thm_2Ellist_2ELLIST__BISIMULATION0 (TMHtX-Pvj45NxecEWstrwfnCaUURk6v1CTqQ)

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Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Let $ty\_2Eone\_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Eone\_2Eone \tag{1}$$

Definition 3 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.\mathbf{if}\ (\exists x \in A.p\ (ap\ P\ x))\ \mathbf{then}\ (the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

Definition 4 *We define* c_2Eone_2Eone *to be* $(ap\ (\mathsf{c\_2Emin\_2E\_40}\ ty\_2Eone\_2Eone)\ (\lambda V0x \in ty\_2Eone\_2E$

Definition 5 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 6 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 7 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 8 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \mathsf{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \mathsf{c\_2Ebool\_2EF}$

Definition 9 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Let $ty\_2Esum\_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Esum\_2Esum\ A0\ A1) \tag{2}$$

Let $c\_2Esum\_2EABS\_\_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Esum\_2EABS\_\_sum\ A\_27a\ A\_27b \in ((ty\_2Esum\_2Esum\ A\_27a\ A\_27b)^{(((2^{A\_27b})^{A\_27a})^2)}) \tag{3}$$

Definition 10 *We define* c_2Esum_2EINR *to be* $\lambda A\_27a:\iota.\lambda A\_27b:\iota.\lambda V0e\in A\_27b.(ap\ (c\_2Esum\_2EABS$

Let $ty\_2Eoption\_2Eoption:\iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0\Rightarrow nonempty\ (ty\_2Eoption\_2Eoption\ A0) \tag{4}$$

Let $c\_2Eoption\_2Eoption\_\_ABS:\iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow c\_2Eoption\_2Eoption\_\_ABS\ A\_27a\in ((ty\_2Eoption\_2Eoption\ A\_27a)^{(ty\_2Esum\_2Esum\ A\_27a\ ty\_2Eone\_2Eone)}) \tag{5}$$

Definition 11 *We define* c_2Eoption_2ENONE *to be* $\lambda A\_27a:\iota.(ap\ (c\_2Eoption\_2Eoption\_\_ABS\ A\_27a)\ (c$

Let $ty\_2Enum\_2Enum:\iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{6}$$

Let $ty\_2Ellist\_2Ellist:\iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0\Rightarrow nonempty\ (ty\_2Ellist\_2Ellist\ A0) \tag{7}$$

Let $c\_2Ellist\_2Ellist\_\_abs:\iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow c\_2Ellist\_2Ellist\_\_abs\ A\_27a\in ((ty\_2Ellist\_2Ellist\ A\_27a)^{((ty\_2Eoption\_2Eoption\ A\_27a)^{ty\_2Enum\_2Enum})}) \tag{8}$$

Definition 12 *We define* c_2Ellist_2ELNIL *to be* $\lambda A\_27a:\iota.(ap\ (c\_2Ellist\_2Ellist\_\_abs\ A\_27a)\ (\lambda V0n\in ty\_$

Let $c\_2Enum\_2EZERO\_\_REP:\iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP\in omega \tag{9}$$

Let $c\_2Enum\_2EABS\_\_num:\iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num\in (ty\_2Enum\_2Enum^{omega}) \tag{10}$$

Definition 13 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Definition 14 *We define* c_2Earithmetic_2EZERO *to be* c_2Enum_2E0.

Let $c\_2Enum\_2EREP\_\_num:\iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num\in (omega^{ty\_2Enum\_2Enum}) \tag{11}$$

Let $c\_2Enum\_2ESUC\_\_REP:\iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP\in (omega^{omega}) \tag{12}$$

Definition 15 *We define* c_2Enum_2ESUC *to be* $\lambda V0m\in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num$

Let $c\_2Earithmetic\_2E\_2B : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{13}$$

Definition 16 *We define* c_2Earithmetic_2EBIT1 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

Definition 17 *We define* c_2Earithmetic_2ENUMERAL *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x.$

Let $c\_2Earithmetic\_2E\_2D : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{14}$$

Let $c\_2Ellist\_2Ellist\_\_rep : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ellist\_2Ellist\_\_rep\ A\_27a \in (((ty\_2Eoption\_2Eoption\ A\_27a)^{ty\_2Enum\_2Enum})^{(ty\_2Ellist\_2Ellist\ A\_27a)}) \tag{15}$$

