thm_2Ellist_2ELHD__EQ__NONE (TMRGQGpDuRCZnHHi8gfK5op9PgUt8sg2zx7)

October 26, 2020

Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 4 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \tag{1}$$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{2}$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \tag{3}$$

Definition 5 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Definition 6 *We define* c_2Earithmetic_2EZERO *to be* c_2Enum_2E0.

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \tag{4}$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \tag{5}$$

Definition 7 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num\ ($

Let $c\_2Earithmetic\_2E\_2B : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (6)$$

Definition 8 *We define* c_2Earithmetic_2EBIT1 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic\_$

Definition 9 *We define* c_2Earithmetic_2ENUMERAL *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x$.

Let $c\_2Earithmetic\_2E\_2D : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (7)$$

Definition 10 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 11 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap$ (c_2Ebool_2E_21 2) $(\lambda V2t \in$

Definition 12 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

Definition 13 *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

Let $ty\_2Eone\_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Eone\_2Eone \quad (8)$$

Let $ty\_2Esum\_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Esum\_2Esum\ A0\ A1) \quad (9)$$

Let $c\_2Esum\_2EABS\_\_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Esum\_2EABS\_\_sum\ A\_27a\ A\_27b \in ((ty\_2Esum\_2Esum\ A\_27a\ A\_27b)^{(((2^{A\_27b})^{A\_27a})^{2})}) \quad (10)$$

Definition 14 *We define* c_2Esum_2EINL *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0e \in A\_27a.(ap\ (c\_2Esum\_2EABS$

Let $ty\_2Eoption\_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Eoption\_2Eoption\ A0) \quad (11)$$

Let $c\_2Eoption\_2Eoption\_\_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Eoption\_2Eoption\_\_ABS\ A\_27a \in ((ty\_2Eoption\_2Eoption\ A\_27a)^{(ty\_2Esum\_2Esum\ A\_27a\ ty\_2Eone\_2Eone)}) \quad (12)$$

Definition 15 *We define* c_2Eoption_2ESOME *to be* $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.(ap\ (c\_2Eoption\_2Eoption\_\_$

Definition 16 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap$ (c_2Emin_2E_40

Definition 17 *We define* c_2Eone_2Eone *to be* $(ap$ (c_2Emin_2E_40 $ty\_2Eone\_2Eone)\ (\lambda V0x \in ty\_2Eone\_2$

Definition 18 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap$ c_2Emin_2E_3D_3D_3E $V0t)$ c_2Ebool_2E

Definition 19 *We define* c_2Esum_2EINR *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0e \in A\_27b.(ap\ (c\_2Esum\_2EABS$

Definition 20 *We define* c_2Eoption_2ENONE *to be* $\lambda A\_27a : \iota.(ap\ (c\_2Eoption\_2Eoption\_\_ABS\ A\_27a)\ ($

Definition 21 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap$ (c_2Ebool_2E_21 2) $(\lambda V2t \in$

Definition 22 *We define* c_2Ellist_2Ellrep__ok *to be* $\lambda A\_27a : \iota.(\lambda V0a0 \in ((ty\_2Eoption\_2Eoption\ A\_27a)^{ty}$

Let $ty\_2Ellist\_2Ellist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Ellist\_2Ellist\ A0) \tag{13}$$

Let $c\_2Ellist\_2Ellist\_\_abs : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ellist\_2Ellist\_\_abs\ A\_27a \in ((ty\_2Ellist\_2Ellist\ A\_27a)^{((ty\_2Eoption\_2Eoption\ A\_27a)^{ty\_2Enum\_2Enum})}) \tag{14}$$

Let $c\_2Ellist\_2Ellist\_\_rep : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ellist\_2Ellist\_\_rep\ A\_27a \in (((ty\_2Eoption\_2Eoption\ A\_27a)^{ty\_2Enum\_2Enum})^{(ty\_2Ellist\_2Ellist\ A\_27a)}) \tag{15}$$

Definition 23 *We define* c_2Ellist_2ELHD *to be* $\lambda A\_27a : \iota.\lambda V0ll \in (ty\_2Ellist\_2Ellist\ A\_27a).(ap\ (ap\ (c\_2$

Let $c\_2Eoption\_2Eoption\_\_CASE : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Eoption\_2Eoption\_\_CASE\ A\_27a\ A\_27b \in (((A\_27b^{(A\_27b^{A\_27a})})^{A\_27b})^{(ty\_2Eoption\_2Eoption\ A\_27a)}) \tag{16}$$

Definition 24 *We define* c_2Ellist_2ELTL *to be* $\lambda A\_27a : \iota.\lambda V0ll \in (ty\_2Ellist\_2Ellist\ A\_27a).(ap\ (ap\ (ap$

Definition 25 *We define* c_2Ellist_2ELNIL *to be* $\lambda A\_27a : \iota.(ap\ (c\_2Ellist\_2Ellist\_\_abs\ A\_27a)\ (\lambda V0n \in ty\_$

Definition 26 *We define* c_2Ellist_2ELCONS *to be* $\lambda A\_27a : \iota.\lambda V0h \in A\_27a.\lambda V1t \in (ty\_2Ellist\_2Ellist\ A$

