thm_2Ellist_2ELLIST__BISIMULATION (TMb2RPRGGi1Tj5hRNCjg6pZD8KUmBL5TZt4)

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Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$
of type $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \qquad (1)$$

Let $c\_2Earithmetic\_2E\_2D : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \qquad (2)$$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 4 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 5 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$
of type ι.

Definition 6 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Definition 7 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.\textbf{if}\ (\exists x \in A.p\ (ap\ P\ x))\ \textbf{then}\ (the\ (\lambda x.x \in A \wedge p$
of type $\iota \Rightarrow \iota$.

Definition 8 *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.(a$

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \qquad (3)$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \qquad (4)$$

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Definition 9 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Definition 10 *We define* c_2Earithmetic_2EZERO *to be* c_2Enum_2E0.

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \tag{5}$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \tag{6}$$

Definition 11 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num$

Let $c\_2Earithmetic\_2E\_2B : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{7}$$

Definition 12 *We define* c_2Earithmetic_2EBIT1 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

Definition 13 *We define* c_2Earithmetic_2ENUMERAL *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x$.

Let $ty\_2Eone\_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Eone\_2Eone \tag{8}$$

Let $ty\_2Esum\_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Esum\_2Esum \\ A0\ A1) \tag{9}$$

Let $c\_2Esum\_2EABS\_\_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Esum\_2EABS\_\_sum \\ A\_27a\ A\_27b \in ((ty\_2Esum\_2Esum\ A\_27a\ A\_27b)^{(((2^{A\_27b})^{A\_27a})^2)}) \tag{10}$$

Definition 14 *We define* c_2Esum_2EINL *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0e \in A\_27a.(ap\ (c\_2Esum\_2EABS$

Let $ty\_2Eoption\_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Eoption\_2Eoption\ A0) \tag{11}$$

Let $c\_2Eoption\_2Eoption\_\_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Eoption\_2Eoption\_\_ABS\ A\_27a \in \\ ((ty\_2Eoption\_2Eoption\ A\_27a)^{(ty\_2Esum\_2Esum\ A\_27a\ ty\_2Eone\_2Eone)}) \tag{12}$$

Definition 15 *We define* c_2Eoption_2ESOME *to be* $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.(ap\ (c\_2Eoption\_2Eoption\_\_$

Definition 16 *We define* c_2Ebool_2E_3F *to be* $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ $ c_2Emin_2E_40

Definition 17 *We define* c_2Eone_2Eone *to be* $(ap\ ($c_2Emin_2E_40 $ty\_2Eone\_2Eone)\ (\lambda V0x \in ty\_2Eone\_2$

Definition 18 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ $c_2Emin_2E_3D_3D_3E $V0t)\ $c_2Ebool_2E

Definition 19 *We define* c_2Esum_2EINR *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0e \in A\_27b.(ap\ ($c_2Esum_2EABS

Definition 20 *We define* c_2Eoption_2ENONE *to be* $\lambda A\_27a : \iota.(ap\ (c\_2Eoption\_2Eoption\_\_ABS\ A\_27a)\ ($

Definition 21 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ ($c_2Ebool_2E_21 $2)\ (\lambda V2t \in$

Definition 22 *We define* c_2Ellist_2Elrep__ok *to be* $\lambda A\_27a : \iota.(\lambda V0a0 \in ((ty\_2Eoption\_2Eoption\ A\_27a)^{ty-}$

Let $ty\_2Ellist\_2Ellist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Ellist\_2Ellist\ A0) \qquad (13)$$

Let $c\_2Ellist\_2Ellist\_\_rep : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ellist\_2Ellist\_\_rep\ A\_27a \in$$
$$(((ty\_2Eoption\_2Eoption\ A\_27a)^{ty\_2Enum\_2Enum})^{(ty\_2Ellist\_2Ellist\ A\_27a)})$$
$$(14)$$

Let $c\_2Ellist\_2Ellist\_\_abs : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ellist\_2Ellist\_\_abs\ A\_27a \in$$
$$((ty\_2Ellist\_2Ellist\ A\_27a)^{((ty\_2Eoption\_2Eoption\ A\_27a)^{ty\_2Enum\_2Enum})}) \qquad (15)$$

Definition 23 *We define* c_2Ellist_2ELHD *to be* $\lambda A\_27a : \iota.\lambda V0ll \in (ty\_2Ellist\_2Ellist\ A\_27a).(ap\ (ap\ (c\_2$

