

thm_2Ellist_2ELLENGTH_THM
 (TMHvZBLbRQFncBDggseN-
 geRev7A5q2P5Rkp)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{1}$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{2}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{3}$$

Definition 3 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 4 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \tag{4}$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \tag{5}$$

Definition 5 We define $c_2Ebool_2E_21$ to be $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a}))$

Definition 6 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num\ (c_2Enum_2ESUC_REP\ m))$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (6)$$

Definition 7 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_2Earithmetic_2E_2B))$

Definition 8 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let $c_2Earithmetic_2E_2D : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2D \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (7)$$

Let $ty_2Eoption_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Eoption_2Eoption A0) \quad (8)$$

Let $ty_2Ellist_2Ellist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Ellist_2Ellist A0) \quad (9)$$

Let $c_2Ellist_2Ellist_rep : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow c_2Ellist_2Ellist_rep A_27a \in \\ & (((ty_2Eoption_2Eoption A_27a)^{ty_2Enum_2Enum})^{(ty_2Ellist_2Ellist A_27a)}) \end{aligned} \quad (10)$$

Let $ty_2Eone_2Eone : \iota$ be given. Assume the following.

$$nonempty ty_2Eone_2Eone \quad (11)$$

Definition 9 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 10 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2)) (\lambda V2t \in 2)))$

Let $ty_2Esum_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Esum_2Esum A0 A1) \quad (12)$$

Let $c_2Esum_2EABS_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Esum_2EABS_sum \\ & A_27a A_27b \in ((ty_2Esum_2Esum A_27a A_27b)^{((2^{A_27b})^{A_27a})^2}) \end{aligned} \quad (13)$$

Definition 11 We define c_2Esum_2EINL to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0e \in A_27a.(ap (c_2Esum_2EABS_sum))$

Let $c_2Eoption_2Eoption_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Eoption_2Eoption_ABS A_27a \in ((ty_2Eoption_2Eoption A_27a)^{(ty_2Esum_2Esum A_27a ty_2Eone_2Eone)}) \quad (14)$$

Definition 12 We define `c_2Eoption_2ESOME` to be $\lambda A.27a : \iota. \lambda V0x \in A.27a. (ap (c_2Eoption_2Eoption_2ESOME A.27a) V0x)$

Definition 13 We define `c_2Ebool_2EF` to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 14 We define `c_2Emin_2E_40` to be $\lambda A. \lambda P \in 2^A. \mathbf{if} (\exists x \in A. P x) \mathbf{then} (the (\lambda x. x \in A \wedge P x) of type \iota \Rightarrow \iota)$.

Definition 15 We define `c_2Ebool_2ECOND` to be $\lambda A.27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A.27a. (\lambda V2t2 \in A.27a. (ap (c_2Ebool_2E_21 2) V2t2)))) V0t)$

Let `c_2Ellist_2Ellist_abs` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.27a. nonempty A.27a \Rightarrow c_2Ellist_2Ellist_abs A.27a \in ((ty_2Ellist_2Ellist A.27a)^{(ty_2Eoption_2Eoption A.27a)^{ty_2Enum_2Enum}}) \quad (15)$$

Definition 16 We define `c_2Ellist_2ELCONS` to be $\lambda A.27a : \iota. \lambda V0h \in A.27a. \lambda V1t \in (ty_2Ellist_2Ellist A.27a) (ap (c_2Ellist_2Ellist_abs A.27a) V1t) \mathbin{\&\&} V0h$

Definition 17 We define `c_2Eone_2Eone` to be $(ap (c_2Emin_2E_40 ty_2Eone_2Eone) (\lambda V0x \in ty_2Eone_2Eone.V0x))$

Definition 18 We define `c_2Ebool_2E_7E` to be $(\lambda V0t \in 2. (ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_21 2) V0t)$