Assume the following.

$$True \tag{17}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \tag{18}$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \tag{19}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t)))))))) \tag{20}$$

Assume the following.

$$((\forall V0t \in 2.((\neg (\neg (p\ V0t))) \Leftrightarrow (p\ V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \tag{21}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{22}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{23}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg (p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg (p\ V0t))))))) \tag{24}$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1y \in 2.(\forall V2z \in 2.(\forall V3w \in 2.((((p\ V0x) \Rightarrow (p\ V1y)) \wedge ((p\ V2z) \Rightarrow (p\ V3w))) \Rightarrow (((p\ V0x) \wedge (p\ V2z)) \Rightarrow ((p\ V1y) \wedge (p\ V3w)))))))) \tag{25}$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1y \in 2.(\forall V2z \in 2.(\forall V3w \in 2.((((p\ V0x) \Rightarrow (p\ V1y)) \wedge ((p\ V2z) \Rightarrow (p\ V3w))) \Rightarrow (((p\ V0x) \vee (p\ V2z)) \Rightarrow ((p\ V1y) \vee (p\ V3w)))))))) \tag{26}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).(\forall V1Q \in (2^{A\_27a}).((\forall V2x \in A\_27a.((p\ (ap\ V0P\ V2x)) \Rightarrow (p\ (ap\ V1Q\ V2x)))) \Rightarrow ((\exists V3x \in A\_27a.(p\ (ap\ V0P\ V3x))) \Rightarrow (\exists V4x \in A\_27a.(p\ (ap\ V1Q\ V4x))))))) \tag{27}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow ((\forall V0a \in (ty\_2Ellist\_2Ellist\ A\_27a).((ap\ (c\_2Ellist\_2Ellist\_\_abs\ A\_27a)\ (ap\ (c\_2Ellist\_2Ellist\_\_rep\ A\_27a)\ V0a)) = V0a)) \wedge (\forall V1r \in ((ty\_2Eoption\_2Eoption\ A\_27a)^{ty\_2Enum\_2Enum}).((p\ (ap\ (\mathsf{c\_2Ellist\_2Elrep\_\_ok}\ A\_27a)\ V1r)) \Leftrightarrow ((ap\ (c\_2Ellist\_2Ellist\_\_rep\ A\_27a)\ (ap\ (c\_2Ellist\_2Ellist\_\_abs\ A\_27a)\ V1r)) = V1r)))) \tag{28}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0h \in A\_27a.(\forall V1t \in (ty\_2Ellist\_2Ellist\ A\_27a).(((ap\ (\mathsf{c\_2Ellist\_2ELHD}\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Ellist\_2ELCONS}\ A\_27a)\ V0h)\ V1t)) = (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V0h)) \wedge ((ap\ (\mathsf{c\_2Ellist\_2ELTL}\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Ellist\_2ELCONS}\ A\_27a)\ V0h)\ V1t)) = (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ (ty\_2Ellist\_2Ellist\ A\_27a))\ V1t))))) \tag{29}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0l \in (ty\_2Ellist\_2Ellist\ A\_27a).((V0l = (\mathsf{c\_2Ellist\_2ELNIL}\ A\_27a)) \vee (\exists V1h \in A\_27a.(\exists V2t \in (ty\_2Ellist\_2Ellist\ A\_27a).(V0l = (ap\ (ap\ (\mathsf{c\_2Ellist\_2ELCONS}\ A\_27a)\ V1h)\ V2t)))))) \tag{30}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0h \in A\_27a.(\forall V1t \in (ty\_2Ellist\_2Ellist\ A\_27a).((\neg((ap\ (ap\ (\mathsf{c\_2Ellist\_2ELCONS}\ A\_27a)\ V0h)\ V1t) = (\mathsf{c\_2Ellist\_2ELNIL}\ A\_27a))) \wedge (\neg((\mathsf{c\_2Ellist\_2ELNIL}\ A\_27a) = (ap\ (ap\ (\mathsf{c\_2Ellist\_2ELCONS}\ A\_27a)\ V0h)\ V1t)))))) \tag{31}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.(((ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V0x) = (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V1y)) \Leftrightarrow (V0x = V1y)))) \tag{32}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\neg((\mathsf{c\_2Eoption\_2ENONE}\ A\_27a) = (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V0x)))) \tag{33}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0ll \in (ty\_2Ellist\_2Ellist\ A\_27a).((((ap\ (\mathsf{c\_2Ellist\_2ELHD}\ A\_27a)\ V0ll) = (\mathsf{c\_2Eoption\_2ENONE}\ A\_27a)) \Leftrightarrow (V0ll = (\mathsf{c\_2Ellist\_2ELNIL}\ A\_27a))) \wedge (((\mathsf{c\_2Eoption\_2ENONE}\ A\_27a) = (ap\ (\mathsf{c\_2Ellist\_2ELHD}\ A\_27a)\ V0ll)) \Leftrightarrow (V0ll = (\mathsf{c\_2Ellist\_2ELNIL}\ A\_27a)))))$$