Let $c\_2Eoption\_2Eoption\_\_CASE : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Eoption\_2Eoption\_\_CASE$$
$$A\_27a\ A\_27b \in (((A\_27b^{(A\_27b^{A\_27a})})^{A\_27b})^{(ty\_2Eoption\_2Eoption\ A\_27a)})$$
$$(16)$$

Definition 24 *We define* c_2Ellist_2ELTL *to be* $\lambda A_27a : \iota.\lambda V0ll \in (ty_2Ellist_2Ellist\ A_27a).(ap\ (ap\ (ap\ $

Definition 25 *We define* c_2Ellist_2ELCONS *to be* $\lambda A\_27a : \iota.\lambda V0h \in A\_27a.\lambda V1t \in (ty\_2Ellist\_2Ellist\ A$

Definition 26 *We define* c_2Ellist_2ELNIL *to be* $\lambda A\_27a : \iota.(ap\ (c\_2Ellist\_2Ellist\_\_abs\ A\_27a)\ (\lambda V0n \in ty\_$

Let $c\_2Eoption\_2ETHE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Eoption\_2ETHE\ A\_27a \in (A\_27a^{(ty\_2Eoption\_2Eoption\ A\_27a)})$$
$$(17)$$

Assume the following.
$$True \qquad (18)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p \\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2))))))) \tag{19}$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \tag{20}$$

Assume the following.

$$(\forall V0t \in 2.((p\ V0t) \lor (\neg(p\ V0t)))) \tag{21}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\exists V1x \in \\ A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \tag{22}$$

Assume the following.

$$(\forall V0t \in 2.(((True \land (p\ V0t)) \Leftrightarrow (p\ V0t)) \land ((((p\ V0t) \land True) \Leftrightarrow \\ (p\ V0t)) \land (((False \land (p\ V0t)) \Leftrightarrow False) \land ((((p\ V0t) \land False) \Leftrightarrow False) \land \\ (((p\ V0t) \land (p\ V0t)) \Leftrightarrow (p\ V0t)))))))) \tag{23}$$

Assume the following.

$$(\forall V0t \in 2.(((True \lor (p\ V0t)) \Leftrightarrow True) \land ((((p\ V0t) \lor True) \Leftrightarrow True) \land \\ (((False \lor (p\ V0t)) \Leftrightarrow (p\ V0t)) \land ((((p\ V0t) \lor False) \Leftrightarrow (p\ V0t)) \land (((p\ V0t) \lor \\ (p\ V0t)) \Leftrightarrow (p\ V0t)))))))$$
$$\tag{24}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \land ((((p\ V0t) \Rightarrow True) \Leftrightarrow \\ True) \land (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \land ((((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \land ((\\ (p\ V0t) \Rightarrow False) \Leftrightarrow (\neg(p\ V0t)))))))) \tag{25}$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \land (((\neg True) \Leftrightarrow False) \land \\ ((\neg False) \Leftrightarrow True))) \tag{26}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(V0x = V0x)) \tag{27}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow \\ True)) \tag{28}$$

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Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in \\ A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{29}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \land ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow \\ (p\ V0t)) \land (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg(p\ V0t))) \land (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg(\\ p\ V0t)))))))) \tag{30}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p\ V0A) \lor (\\ (p\ V1B) \land (p\ V2C))) \Leftrightarrow (((p\ V0A) \lor (p\ V1B)) \land ((p\ V0A) \lor (p\ V2C))))))) \tag{31}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.((((p\ V1B) \land \\ (p\ V2C)) \lor (p\ V0A)) \Leftrightarrow (((p\ V1B) \lor (p\ V0A)) \land ((p\ V2C) \lor (p\ V0A))))))) \tag{32}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \Rightarrow \\ ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \land (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \tag{33}$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x\_27 \in 2.(\forall V2y \in 2.(\forall V3y\_27 \in \\ 2.((((p\ V0x) \Leftrightarrow (p\ V1x\_27)) \land ((p\ V1x\_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y\_27)))) \Rightarrow \\ (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x\_27) \Rightarrow (p\ V3y\_27)))))))) \tag{34}$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1y \in 2.(\forall V2z \in 2.(\forall V3w \in \\ 2.((((p\ V0x) \Rightarrow (p\ V1y)) \land ((p\ V2z) \Rightarrow (p\ V3w))) \Rightarrow (((p\ V0x) \land (p\ V2z)) \Rightarrow \\ ((p\ V1y) \land (p\ V3w))))))))) \tag{35}$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1y \in 2.(\forall V2z \in 2.(\forall V3w \in \\ 2.((((p\ V0x) \Rightarrow (p\ V1y)) \land ((p\ V2z) \Rightarrow (p\ V3w))) \Rightarrow (((p\ V0x) \lor (p\ V2z)) \Rightarrow \\ ((p\ V1y) \lor (p\ V3w))))))))) \tag{36}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).(\forall V1Q \in \\ (2^{A\_27a}).((\forall V2x \in A\_27a.((p\ (ap\ V0P\ V2x)) \Rightarrow (p\ (ap\ V1Q\ V2x)))) \Rightarrow \\ ((\exists V3x \in A\_27a.(p\ (ap\ V0P\ V3x))) \Rightarrow (\exists V4x \in A\_27a.(p\ (\\ ap\ V1Q\ V4x))))))) \tag{37}$$

