thm_2Ellist_2ELHD__LUNFOLD (TMd7D9GKkmWWmnzPj4fpLDtw8jR3PyqWGYY)

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Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \tag{1}$$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{2}$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \tag{3}$$

Definition 3 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Definition 4 *We define* c_2Earithmetic_2EZERO *to be* c_2Enum_2E0.

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \tag{4}$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \tag{5}$$

Definition 5 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 6 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num\ ($

Let $c\_2Earithmetic\_2E\_2B : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{6}$$

Definition 7 *We define* c_2Earithmetic_2EBIT1 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic\_$

Definition 8 *We define* c_2Earithmetic_2ENUMERAL *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x$.

Let $ty\_2Eoption\_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Eoption\_2Eoption\ A0) \tag{7}$$

Let $ty\_2Ellist\_2Ellist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Ellist\_2Ellist\ A0) \tag{8}$$

Let $c\_2Ellist\_2Ellist\_\_rep : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ellist\_2Ellist\_\_rep\ A\_27a \in (((ty\_2Eoption\_2Eoption\ A\_27a)^{ty\_2Enum\_2Enum})^{(ty\_2Ellist\_2Ellist\ A\_27a)}) \tag{9}$$

Let $c\_2Ellist\_2Ellist\_\_abs : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ellist\_2Ellist\_\_abs\ A\_27a \in ((ty\_2Ellist\_2Ellist\ A\_27a)^{((ty\_2Eoption\_2Eoption\ A\_27a)^{ty\_2Enum\_2Enum})}) \tag{10}$$

Let $ty\_2Eone\_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Eone\_2Eone \tag{11}$$

Definition 9 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 10 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in$

Let $ty\_2Esum\_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Esum\_2Esum\ A0\ A1) \tag{12}$$

Let $c\_2Esum\_2EABS\_\_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Esum\_2EABS\_\_sum\ A\_27a\ A\_27b \in ((ty\_2Esum\_2Esum\ A\_27a\ A\_27b)^{(((2^{A\_27b})^{A\_27a})^{2})}) \tag{13}$$

Definition 11 *We define* c_2Esum_2EINL *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0e \in A\_27a.(ap\ (c\_2Esum\_2EABS$

Let $c\_2Eoption\_2Eoption\_\_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Eoption\_2Eoption\_\_ABS\ A\_27a \in ((ty\_2Eoption\_2Eoption\ A\_27a)^{(ty\_2Esum\_2Esum\ A\_27a\ ty\_2Eone\_2Eone)}) \tag{14}$$

Definition 12 *We define* c_2Eoption_2ESOME *to be* $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.(ap\ (c\_2Eoption\_2Eoption\_\_$

Definition 13 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^{A}.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$
of type $\iota \Rightarrow \iota$.

Definition 14 *We define* c_2Eone_2Eone *to be* $(ap$ (c_2Emin_2E_40 $ty\_2Eone\_2Eone)$ $(\lambda V0x \in ty\_2Eone\_2$

Definition 15 *We define* c_2Ebool_2EF *to be* $(ap$ (c_2Ebool_2E_21 $2)$ $(\lambda V0t \in 2.V0t))$.

Definition 16 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap$ c_2Emin_2E_3D_3D_3E $V0t)$ c_2Ebool_2E

Definition 17 *We define* c_2Esum_2EINR *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0e \in A\_27b.(ap\ (c\_2Esum\_2EABS$

Definition 18 *We define* c_2Eoption_2ENONE *to be* $\lambda A\_27a : \iota.(ap\ (c\_2Eoption\_2Eoption\_\_ABS\ A\_27a)\ ($

Definition 19 *We define* c_2Ellist_2ELHD *to be* $\lambda A\_27a : \iota.\lambda V0ll \in (ty\_2Ellist\_2Ellist\ A\_27a).(ap\ (ap\ (c\_2$

Let $c\_2Eoption\_2Eoption\_\_CASE : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Eoption\_2Eoption\_\_CASE\ A\_27a\ A\_27b \in (((A\_27b^{(A\_27b^{A\_27a})})^{A\_27b})^{(ty\_2Eoption\_2Eoption\ A\_27a)}) \tag{15}$$

Definition 20 *We define* c_2Ellist_2ELTL *to be* $\lambda A\_27a : \iota.\lambda V0ll \in (ty\_2Ellist\_2Ellist\ A\_27a).(ap\ (ap\ (ap$

Let $c\_2Eoption\_2EOPTION\_\_JOIN : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Eoption\_2EOPTION\_\_JOIN\ A\_27a \in ((ty\_2Eoption\_2Eoption\ A\_27a)^{(ty\_2Eoption\_2Eoption\ (ty\_2Eoption\_2Eoption\ A\_27a))}) \tag{16}$$

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{17}$$

Let $c\_2Epair\_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2ESND\ A\_27a\ A\_27b \in (A\_27b^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \tag{18}$$

Let $c\_2Epair\_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EFST\ A\_27a\ A\_27b \in (A\_27a^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \tag{19}$$

Definition 21 *We define* c_2Epair_2Epair__CASE *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0p \in (ty\_2Epair\_$

Let $c\_2Eoption\_2EOPTION\_\_MAP : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Eoption\_2EOPTION\_\_MAP\ A\_27a\ A\_27b \in (((ty\_2Eoption\_2Eoption\ A\_27b)^{(ty\_2Eoption\_2Eoption\ A\_27a)})^{(A\_27b^{A\_27a})}) \tag{20}$$

Definition 22 *We define* c_2Ecombin_2EK *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0x \in A\_27a.(\lambda V1y \in A\_27b.V0x)$

