

thm_2Ellist_2ELMAP__LUNFOLD

(TMLSshSkE2s7DSupQQA2cjVyVYCLe4NLm6f)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 4 We define $c_2Ebool_2E_2F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{1}$$

Let $c_2Earithmetic_2E_2D : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2D \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \tag{2}$$

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p \Rightarrow q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))$

Definition 7 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A \wedge p x))$ of type $\iota \Rightarrow \iota$.

Definition 8 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.(ap (c_2Emin_2E_40$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{3}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{4}$$

Definition 9 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 10 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (5)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (6)$$

Definition 11 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum)^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (7)$$

Definition 12 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ (ap\ c_2Earithmetic$

Definition 13 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let $ty_2Eone_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eone_2Eone \quad (8)$$

Let $ty_2Esum_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Esum_2Esum\ A0\ A1) \quad (9)$$

Let $c_2Esum_2EABS_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Esum_2EABS_sum\ A_27a\ A_27b \in ((ty_2Esum_2Esum\ A_27a\ A_27b)^{((2^{A_27b})^{A_27a})^2}) \quad (10)$$

Definition 14 We define c_2Esum_2EINL to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0e \in A_27a.(ap\ (c_2Esum_2EABS$

Let $ty_2Eoption_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Eoption_2Eoption\ A0) \quad (11)$$

Let $c_2Eoption_2Eoption_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Eoption_2Eoption_ABS\ A_27a \in ((ty_2Eoption_2Eoption\ A_27a)^{(ty_2Esum_2Esum\ A_27a\ ty_2Eone_2Eone)}) \quad (12)$$

Definition 15 We define $c_2Eoption_2ESOME$ to be $\lambda A_27a : \iota.\lambda V0x \in A_27a.(ap\ (c_2Eoption_2Eoption$

Definition 16 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap\ V0P\ (ap\ (c_2Emin_2E_40$

Definition 17 We define c_2Eone_2Eone to be $(ap\ (c_2Emin_2E_40\ ty_2Eone_2Eone)\ (\lambda V0x \in ty_2Eone_2E$

Definition 18 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2. (ap\ (ap\ c_2Emin_2E_3D_3D_3E\ V0t)\ c_2Ebool_2E$

Definition 19 We define c_2Esum_2EINR to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0e \in A_27b. (ap\ (c_2Esum_2EABS$

Definition 20 We define $c_2Eoption_2ENONE$ to be $\lambda A_27a : \iota. (ap\ (c_2Eoption_2Eoption_ABS\ A_27a)\ ($

Definition 21 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in$

Definition 22 We define $c_2Ellist_2Elrep_ok$ to be $\lambda A_27a : \iota. (\lambda V0a0 \in ((ty_2Eoption_2Eoption\ A_27a)^{ty-$

Let $ty_2Ellist_2Ellist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. nonempty\ A0 \Rightarrow nonempty\ (ty_2Ellist_2Ellist\ A0) \quad (13)$$

Let $c_2Ellist_2Ellist_rep : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow c_2Ellist_2Ellist_rep\ A_27a \in \\ (((ty_2Eoption_2Eoption\ A_27a)^{ty_2Eenum_2Eenum})^{(ty_2Ellist_2Ellist\ A_27a)}) \quad (14)$$

Let $c_2Ellist_2Ellist_abs : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow c_2Ellist_2Ellist_abs\ A_27a \in \\ ((ty_2Ellist_2Ellist\ A_27a)^{((ty_2Eoption_2Eoption\ A_27a)^{ty_2Eenum_2Eenum})}) \quad (15)$$

Let $c_2Ellist_2ELMAP : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow \forall A_27b. nonempty\ A_27b \Rightarrow c_2Ellist_2ELMAP \\ A_27a\ A_27b \in (((ty_2Ellist_2Ellist\ A_27b)^{(ty_2Ellist_2Ellist\ A_27a)})^{(A_27b^{A_27a})}) \quad (16)$$