Definition 19 We define `c_2Esum_2EINR` to be $\lambda A.27a : \iota. \lambda A.27b : \iota. \lambda V0e \in A.27b. (ap (c_2Esum_2EABS A.27a A.27b) V0e)$

Definition 20 We define `c_2Eoption_2ENONE` to be $\lambda A.27a : \iota. (ap (c_2Eoption_2Eoption_ABS A.27a) (ap (c_2Ebool_2E_21 2) V0t))$

Definition 21 We define `c_2Ellist_2ELNIL` to be $\lambda A.27a : \iota. (ap (c_2Ellist_2Ellist_abs A.27a) (\lambda V0n \in ty_2Ellist_2Ellist A.27a.V0n))$

Definition 22 We define `c_2Ebool_2E_3F` to be $\lambda A.27a : \iota. (\lambda V0P \in (2^{A-27a}). (ap V0P (ap (c_2Emin_2E_40 A.27a) V0P)))$

Definition 23 We define `c_2Ebool_2E_5C_2F` to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2. (ap (c_2Ebool_2E_21 2) V2t) V1t2)))) V0t1)$

Definition 24 We define `c_2Ellist_2Ellength_rel` to be $\lambda A.27a : \iota. (\lambda V0a0 \in (ty_2Ellist_2Ellist A.27a). (\lambda V1a1 \in (ty_2Ellist_2Ellist A.27a). (ap (c_2Ellist_2Ellist_abs A.27a) V1a1) \mathbin{\&\&} V0a0)))$

Definition 25 We define `c_2Ellist_2ELFINITE` to be $\lambda A.27a : \iota. (\lambda V0a0 \in (ty_2Ellist_2Ellist A.27a). (ap (c_2Ellist_2Ellist_abs A.27a) V0a0) \mathbin{\&\&} V0a0))$

Definition 26 We define `c_2Ellist_2ELLENGTH` to be $\lambda A.27a : \iota. \lambda V0ll \in (ty_2Ellist_2Ellist A.27a). (ap (c_2Ellist_2Ellist_abs A.27a) V0ll) \mathbin{\&\&} V0ll$

Let `c_2Eoption_2EOPTION_MAP` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.27a. nonempty A.27a \Rightarrow \forall A.27b. nonempty A.27b \Rightarrow c_2Eoption_2EOPTION_MAP A.27a A.27b \in (((ty_2Eoption_2Eoption A.27b)^{(ty_2Eoption_2Eoption A.27a)})(A.27b^{A-27a})) \quad (16)$$

Assume the following.

$$True \quad (17)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (18)$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p V0t))) \quad (19)$$

Assume the following.

$$(\forall V0t \in 2. ((p V0t) \vee (\neg(p V0t)))) \quad (20)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0t \in 2. ((\exists V1x \in A_27a. (p V0t)) \Leftrightarrow (p V0t))) \quad (21)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \end{aligned} \quad (22)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \vee (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \vee True) \Leftrightarrow True) \wedge \\ & (((False \vee (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee False) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee \\ & (p V0t)) \Leftrightarrow (p V0t)))))) \end{aligned} \quad (23)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((\\ & (p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))) \end{aligned} \quad (24)$$

Assume the following.