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Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).(\forall V1a \in A\_27a.((\exists V2x \in A\_27a.((V2x = V1a) \land (p\ (ap\ V0P\ V2x)))) \Leftrightarrow (p\ (ap\ V0P\ V1a))))) \tag{38}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow ((\forall V0a \in (ty\_2Ellist\_2Ellist\ A\_27a).((ap\ (c\_2Ellist\_2Ellist\_abs\ A\_27a)\ (ap\ (c\_2Ellist\_2Ellist\_rep\ A\_27a)\ V0a)) = V0a)) \land (\forall V1r \in ((ty\_2Eoption\_2Eoption\ A\_27a)^{ty\_2Enum\_2Enum}). ((p\ (ap\ (c\_2Ellist\_2Elrep\_ok\ A\_27a)\ V1r)) \Leftrightarrow ((ap\ (c\_2Ellist\_2Ellist\_rep\ A\_27a)\ (ap\ (c\_2Ellist\_2Ellist\_abs\ A\_27a)\ V1r)) = V1r))))$$
$$\tag{39}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0h \in A\_27a.(\forall V1t \in (ty\_2Ellist\_2Ellist\ A\_27a).(((ap\ (c\_2Ellist\_2ELHD\ A\_27a)\ (ap\ (ap\ (c\_2Ellist\_2ELCONS\ A\_27a)\ V0h)\ V1t)) = (ap\ (c\_2Eoption\_2ESOME\ A\_27a)\ V0h)) \land ((ap\ (c\_2Ellist\_2ELTL\ A\_27a)\ (ap\ (ap\ (c\_2Ellist\_2ELCONS\ A\_27a)\ V0h)\ V1t)) = (ap\ (c\_2Eoption\_2ESOME\ (ty\_2Ellist\_2Ellist\ A\_27a))\ V1t)))))$$
$$\tag{40}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0l \in (ty\_2Ellist\_2Ellist\ A\_27a).((V0l = (c\_2Ellist\_2ELNIL\ A\_27a)) \lor (\exists V1h \in A\_27a. (\exists V2t \in (ty\_2Ellist\_2Ellist\ A\_27a).(V0l = (ap\ (ap\ (c\_2Ellist\_2ELCONS\ A\_27a)\ V1h)\ V2t))))))$$
$$\tag{41}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0h \in A\_27a.(\forall V1t \in (ty\_2Ellist\_2Ellist\ A\_27a).((\neg((ap\ (ap\ (c\_2Ellist\_2ELCONS\ A\_27a)\ V0h)\ V1t) = (c\_2Ellist\_2ELNIL\ A\_27a))) \land (\neg((c\_2Ellist\_2ELNIL\ A\_27a) = (ap\ (ap\ (c\_2Ellist\_2ELCONS\ A\_27a)\ V0h)\ V1t)))))) \tag{42}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0h1 \in A\_27a.(\forall V1t1 \in (ty\_2Ellist\_2Ellist\ A\_27a).(\forall V2h2 \in A\_27a.(\forall V3t2 \in (ty\_2Ellist\_2Ellist\ A\_27a).(((ap\ (ap\ (c\_2Ellist\_2ELCONS\ A\_27a)\ V0h1)\ V1t1) = (ap\ (ap\ (c\_2Ellist\_2ELCONS\ A\_27a)\ V2h2)\ V3t2)) \Leftrightarrow ((V0h1 = V2h2) \land (V1t1 = V3t2))))))) \tag{43}$$

