

thm\_2Ellist\_2ELHDTL\_EQ\_SOME  
(TMVyNprRksfdyUDgfCCP6jfu7k5yobccjUs)

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**Definition 1** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 2** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 3** We define  $c\_2Ebool\_2E\_2T$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$ .

**Definition 4** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a}))$

**Definition 5** We define  $c\_2Ebool\_2E\_2F$  to be  $(ap (c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 6** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) \text{ then } (the (\lambda x.x \in A \wedge p (ap P x)))$  of type  $\iota \Rightarrow \iota$ .

**Definition 7** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap V0P (ap (c\_2Emin\_2E\_40 A$

**Definition 8** We define  $c\_2Ebool\_2E\_5C\_2E\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V2t)))$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \tag{1}$$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{2}$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \tag{3}$$

**Definition 9** We define  $c\_2Enum\_2E0$  to be  $(ap c\_2Enum\_2EABS\_num c\_2Enum\_2EZERO\_REP)$ .

Let  $ty\_2Eoption\_2Eoption : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Eoption\_2Eoption\ A0) \quad (4)$$

Let  $ty\_2Ellist\_2Ellist : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Ellist\_2Ellist\ A0) \quad (5)$$

Let  $c\_2Ellist\_2Ellist\_rep : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ellist\_2Ellist\_rep\ A\_27a \in ((ty\_2Eoption\_2Eoption\ A\_27a)^{ty\_2Enum\_2Enum})^{(ty\_2Ellist\_2Ellist\ A\_27a)} \quad (6)$$

**Definition 10** We define  $c\_2Ellist\_2ELHD$  to be  $\lambda A\_27a : \iota.\lambda V0l \in (ty\_2Ellist\_2Ellist\ A\_27a).(ap\ (ap\ (c\_2Ellist\_2Ellist\_rep\ A\_27a)\ V0l)\ l)$

**Definition 11** We define  $c\_2Earithmetic\_2EZERO$  to be  $c\_2Enum\_2E0$ .

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \quad (7)$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \quad (8)$$

**Definition 12** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_num\ m)$

Let  $c\_2Earithmetic\_2E\_2B : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum)^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum} \quad (9)$$

**Definition 13** We define  $c\_2Earithmetic\_2EBIT1$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic\_2E\_2B\ n)\ 1)$

**Definition 14** We define  $c\_2Earithmetic\_2ENUMERAL$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.V0x$ .

Let  $c\_2Ellist\_2Ellist\_abs : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ellist\_2Ellist\_abs\ A\_27a \in ((ty\_2Ellist\_2Ellist\ A\_27a)^{(ty\_2Eoption\_2Eoption\ A\_27a)^{ty\_2Enum\_2Enum}}) \quad (10)$$

Let  $ty\_2Eone\_2Eone : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Eone\_2Eone \quad (11)$$

**Definition 15** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_2Ebool\_2E\_21\ 2)\ t2)\ t1))$

Let  $ty\_2Esum\_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Esum\_2Esum\ A0\ A1) \quad (12)$$

Let  $c\_2Esum\_2EABS\_sum : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Esum\_2EABS\_sum\ A\_27a\ A\_27b \in ((ty\_2Esum\_2Esum\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})^2}) \quad (13)$$

**Definition 16** We define  $c\_2Esum\_2EINL$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0e \in A\_27a. (ap\ (c\_2Esum\_2EABS\_sum\ A\_27a\ A\_27b)\ V0e)$

Let  $c\_2Eoption\_2Eoption\_ABS : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Eoption\_2Eoption\_ABS\ A\_27a \in ((ty\_2Eoption\_2Eoption\ A\_27a)^{ty\_2Esum\_2Esum\ A\_27a\ ty\_2Eone\_2Eone}) \quad (14)$$

**Definition 17** We define  $c\_2Eoption\_2ESOME$  to be  $\lambda A\_27a : \iota. \lambda V0x \in A\_27a. (ap\ (c\_2Eoption\_2Eoption\_ABS\ A\_27a)\ V0x)$

**Definition 18** We define  $c\_2Eone\_2Eone$  to be  $(ap\ (c\_2Emin\_2E\_40\ ty\_2Eone\_2Eone)\ (\lambda V0x \in ty\_2Eone\_2Eone. V0x))$

**Definition 19** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2. (ap\ (ap\ c\_2Emin\_2E\_3D\_3D\_3E\ V0t)\ c\_2Ebool\_2E\_7E))$

**Definition 20** We define  $c\_2Esum\_2EINR$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0e \in A\_27b. (ap\ (c\_2Esum\_2EABS\_sum\ A\_27a\ A\_27b)\ V0e)$

**Definition 21** We define  $c\_2Eoption\_2ENONE$  to be  $\lambda A\_27a : \iota. (ap\ (c\_2Eoption\_2Eoption\_ABS\ A\_27a)\ 0)$

Let  $c\_2Eoption\_2Eoption\_CASE : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Eoption\_2Eoption\_CASE\ A\_27a\ A\_27b \in (((A\_27b^{(A\_27b^{A\_27a})})^{A\_27b})^{ty\_2Eoption\_2Eoption\ A\_27a}) \quad (15)$$

**Definition 22** We define  $c\_2Ellist\_2ELTL$  to be  $\lambda A\_27a : \iota. \lambda V0ll \in (ty\_2Ellist\_2Ellist\ A\_27a). (ap\ (ap\ (ap\ c\_2Eoption\_2ENONE\ V0ll)\ c\_2Eoption\_2Eoption\_CASE\ A\_27a)\ V0ll)$

