ty\_2Ellist\_2Ellist A\_27a)) V1t)))) \end{aligned} \quad (20)$$

Assume the following.

$$\begin{aligned} \forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in \\ A_27a.(((ap(c_2Eoption_2ESOME A_27a) V0x) = (ap(c_2Eoption_2ESOME \\ A_27a) V1y)) \Leftrightarrow (V0x = V1y)))) \end{aligned} \quad (21)$$

**Theorem 1**

$$\begin{aligned} \forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0h \in A_27a.(\forall V1t \in \\ (ty_2Ellist_2Ellist A_27a).(((ap(c_2Ellist_2ELHD A_27a) (ap \\ (ap(c_2Ellist_2ELCONS A_27a) V0h) V1t)) = (ap(c_2Eoption_2ESOME \\ A_27a) V0h)) \wedge ((ap(c_2Ellist_2ELTL A_27a) (ap(ap(c_2Ellist_2ELCONS \\ A_27a) V0h) V1t)) = (ap(c_2Eoption_2ESOME (ty_2Ellist_2Ellist \\ A_27a)) V1t))))))) \end{aligned}$$