Definition 23 *We define* c_2Ecombin_2Eo *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in (A\_27b^{A\_27c}).\lambda V1$

Definition 24 *We define* c_2Epair_2EUNCURRY *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in ((A\_27c^{A\_27}$

Let $c\_2Eoption\_2EOPTION\_\_BIND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Eoption\_2EOPTION\_\_BIND\ A\_27a\ A\_27b \in (((ty\_2Eoption\_2Eoption\ A\_27a)^{((ty\_2Eoption\_2Eoption\ A\_27a)^{A\_27b})})^{(ty\_2Eoption\_2Eoption\ A\_27} \tag{21}$$

Let $c\_2Earithmetic\_2EFUNPOW : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Earithmetic\_2EFUNPOW\ A\_27a \in (((A\_27a^{A\_27a})^{ty\_2Enum\_2Enum})^{(A\_27a^{A\_27a})}) \tag{22}$$

Definition 25 *We define* c_2Ellist_2ELUNFOLD *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in ((ty\_2Eoption\_2Eoptio$

Let $c\_2Ellist\_2ELNTH : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ellist\_2ELNTH\ A\_27a \in (((ty\_2Eoption\_2Eoption\ A\_27a)^{(ty\_2Ellist\_2Ellist\ A\_27a)})^{ty\_2Enum\_2Enum}) \tag{23}$$

Assume the following.

$$True \tag{24}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{25}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{26}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow ((\forall V0ll \in (ty\_2Ellist\_2Ellist\ A\_27a).((ap\ (ap\ (c\_2Ellist\_2ELNTH\ A\_27a)\ \mathsf{c\_2Enum\_2E0})\ V0ll) = (ap\ (\mathsf{c\_2Ellist\_2ELHD}\ A\_27a)\ V0ll))) \wedge (\forall V1n \in ty\_2Enum\_2Enum.(\forall V2ll \in (ty\_2Ellist\_2Ellist\ A\_27a).((ap\ (ap\ (c\_2Ellist\_2ELNTH\ A\_27a)\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V1n))\ V2ll) = (ap\ (c\_2Eoption\_2EOPTION\_\_JOIN\ A\_27a)\ (ap\ (ap\ (c\_2Eoption\_2EOPTION\_\_MAP\ (ty\_2Ellist\_2Ellist\ A\_27a)\ (ty\_2Eoption\_2Eoption\ A\_27a))\ (ap\ (c\_2Ellist\_2ELNTH\ A\_27a)\ V1n))\ (ap\ (\mathsf{c\_2Ellist\_2ELTL}\ A\_27a)\ V2ll))))))) \tag{27}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0f \in ((ty\_2Eoption\_2Eoption\ (ty\_2Epair\_2Eprod\ A\_27b\ A\_27a))^{A\_27b}).(\forall V1x \in A\_27b.(\forall V2n \in ty\_2Enum\_2Enum. (((ap\ (ap\ (c\_2Ellist\_2ELNTH\ A\_27a)\ \mathsf{c\_2Enum\_2E0})\ (ap\ (ap\ (\mathsf{c\_2Ellist\_2ELUNFOLD}\ A\_27a\ A\_27b)\ V0f)\ V1x)) = (ap\ (ap\ (c\_2Eoption\_2EOPTION\_\_MAP\ (ty\_2Epair\_2Eprod\ A\_27b\ A\_27a)\ A\_27a)\ (c\_2Epair\_2ESND\ A\_27b\ A\_27a))\ (ap\ V0f\ V1x))) \wedge ((ap\ (ap\ (c\_2Ellist\_2ELNTH\ A\_27a)\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V2n))\ (ap\ (ap\ (\mathsf{c\_2Ellist\_2ELUNFOLD}\ A\_27a\ A\_27b)\ V0f)\ V1x)) = (ap\ (ap\ (ap\ (c\_2Eoption\_2Eoption\_\_CASE\ (ty\_2Epair\_2Eprod\ A\_27b\ A\_27a)\ (ty\_2Eoption\_2Eoption\ A\_27a))\ (ap\ V0f\ V1x))\ (\mathsf{c\_2Eoption\_2ENONE}\ A\_27a))\ (\lambda V3v \in (ty\_2Epair\_2Eprod\ A\_27b\ A\_27a).(ap\ (ap\ (\mathsf{c\_2Epair\_2Epair\_\_CASE}\ (ty\_2Eoption\_2Eoption\ A\_27a)\ A\_27b\ A\_27a)\ V3v)\ (\lambda V4tx \in A\_27b.(\lambda V5hx \in A\_27a.(ap\ (ap\ (c\_2Ellist\_2ELNTH\ A\_27a)\ V2n)\ (ap\ (ap\ (\mathsf{c\_2Ellist\_2ELUNFOLD}\ A\_27a\ A\_27b)\ V0f)\ V4tx)))))))))))) \tag{28}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0f \in ((ty\_2Eoption\_2Eoption\ (ty\_2Epair\_2Eprod\ A\_27b\ A\_27a))^{A\_27b}).(\forall V1x \in A\_27b.((ap\ (\mathsf{c\_2Ellist\_2ELHD}\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Ellist\_2ELUNFOLD}\ A\_27a\ A\_27b)\ V0f)\ V1x)) = (ap\ (ap\ (c\_2Eoption\_2EOPTION\_\_MAP\ (ty\_2Epair\_2Eprod\ A\_27b\ A\_27a)\ A\_27a)\ (c\_2Epair\_2ESND\ A\_27b\ A\_27a))\ (ap\ V0f\ V1x)))))$$