$$\begin{aligned} & ((\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t)) \wedge ((\neg True) \Leftrightarrow False) \wedge \\ & ((\neg False) \Leftrightarrow True))) \end{aligned} \quad (25)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (26)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (27)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(\\ & p V0t)))))) \end{aligned} \quad (28)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t1 \in A_27a. (\forall V1t2 \in \\ A_27a. ((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ c_2Ebool_2ET)\ V0t1) \\ V1t2) = V0t1) \wedge ((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ c_2Ebool_2EF) \\ V0t1)\ V1t2) = V1t2)))) \end{aligned} \quad (29)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. ((ap\ (c_2Emin_2E_40 \\ A_27a)\ (\lambda V1y \in A_27a. (ap\ (ap\ (c_2Emin_2E_3D\ A_27a)\ V1y)\ V0x))) = \\ V0x)) \end{aligned} \quad (30)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (\\ 2^{A_27a}). ((\forall V2x \in A_27a. (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow (\forall V3x \in \\ A_27a. ((p\ V0P) \wedge (p\ (ap\ V1Q\ V3x)))))) \end{aligned} \quad (31)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0Q \in 2. (\forall V1P \in (\\ 2^{A_27a}). ((\forall V2x \in A_27a. ((p\ (ap\ V1P\ V2x)) \vee (p\ V0Q))) \Leftrightarrow ((\forall V3x \in \\ A_27a. (p\ (ap\ V1P\ V3x)) \vee (p\ V0Q)))))) \end{aligned} \quad (32)$$

Assume the following.

$$\begin{aligned} (\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p\ V0t1) \Rightarrow \\ ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \end{aligned} \quad (33)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in 2. \\ (\forall V2x \in A_27a. (\forall V3x_27 \in A_27a. (\forall V4y \in A_27a. \\ (\forall V5y_27 \in A_27a. (((p\ V0P) \Leftrightarrow (p\ V1Q)) \wedge (((p\ V1Q) \Rightarrow (V2x = V3x_27)) \wedge \\ ((\neg(p\ V1Q)) \Rightarrow (V4y = V5y_27)))))) \Rightarrow ((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a) \\ V0P)\ V2x)\ V4y) = (ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ V1Q)\ V3x_27) \\ V5y_27)))))) \end{aligned} \quad (34)$$

Assume the following.

$$\begin{aligned} (\forall V0x \in 2. (\forall V1y \in 2. (\forall V2z \in 2. (\forall V3w \in \\ 2. (((p\ V0x) \Rightarrow (p\ V1y)) \wedge ((p\ V2z) \Rightarrow (p\ V3w))) \Rightarrow (((p\ V0x) \wedge (p\ V2z)) \Rightarrow \\ ((p\ V1y) \wedge (p\ V3w)))))) \end{aligned} \quad (35)$$

Assume the following.