Definition 23 We define $c_2Ellist_2ELCONS$ to be $\lambda A_27a : \iota. \lambda V0h \in A_27a. \lambda V1t \in (ty_2Ellist_2Ellist\ A$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. nonempty\ A0 \Rightarrow \forall A1. nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod \\ A0\ A1) \quad (17)$$

Definition 24 We define $c_2Ecombin_2EK$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. (\lambda V0x \in A_27a. (\lambda V1y \in A_27b. V0x)$

Definition 25 We define $c_2Ecombin_2Eo$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda A_27c : \iota. \lambda V0f \in (A_27b^{A_27c}). \lambda V1g$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow \forall A_27b. nonempty\ A_27b \Rightarrow c_2Epair_2ESND \\ A_27a\ A_27b \in (A_27b^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \quad (18)$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow \forall A_27b. nonempty\ A_27b \Rightarrow c_2Epair_2EFST \\ A_27a\ A_27b \in (A_27a^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \quad (19)$$

Definition 26 We define $c_2Epair_2EUNCURRY$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in ((A_27c^{A_27a})$
Let $c_2Eoption_2EOPTION_BIND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Eoption_2EOPTION_BIND \\ & A_27a\ A_27b \in (((ty_2Eoption_2Eoption\ A_27a)^{(ty_2Eoption_2Eoption\ A_27a)^{A_27b}})^{(ty_2Eoption_2Eoption\ A_27a)}) \end{aligned} \quad (20)$$

Let $c_2Earithmetic_2EFUNPOW : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow c_2Earithmetic_2EFUNPOW\ A_27a \in \\ & (((A_27a^{A_27a})^{ty_2Enum_2Enum})^{(A_27a^{A_27a})}) \end{aligned} \quad (21)$$

Let $c_2Eoption_2EOPTION_MAP : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Eoption_2EOPTION_MAP \\ & A_27a\ A_27b \in (((ty_2Eoption_2Eoption\ A_27b)^{(ty_2Eoption_2Eoption\ A_27a)})^{(A_27b^{A_27a})}) \end{aligned} \quad (22)$$

Definition 27 We define $c_2Ellist_2ELUNFOLD$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in ((ty_2Eoption_2Eoption$

Definition 28 We define $c_2Ellist_2ELHD$ to be $\lambda A_27a : \iota.\lambda V0ll \in (ty_2Ellist_2Ellist\ A_27a).(ap\ (ap\ (c_2$

Let $c_2Eoption_2Eoption_CASE : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Eoption_2Eoption_CASE \\ & A_27a\ A_27b \in (((A_27b^{(A_27b^{A_27a})})^{A_27b})^{(ty_2Eoption_2Eoption\ A_27a)}) \end{aligned} \quad (23)$$

Definition 29 We define $c_2Ellist_2ELTL$ to be $\lambda A_27a : \iota.\lambda V0ll \in (ty_2Ellist_2Ellist\ A_27a).(ap\ (ap\ (ap$

Definition 30 We define $c_2Ellist_2ELNIL$ to be $\lambda A_27a : \iota.(ap\ (c_2Ellist_2Ellist_abs\ A_27a)\ (\lambda V0n \in ty$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod \\ & A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \end{aligned} \quad (24)$$

Definition 31 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap\ (c_2$

Assume the following.

$$True \quad (25)$$

Assume the following.

$$\begin{aligned} & (\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p \\ & V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \end{aligned} \quad (26)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \quad (27)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2. ((\exists V1x \in A_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \quad (28)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \end{aligned} \quad (29)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \vee (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \vee True) \Leftrightarrow True) \wedge \\ & (((False \vee (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \vee False) \Leftrightarrow (p\ V0t)) \wedge ((p\ V0t) \vee \\ & (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \end{aligned} \quad (30)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((\\ & (p\ V0t) \Rightarrow False) \Leftrightarrow \neg(p\ V0t)))))) \end{aligned} \quad (31)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (32)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (33)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg(\\ & p\ V0t)))))) \end{aligned} \quad (34)$$

Assume the following.