Definition 18 *We define* c_2Esum_2EINL *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0e \in A\_27a.(ap\ (c\_2Esum\_2EABS$

Definition 19 *We define* c_2Eoption_2ESOME *to be* $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.(ap\ (c\_2Eoption\_2Eoption\_\_$

Definition 20 *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

Definition 21 *We define* c_2Ellist_2ELCONS *to be* $\lambda A\_27a : \iota.\lambda V0h \in A\_27a.\lambda V1t \in (ty\_2Ellist\_2Ellist\ A$

Definition 22 *We define* c_2Ecombin_2Eo *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in (A\_27b^{A\_27c}).\lambda V1g$

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{16}$$

Let $c\_2Epair\_2EABS\_\_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})}) \tag{17}$$

Definition 23 *We define* c_2Epair_2E_2C *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2$

Let $c\_2Eoption\_2Eoption\_\_CASE : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Eoption\_2Eoption\_\_CASE\ A\_27a\ A\_27b \in (((A\_27b^{(A\_27b^{A\_27a})})^{A\_27b})^{(ty\_2Eoption\_2Eoption\ A\_27a)}) \tag{18}$$

Definition 24 *We define* c_2Ellist_2ELTL__HD *to be* $\lambda A\_27a : \iota.\lambda V0ll \in (ty\_2Ellist\_2Ellist\ A\_27a).(ap\ (ap$

Definition 25 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V2t \in$

Definition 26 *We define* c_2Ebool_2E_3F *to be* $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ ($c_2Emin_2E_40

Definition 27 *We define* c_2Ecombin_2EK *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0x \in A\_27a.(\lambda V1y \in A\_27b.V0x)$

Let $c\_2Epair\_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2ESND\ A\_27a\ A\_27b \in (A\_27b^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \tag{19}$$

Let $c\_2Epair\_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EFST\ A\_27a\ A\_27b \in (A\_27a^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \tag{20}$$

Definition 28 *We define* c_2Epair_2EUNCURRY *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in ((A\_27c^{A\_27}$

Let $c\_2Eoption\_2EOPTION\_\_BIND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Eoption\_2EOPTION\_\_BIND\ A\_27a\ A\_27b \in (((ty\_2Eoption\_2Eoption\ A\_27a)^{((ty\_2Eoption\_2Eoption\ A\_27a)^{A\_27b})})^{(ty\_2Eoption\_2Eoption\ A\_27} \tag{21}$$

Let $c\_2Earithmetic\_2EFUNPOW : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Earithmetic\_2EFUNPOW\ A\_27a \in (((A\_27a^{A\_27a})^{ty\_2Enum\_2Enum})^{(A\_27a^{A\_27a})}) \tag{22}$$

Let $c\_2Eoption\_2EOPTION\_\_MAP : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Eoption\_2EOPTION\_\_MAP\ A\_27a\ A\_27b \in (((ty\_2Eoption\_2Eoption\ A\_27b)^{(ty\_2Eoption\_2Eoption\ A\_27a)})^{(A\_27b^{A\_27a})}) \tag{23}$$

Definition 29 *We define* c_2Ellist_2ELUNFOLD *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in ((ty\_2Eoption\_2Eoptio$

Assume the following.