$$\begin{aligned} (\forall V0x \in 2. (\forall V1y \in 2. (\forall V2z \in 2. (\forall V3w \in \\ 2. (((p\ V0x) \Rightarrow (p\ V1y)) \wedge ((p\ V2z) \Rightarrow (p\ V3w))) \Rightarrow (((p\ V0x) \vee (p\ V2z)) \Rightarrow \\ ((p\ V1y) \vee (p\ V3w)))))) \end{aligned} \quad (36)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). (\forall V1Q \in \\ (2^{A_27a}). ((\forall V2x \in A_27a. ((p\ (ap\ V0P\ V2x)) \Rightarrow (p\ (ap\ V1Q\ V2x)))))) \Rightarrow \\ ((\exists V3x \in A_27a. (p\ (ap\ V0P\ V3x))) \Rightarrow (\exists V4x \in A_27a. (p\ (\\ ap\ V1Q\ V4x)))))) \end{aligned} \quad (37)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). (\forall V1a \in \\ A_27a. ((\exists V2x \in A_27a. ((V2x = V1a) \wedge (p\ (ap\ V0P\ V2x)))))) \Leftrightarrow (p\ (\\ ap\ V0P\ V1a)))) \end{aligned} \quad (38)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow ((\forall V0t1 \in A_27a. (\forall V1t2 \in \\ A_27a. ((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ c_2Ebool_2ET)\ V0t1) \\ V1t2) = V0t1))) \wedge (\forall V2t1 \in A_27a. (\forall V3t2 \in A_27a. ((ap \\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ c_2Ebool_2EF)\ V2t1)\ V3t2) = V3t2)))) \end{aligned} \quad (39)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0h \in A_27a. (\forall V1t \in \\ (ty_2Ellist_2Ellist\ A_27a). ((\neg((ap\ (ap\ (c_2Ellist_2ELCONS\ A_27a) \\ V0h)\ V1t) = (c_2Ellist_2ELNIL\ A_27a)))) \wedge (\neg((c_2Ellist_2ELNIL \\ A_27a) = (ap\ (ap\ (c_2Ellist_2ELCONS\ A_27a)\ V0h)\ V1t)))))) \end{aligned} \quad (40)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0h1 \in A_27a. (\forall V1t1 \in \\ (ty_2Ellist_2Ellist\ A_27a). (\forall V2h2 \in A_27a. (\forall V3t2 \in \\ (ty_2Ellist_2Ellist\ A_27a). (((ap\ (ap\ (c_2Ellist_2ELCONS\ A_27a) \\ V0h1)\ V1t1) = (ap\ (ap\ (c_2Ellist_2ELCONS\ A_27a)\ V2h2)\ V3t2))) \Leftrightarrow ((\\ V0h1 = V2h2) \wedge (V1t1 = V3t2)))))) \end{aligned} \quad (41)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ ((p\ (ap\ (c_2Ellist_2ELFINITE\ A_27a)\ (c_2Ellist_2ELNIL\ A_27a))) \Leftrightarrow \\ True) \wedge (\forall V0h \in A_27b. (\forall V1t \in (ty_2Ellist_2Ellist \\ A_27b). ((p\ (ap\ (c_2Ellist_2ELFINITE\ A_27b)\ (ap\ (ap\ (c_2Ellist_2ELCONS \\ A_27b)\ V0h)\ V1t))) \Leftrightarrow (p\ (ap\ (c_2Ellist_2ELFINITE\ A_27b)\ V1t)))))) \end{aligned} \quad (42)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in \\ A_27a. (((ap\ (c_2Eoption_2ESOME\ A_27a)\ V0x) = (ap\ (c_2Eoption_2ESOME \\ A_27a)\ V1y))) \Leftrightarrow (V0x = V1y)))) \end{aligned} \quad (43)$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0P \in 2. (\forall V1x \in A.27a. \\
& (\forall V2y \in A.27a. (((ap\ (ap\ (ap\ (c.2Ebool.2ECOND\ (ty.2Eoption.2Eoption \\
& A.27a))\ V0P)\ (ap\ (c.2Eoption.2ESOME\ A.27a)\ V1x))\ (c.2Eoption.2ENONE \\
& A.27a)) = (c.2Eoption.2ENONE\ A.27a)) \Leftrightarrow (\neg(p\ V0P)))) \wedge (((ap\ (ap\ (\\
& ap\ (c.2Ebool.2ECOND\ (ty.2Eoption.2Eoption\ A.27a))\ V0P)\ (c.2Eoption.2ENONE \\
& A.27a))\ (ap\ (c.2Eoption.2ESOME\ A.27a)\ V1x)) = (c.2Eoption.2ENONE \\
& A.27a)) \Leftrightarrow (p\ V0P)) \wedge (((ap\ (ap\ (ap\ (c.2Ebool.2ECOND\ (ty.2Eoption.2Eoption \\
& A.27a))\ V0P)\ (ap\ (c.2Eoption.2ESOME\ A.27a)\ V1x))\ (c.2Eoption.2ENONE \\
& A.27a)) = (ap\ (c.2Eoption.2ESOME\ A.27a)\ V2y)) \Leftrightarrow ((p\ V0P) \wedge (V1x = V2y))) \wedge \\
& (((ap\ (ap\ (ap\ (c.2Ebool.2ECOND\ (ty.2Eoption.2Eoption\ A.27a)) \\
& V0P)\ (c.2Eoption.2ENONE\ A.27a))\ (ap\ (c.2Eoption.2ESOME\ A.27a) \\
& V1x)) = (ap\ (c.2Eoption.2ESOME\ A.27a)\ V2y)) \Leftrightarrow ((\neg(p\ V0P)) \wedge (V1x = \\
& V2y)))))))))
\end{aligned} \tag{44}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\
& \forall V0f \in (A.27b^{A.27a}). (\forall V1x \in (ty.2Eoption.2Eoption \\
& A.27a). (\forall V2y \in A.27b. (((ap\ (ap\ (c.2Eoption.2EOPTION_MAP \\
& A.27a\ A.27b)\ V0f)\ V1x) = (ap\ (c.2Eoption.2ESOME\ A.27b)\ V2y)) \Leftrightarrow (\exists V3z \in \\
& A.27a. ((V1x = (ap\ (c.2Eoption.2ESOME\ A.27a)\ V3z)) \wedge (V2y = (ap\ V0f \\
& V3z)))))))))
\end{aligned} \tag{45}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\
& \forall V0f \in (A.27a^{A.27b}). (\forall V1x \in (ty.2Eoption.2Eoption \\
& A.27b). (((ap\ (ap\ (c.2Eoption.2EOPTION_MAP\ A.27b\ A.27a)\ V0f) \\
& V1x) = (c.2Eoption.2ENONE\ A.27a)) \Leftrightarrow (V1x = (c.2Eoption.2ENONE\ A.27b))) \wedge \\
& (((c.2Eoption.2ENONE\ A.27a) = (ap\ (ap\ (c.2Eoption.2EOPTION_MAP \\
& A.27b\ A.27a)\ V0f)\ V1x)) \Leftrightarrow (V1x = (c.2Eoption.2ENONE\ A.27b))))))
\end{aligned} \tag{46}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty.2Enum.2Enum. (\forall V1n \in ty.2Enum.2Enum. (\\
& ((ap\ c.2Enum.2ESUC\ V0m) = (ap\ c.2Enum.2ESUC\ V1n)) \Leftrightarrow (V0m = V1n))))
\end{aligned} \tag{47}$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \tag{48}$$