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Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0ll \in (ty\_2Ellist\_2Ellist$$
$$A\_27a).((((ap\ (\mathsf{c\_2Ellist\_2ELHD}\ A\_27a)\ V0ll) = (\mathsf{c\_2Eoption\_2ENONE}$$
$$A\_27a)) \Leftrightarrow (V0ll = (\mathsf{c\_2Ellist\_2ELNIL}\ A\_27a))) \wedge (((\mathsf{c\_2Eoption\_2ENONE}$$
$$A\_27a) = (ap\ (\mathsf{c\_2Ellist\_2ELHD}\ A\_27a)\ V0ll)) \Leftrightarrow (V0ll = (\mathsf{c\_2Ellist\_2ELNIL}$$
$$A\_27a)))))$$

$$(44)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0ll1 \in (ty\_2Ellist\_2Ellist$$
$$A\_27a).(\forall V1ll2 \in (ty\_2Ellist\_2Ellist\ A\_27a).((V0ll1 =$$
$$V1ll2) \Leftrightarrow (\exists V2R \in ((2^{(ty\_2Ellist\_2Ellist\ A\_27a)})^{(ty\_2Ellist\_2Ellist\ A\_27a)}).$$
$$((p\ (ap\ (ap\ V2R\ V0ll1)\ V1ll2)) \wedge (\forall V3ll3 \in (ty\_2Ellist\_2Ellist$$
$$A\_27a).(\forall V4ll4 \in (ty\_2Ellist\_2Ellist\ A\_27a).((p\ (ap\ (ap$$
$$V2R\ V3ll3)\ V4ll4)) \Rightarrow (((V3ll3 = (\mathsf{c\_2Ellist\_2ELNIL}\ A\_27a)) \wedge (V4ll4 = \quad (45)$$
$$(\mathsf{c\_2Ellist\_2ELNIL}\ A\_27a))) \vee (\exists V5h \in A\_27a.(\exists V6t1 \in$$
$$(ty\_2Ellist\_2Ellist\ A\_27a).(\exists V7t2 \in (ty\_2Ellist\_2Ellist$$
$$A\_27a).((V3ll3 = (ap\ (ap\ \mathsf{c\_2Ellist\_2ELCONS}\ A\_27a)\ V5h)\ V6t1)) \wedge$$
$$((V4ll4 = (ap\ (ap\ \mathsf{c\_2Ellist\_2ELCONS}\ A\_27a)\ V5h)\ V7t2)) \wedge (p\ (ap\ ($$
$$ap\ V2R\ V6t1)\ V7t2)))))))))))))))))$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ($$
$$(\forall V0v \in A\_27b.(\forall V1f \in (A\_27b^{A\_27a}).((ap\ (ap\ (ap\ (\mathsf{c\_2Eoption\_2Eoption\_CASE}$$
$$A\_27a\ A\_27b)\ (\mathsf{c\_2Eoption\_2ENONE}\ A\_27a))\ V0v)\ V1f) = V0v))) \wedge (\forall V2x \in$$
$$A\_27a.(\forall V3v \in A\_27b.(\forall V4f \in (A\_27b^{A\_27a}).((ap\ (ap$$
$$(ap\ (\mathsf{c\_2Eoption\_2Eoption\_CASE}\ A\_27a\ A\_27b)\ (ap\ (\mathsf{c\_2Eoption\_2ESOME}$$
$$A\_27a)\ V2x))\ V3v)\ V4f) = (ap\ V4f\ V2x))))))$$

$$(46)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in$$
$$A\_27a.(((ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V0x) = (ap\ (\mathsf{c\_2Eoption\_2ESOME}$$
$$A\_27a)\ V1y)) \Leftrightarrow (V0x = V1y))))$$

$$(47)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\neg ((\mathsf{c\_2Eoption\_2ENONE}$$
$$A\_27a) = (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V0x)))) \quad (48)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((ap\ (\mathsf{c\_2Eoption\_2ETHE}$$
$$A\_27a)\ (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V0x)) = V0x)) \quad (49)$$

Assume the following.