$$\begin{aligned} & (\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p\ V0t1) \Rightarrow \\ & ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \end{aligned} \quad (35)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in 2. (\forall V1x_27 \in 2. (\forall V2y \in 2. (\forall V3y_27 \in \\ & 2. (((p\ V0x) \Leftrightarrow (p\ V1x_27)) \wedge ((p\ V1x_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y_27)))) \Rightarrow \\ & (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x_27) \Rightarrow (p\ V3y_27)))))) \end{aligned} \quad (36)$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in 2. (\forall V1y \in 2. (\forall V2z \in 2. (\forall V3w \in \\
& 2. (((p \ V0x) \Rightarrow (p \ V1y)) \wedge ((p \ V2z) \Rightarrow (p \ V3w))) \Rightarrow (((p \ V0x) \wedge (p \ V2z)) \Rightarrow \\
& ((p \ V1y) \wedge (p \ V3w))))))
\end{aligned} \tag{37}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in 2. (\forall V1y \in 2. (\forall V2z \in 2. (\forall V3w \in \\
& 2. (((p \ V0x) \Rightarrow (p \ V1y)) \wedge ((p \ V2z) \Rightarrow (p \ V3w))) \Rightarrow (((p \ V0x) \vee (p \ V2z)) \Rightarrow \\
& ((p \ V1y) \vee (p \ V3w))))))
\end{aligned} \tag{38}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). (\forall V1Q \in \\
& (2^{A_27a}). ((\forall V2x \in A_27a. ((p \ (ap \ V0P \ V2x)) \Rightarrow (p \ (ap \ V1Q \ V2x)))) \Rightarrow \\
& ((\exists V3x \in A_27a. (p \ (ap \ V0P \ V3x))) \Rightarrow (\exists V4x \in A_27a. (p \ (\\
& ap \ V1Q \ V4x))))))
\end{aligned} \tag{39}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). (\forall V1a \in \\
& A_27a. ((\exists V2x \in A_27a. ((V2x = V1a) \wedge (p \ (ap \ V0P \ V2x)))) \Leftrightarrow (p \ (\\
& ap \ V0P \ V1a))))
\end{aligned} \tag{40}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty \ A_27a \Rightarrow ((\forall V0a \in (ty_2Ellist_2Ellist \\
& A_27a). ((ap \ (c_2Ellist_2Ellist_abs \ A_27a) \ (ap \ (c_2Ellist_2Ellist_rep \\
& A_27a) \ V0a)) = V0a)) \wedge (\forall V1r \in ((ty_2Eoption_2Eoption \ A_27a)^{ty_2Enum_2Enum}). \\
& ((p \ (ap \ (c_2Ellist_2Elrep_ok \ A_27a) \ V1r)) \Leftrightarrow ((ap \ (c_2Ellist_2Ellist_rep \\
& A_27a) \ (ap \ (c_2Ellist_2Ellist_abs \ A_27a) \ V1r)) = V1r))))
\end{aligned} \tag{41}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0h \in A_27a. (\forall V1t \in \\
& (ty_2Ellist_2Ellist \ A_27a). (((ap \ (c_2Ellist_2ELHD \ A_27a) \ (ap \\
& (ap \ (c_2Ellist_2ELCONS \ A_27a) \ V0h) \ V1t)) = (ap \ (c_2Eoption_2ESOME \\
& A_27a) \ V0h)) \wedge ((ap \ (c_2Ellist_2ELTL \ A_27a) \ (ap \ (ap \ (c_2Ellist_2ELCONS \\