Assume the following.

$$(\forall V0A \in 2. ((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \tag{49}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\
& (((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False))))))
\end{aligned} \tag{50}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg(\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (51)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \quad (52)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee (\neg(p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee (\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \quad (53)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \wedge (p V2r)) \Leftrightarrow (((p V0p) \vee (\neg(p V1q)) \vee (\neg(p V2r))) \wedge (((p V1q) \vee (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))))) \quad (54)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \vee (p V2r)) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \quad (55)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \Rightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((\neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \quad (56)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \quad (57)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p)))) \quad (58)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))))) \quad (59)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V0p)))))) \quad (60)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V1q)))))) \quad (61)$$

Assume the following.

$$(\forall V0p \in 2.((\neg(\neg(p V0p))) \Rightarrow (p V0p))) \quad (62)$$

Theorem 1

$$\begin{aligned} & \forall A_{27a}.nonempty\ A_{27a} \Rightarrow \forall A_{27b}.nonempty\ A_{27b} \Rightarrow (\\ & ((ap\ (c_2Elist_2ELLENGTH\ A_{27a})\ (c_2Elist_2ELNIL\ A_{27a})) = \\ & (ap\ (c_2Eoption_2ESOME\ ty_2Enum_2Enum)\ c_2Enum_2E0)) \wedge (\forall V0h \in \\ & A_{27b}.(\forall V1t \in (ty_2Elist_2Elist\ A_{27b}).((ap\ (c_2Elist_2ELLENGTH \\ A_{27b})\ (ap\ (ap\ (c_2Elist_2ELCONS\ A_{27b})\ V0h)\ V1t)) = (ap\ (ap\ (c_2Eoption_2EOPTION_MAP \\ ty_2Enum_2Enum\ ty_2Enum_2Enum)\ c_2Enum_2ESUC)\ (ap\ (c_2Elist_2ELLENGTH \\ A_{27b})\ V1t)))))) \end{aligned}$$