$$True \tag{24}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{25}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{26}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in (ty\_2Ellist\_2Ellist\ A\_27a).(\forall V1t \in (ty\_2Ellist\_2Ellist\ A\_27a).(\forall V2h \in A\_27a.((((ap\ (\mathsf{c\_2Ellist\_2ELTL\_\_HD}\ A\_27a)\ V0x) = (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ (ty\_2Epair\_2Eprod\ (ty\_2Ellist\_2Ellist\ A\_27a)\ A\_27a))\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (ty\_2Ellist\_2Ellist\ A\_27a)\ A\_27a)\ V1t)\ V2h))) \Leftrightarrow (V0x = (ap\ (ap\ (\mathsf{c\_2Ellist\_2ELCONS}\ A\_27a)\ V2h)\ V1t))) \land (((ap\ (\mathsf{c\_2Ellist\_2ELTL\_\_HD}\ A\_27a)\ V0x) = (\mathsf{c\_2Eoption\_2ENONE}\ (ty\_2Epair\_2Eprod\ (ty\_2Ellist\_2Ellist\ A\_27a)\ A\_27a))) \Leftrightarrow (V0x = (\mathsf{c\_2Ellist\_2ELNIL}\ A\_27a)))))) \quad (27)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0ll \in (ty\_2Ellist\_2Ellist\ A\_27a).((ap\ (ap\ (\mathsf{c\_2Ellist\_2ELUNFOLD}\ A\_27a\ (ty\_2Ellist\_2Ellist\ A\_27a))\ (\mathsf{c\_2Ellist\_2ELTL\_\_HD}\ A\_27a))\ V0ll) = V0ll)) \quad (28)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c.nonempty\ A\_27c \Rightarrow (\forall V0f1 \in ((ty\_2Eoption\_2Eoption\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b))^{A\_27a}).(\forall V1f2 \in ((ty\_2Eoption\_2Eoption\ (ty\_2Epair\_2Eprod\ A\_27c\ A\_27b))^{A\_27c}).(\forall V2x1 \in A\_27a.(\forall V3x2 \in A\_27c.(((ap\ (ap\ (\mathsf{c\_2Ellist\_2ELUNFOLD}\ A\_27b\ A\_27a)\ V0f1)\ V2x1) = (ap\ (ap\ (\mathsf{c\_2Ellist\_2ELUNFOLD}\ A\_27b\ A\_27c)\ V1f2)\ V3x2)) \Leftrightarrow (\exists V4R \in ((2^{A\_27c})^{A\_27a}).((p\ (ap\ (ap\ V4R\ V2x1)\ V3x2)) \land (\forall V5y1 \in A\_27a.(\forall V6y2 \in A\_27c.((p\ (ap\ (ap\ V4R\ V5y1)\ V6y2)) \Rightarrow ((((ap\ V0f1\ V5y1) = (\mathsf{c\_2Eoption\_2ENONE}\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b))) \land ((ap\ V1f2\ V6y2) = (\mathsf{c\_2Eoption\_2ENONE}\ (ty\_2Epair\_2Eprod\ A\_27c\ A\_27b)))) \lor (\exists V7h \in A\_27b.(\exists V8t1 \in A\_27a.(\exists V9t2 \in A\_27c.(((ap\ V0f1\ V5y1) = (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b))\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27b)\ V8t1)\ V7h))) \land (((ap\ V1f2\ V6y2) = (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ (ty\_2Epair\_2Eprod\ A\_27c\ A\_27b))\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27c\ A\_27b)\ V9t2)\ V7h))) \land (p\ (ap\ (ap\ V4R\ V8t1)\ V9t2)))))))))))))))))) \quad (29)$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0ll1 \in (ty\_2Ellist\_2Ellist\ A\_27a).(\forall V1ll2 \in (ty\_2Ellist\_2Ellist\ A\_27a).((V0ll1 = V1ll2) \Leftrightarrow (\exists V2R \in ((2^{(ty\_2Ellist\_2Ellist\ A\_27a)})^{(ty\_2Ellist\_2Ellist\ A\_27a)}).((p\ (ap\ (ap\ V2R\ V0ll1)\ V1ll2)) \land (\forall V3ll3 \in (ty\_2Ellist\_2Ellist\ A\_27a).(\forall V4ll4 \in (ty\_2Ellist\_2Ellist\ A\_27a).((p\ (ap\ (ap\ V2R\ V3ll3)\ V4ll4)) \Rightarrow (((V3ll3 = (\mathsf{c\_2Ellist\_2ELNIL}\ A\_27a)) \land (V4ll4 = (\mathsf{c\_2Ellist\_2ELNIL}\ A\_27a))) \lor (\exists V5h \in A\_27a.(\exists V6t1 \in (ty\_2Ellist\_2Ellist\ A\_27a).(\exists V7t2 \in (ty\_2Ellist\_2Ellist\ A\_27a).((V3ll3 = (ap\ (ap\ (\mathsf{c\_2Ellist\_2ELCONS}\ A\_27a)\ V5h)\ V6t1)) \land ((V4ll4 = (ap\ (ap\ (\mathsf{c\_2Ellist\_2ELCONS}\ A\_27a)\ V5h)\ V7t2)) \land (p\ (ap\ (ap\ V2R\ V6t1)\ V7t2)))))))))))))))$$