$$(\forall V0t \in 2.((\neg (\neg (p\ V0t))) \Leftrightarrow (p\ V0t))) \quad (50)$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \tag{51}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\ (((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False))))) \tag{52}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p\ V0A)) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\ ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False))))) \tag{53}$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p\ V0A)) \Rightarrow False) \Rightarrow (((p\ V0A) \Rightarrow False) \Rightarrow False))) \tag{54}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow (\\ (p\ V1q) \Leftrightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((p\ V1q) \vee (p\ V2r))) \wedge (((p\ V0p) \vee ((\neg(\\ p\ V2r)) \vee (\neg(p\ V1q)))) \wedge (((p\ V1q) \vee ((\neg(p\ V2r)) \vee (\neg(p\ V0p)))) \wedge ((p\ V2r) \vee \\ ((\neg(p\ V1q)) \vee (\neg(p\ V0p))))))))))))$$
$$\tag{55}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow (\\ (p\ V1q) \wedge (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((\neg(p\ V1q)) \vee (\neg(p\ V2r)))) \wedge (((p\ V1q) \vee \\ (\neg(p\ V0p))) \wedge ((p\ V2r) \vee (\neg(p\ V0p)))))))))$$
$$\tag{56}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow (\\ (p\ V1q) \vee (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee (\neg(p\ V1q))) \wedge (((p\ V0p) \vee (\neg(p\ V2r))) \wedge \\ ((p\ V1q) \vee ((p\ V2r) \vee (\neg(p\ V0p)))))))))$$
$$\tag{57}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow (\\ (p\ V1q) \Rightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee (p\ V1q)) \wedge (((p\ V0p) \vee (\neg(p\ V2r))) \wedge ((\\ \neg(p\ V1q)) \vee ((p\ V2r) \vee (\neg(p\ V0p)))))))))) \tag{58}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p\ V0p) \Leftrightarrow (\neg(p\ V1q))) \Leftrightarrow (((p\ V0p) \vee \\ (p\ V1q)) \wedge ((\neg(p\ V1q)) \vee (\neg(p\ V0p))))))) \tag{59}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p) \Rightarrow (p\ V1q))) \Rightarrow (p\ V0p)))) \tag{60}$$

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Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p)\Rightarrow(p\ V1q)))\Rightarrow(\neg(p\ V1q))))) \quad (61)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p)\vee(p\ V1q)))\Rightarrow(\neg(p\ V0p))))) \quad (62)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p)\vee(p\ V1q)))\Rightarrow(\neg(p\ V1q))))) \quad (63)$$

Assume the following.

$$(\forall V0p \in 2.((\neg(\neg(p\ V0p)))\Rightarrow(p\ V0p))) \quad (64)$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0ll1 \in (ty\_2Ellist\_2Ellist$$
$$A\_27a).(\forall V1ll2 \in (ty\_2Ellist\_2Ellist\ A\_27a).((V0ll1 =$$
$$V1ll2)\Leftrightarrow(\exists V2R \in ((2^{(ty\_2Ellist\_2Ellist\ A\_27a)})^{(ty\_2Ellist\_2Ellist\ A\_27a)}).$$
$$((p\ (ap\ (ap\ V2R\ V0ll1)\ V1ll2))\wedge(\forall V3ll3 \in (ty\_2Ellist\_2Ellist$$
$$A\_27a).(\forall V4ll4 \in (ty\_2Ellist\_2Ellist\ A\_27a).((p\ (ap\ (ap$$
$$V2R\ V3ll3)\ V4ll4))\Rightarrow(((V3ll3 = (\mathsf{c\_2Ellist\_2ELNIL}\ A\_27a))\wedge(V4ll4 =$$
$$(\mathsf{c\_2Ellist\_2ELNIL}\ A\_27a)))\vee(((ap\ (\mathsf{c\_2Ellist\_2ELHD}\ A\_27a)\ V3ll3) =$$
$$(ap\ (\mathsf{c\_2Ellist\_2ELHD}\ A\_27a)\ V4ll4))\wedge(p\ (ap\ (ap\ V2R\ (ap\ (\mathsf{c\_2Eoption\_2ETHE}$$
$$(ty\_2Ellist\_2Ellist\ A\_27a))\ (ap\ (\mathsf{c\_2Ellist\_2ELTL}\ A\_27a)\ V3ll3)))$$
$$(ap\ (\mathsf{c\_2Eoption\_2ETHE}\ (ty\_2Ellist\_2Ellist\ A\_27a))\ (ap\ (\mathsf{c\_2Ellist\_2ELTL}$$
$$A\_27a)\ V4ll4)))))))))))))$$