**Definition 23** We define  $c\_2Ellist\_2ELNIL$  to be  $\lambda A\_27a : \iota. (ap\ (c\_2Ellist\_2Ellist\_abs\ A\_27a)\ (\lambda V0n \in ty\_2Eone\_2Eone. V0n))$

Let  $c\_2Earithmetic\_2E\_2D : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (16)$$

**Definition 24** We define  $c\_2Ebool\_2ECOND$  to be  $\lambda A\_27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A\_27a. (\lambda V2t2 \in A\_27a. (c\_2Ebool\_2E\_7E\ V1t1\ V2t2))))$

**Definition 25** We define  $c\_2Ellist\_2ELCONS$  to be  $\lambda A\_27a : \iota. \lambda V0h \in A\_27a. \lambda V1t \in (ty\_2Ellist\_2Ellist\ A\_27a). (ap\ (c\_2Ellist\_2ELTL\ A\_27a)\ V1t)$

Assume the following.

$$True \quad (17)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (18)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p V0t))) \quad (19)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \quad (20)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (21)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \quad (22)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (23)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (24)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0l \in (ty\_2Ellist\_2Ellist A\_27a).((V0l = (c\_2Ellist\_2ELNIL A\_27a)) \vee (\exists V1h \in A\_27a.(\exists V2t \in (ty\_2Ellist\_2Ellist A\_27a).(V0l = (ap (ap (c\_2Ellist\_2ELCONS A\_27a) V1h) V2t)))))) \quad (25)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow ((ap (c\_2Ellist\_2ELHD A\_27a) (c\_2Ellist\_2ELNIL A\_27a)) = (c\_2Eoption\_2ENONE A\_27a)) \wedge (\forall V0h \in A\_27b.(\forall V1t \in (ty\_2Ellist\_2Ellist A\_27b).((ap (c\_2Ellist\_2ELHD A\_27b) (ap (ap (c\_2Ellist\_2ELCONS A\_27b) V0h) V1t)) = (ap (c\_2Eoption\_2ESOME A\_27b) V0h)))) \quad (26)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow ( \\ & ((ap\ (c.2Ellist\_2ELTL\ A.27a)\ (c.2Ellist\_2ELNIL\ A.27a)) = (c.2Eoption\_2ENONE \\ & \quad (ty\_2Ellist\_2Ellist\ A.27a))) \wedge (\forall V0h \in A.27b. (\forall V1t \in \\ & \quad (ty\_2Ellist\_2Ellist\ A.27b). ((ap\ (c.2Ellist\_2ELTL\ A.27b)\ (ap \\ & \quad (ap\ (c.2Ellist\_2ELCONS\ A.27b)\ V0h)\ V1t)) = (ap\ (c.2Eoption\_2ESOME \\ & \quad (ty\_2Ellist\_2Ellist\ A.27b))\ V1t)))))) \end{aligned} \quad (27)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0h \in A.27a. (\forall V1t \in \\ & (ty\_2Ellist\_2Ellist\ A.27a). ((\neg((ap\ (ap\ (c.2Ellist\_2ELCONS\ A.27a)\ \\ & \quad V0h)\ V1t)) = (c.2Ellist\_2ELNIL\ A.27a)))) \wedge (\neg((c.2Ellist\_2ELNIL \\ & \quad A.27a) = (ap\ (ap\ (c.2Ellist\_2ELCONS\ A.27a)\ V0h)\ V1t)))))) \end{aligned} \quad (28)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0h1 \in A.27a. (\forall V1t1 \in \\ & \quad (ty\_2Ellist\_2Ellist\ A.27a). (\forall V2h2 \in A.27a. (\forall V3t2 \in \\ & \quad (ty\_2Ellist\_2Ellist\ A.27a). ((ap\ (ap\ (c.2Ellist\_2ELCONS\ A.27a)\ \\ & \quad V0h1)\ V1t1) = (ap\ (ap\ (c.2Ellist\_2ELCONS\ A.27a)\ V2h2)\ V3t2)) \Leftrightarrow (( \\ & \quad V0h1 = V2h2) \wedge (V1t1 = V3t2)))))) \end{aligned} \quad (29)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0x \in A.27a. (\forall V1y \in \\ & A.27a. (((ap\ (c.2Eoption\_2ESOME\ A.27a)\ V0x) = (ap\ (c.2Eoption\_2ESOME \\ & \quad A.27a)\ V1y)) \Leftrightarrow (V0x = V1y)))) \end{aligned} \quad (30)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0x \in A.27a. (\neg((c.2Eoption\_2ENONE \\ & \quad A.27a) = (ap\ (c.2Eoption\_2ESOME\ A.27a)\ V0x)))) \end{aligned} \quad (31)$$

**Theorem 1**

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0h \in A.27a. (\forall V1t \in \\ & \quad (ty\_2Ellist\_2Ellist\ A.27a). (\forall V2ll \in (ty\_2Ellist\_2Ellist \\ & \quad A.27a). ((V2ll = (ap\ (ap\ (c.2Ellist\_2ELCONS\ A.27a)\ V0h)\ V1t)) \Leftrightarrow ( \\ & \quad ((ap\ (c.2Ellist\_2ELHD\ A.27a)\ V2ll) = (ap\ (c.2Eoption\_2ESOME\ A.27a)\ \\ & \quad V0h)) \wedge ((ap\ (c.2Ellist\_2ELTL\ A.27a)\ V2ll) = (ap\ (c.2Eoption\_2ESOME \\ & \quad (ty\_2Ellist\_2Ellist\ A.27a))\ V1t)))))) \end{aligned}$$