& A_27a) \ V0h) \ V1t)) = (ap \ (c_2Eoption_2ESOME \ (ty_2Ellist_2Ellist \\
& A_27a) \ V1t))))))
\end{aligned} \tag{42}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& (\forall V0f \in (A_27b^{A_27a}).((ap\ (ap\ (c_2Ellist_2ELMAP\ A_27a\ A_27b) \\
& V0f)\ (c_2Ellist_2ELNIL\ A_27a)) = (c_2Ellist_2ELNIL\ A_27b))) \wedge \\
& (\forall V1f \in (A_27b^{A_27a}).(\forall V2h \in A_27a.(\forall V3t \in \\
& (ty_2Ellist_2Ellist\ A_27a).((ap\ (ap\ (c_2Ellist_2ELMAP\ A_27a \\
& A_27b)\ V1f)\ (ap\ (ap\ (c_2Ellist_2ELCONS\ A_27a)\ V2h)\ V3t)) = (ap\ (ap \\
& (c_2Ellist_2ELCONS\ A_27b)\ (ap\ V1f\ V2h))\ (ap\ (ap\ (c_2Ellist_2ELMAP \\
& A_27a\ A_27b)\ V1f)\ V3t))))))))) \\
& \tag{43}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \forall V0f \in ((ty_2Eoption_2Eoption\ (ty_2Epair_2Eprod\ A_27a \\
& A_27b))^{A_27a}).(\forall V1x \in A_27a.(\forall V2v1 \in A_27a.(\forall V3v2 \in \\
& A_27b.(((ap\ V0f\ V1x) = (c_2Eoption_2ENONE\ (ty_2Epair_2Eprod \\
& A_27a\ A_27b)))) \Rightarrow ((ap\ (ap\ (c_2Ellist_2ELUNFOLD\ A_27b\ A_27a)\ V0f) \\
& V1x) = (c_2Ellist_2ELNIL\ A_27b))) \wedge (((ap\ V0f\ V1x) = (ap\ (c_2Eoption_2ESOME \\
& (ty_2Epair_2Eprod\ A_27a\ A_27b))\ (ap\ (ap\ (c_2Epair_2E2C\ A_27a \\
& A_27b)\ V2v1)\ V3v2))) \Rightarrow ((ap\ (ap\ (c_2Ellist_2ELUNFOLD\ A_27b\ A_27a) \\
& V0f)\ V1x) = (ap\ (ap\ (c_2Ellist_2ELCONS\ A_27b)\ V3v2)\ (ap\ (ap\ (c_2Ellist_2ELUNFOLD \\
& A_27b\ A_27a)\ V0f)\ V2v1))))))))) \\
& \tag{44}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \forall V0R \in ((2^{(ty_2Ellist_2Ellist\ A_27b)})^{A_27a}).(\forall V1f \in \\
& ((ty_2Eoption_2Eoption\ (ty_2Epair_2Eprod\ A_27a\ A_27b))^{A_27a}). \\
& (\forall V2s \in A_27a.(\forall V3ll \in (ty_2Ellist_2Ellist\ A_27b). \\
& (((p\ (ap\ (ap\ V0R\ V2s)\ V3ll)) \wedge (\forall V4s \in A_27a.(\forall V5ll \in \\
& (ty_2Ellist_2Ellist\ A_27b).((p\ (ap\ (ap\ V0R\ V4s)\ V5ll)) \Rightarrow (((ap \\
& V1f\ V4s) = (c_2Eoption_2ENONE\ (ty_2Epair_2Eprod\ A_27a\ A_27b)))) \wedge \\
& (V5ll = (c_2Ellist_2ELNIL\ A_27b))) \vee (\exists V6s.27 \in A_27a.(\exists V7x \in \\
& A_27b.(\exists V8ll.27 \in (ty_2Ellist_2Ellist\ A_27b).((ap\ V1f \\
& V4s) = (ap\ (c_2Eoption_2ESOME\ (ty_2Epair_2Eprod\ A_27a\ A_27b)) \\
& (ap\ (ap\ (c_2Epair_2E2C\ A_27a\ A_27b)\ V6s.27)\ V7x))) \wedge (((ap\ (c_2Ellist_2ELHD \\
& A_27b)\ V5ll) = (ap\ (c_2Eoption_2ESOME\ A_27b)\ V7x)) \wedge (((ap\ (c_2Ellist_2ELTL \\
& A_27b)\ V5ll) = (ap\ (c_2Eoption_2ESOME\ (ty_2Ellist_2Ellist\ A_27b)) \\
& V8ll.27)) \wedge (p\ (ap\ (ap\ V0R\ V6s.27)\ V8ll.27))))))))) \Rightarrow ((ap\ (ap \\
& (c_2Ellist_2ELUNFOLD\ A_27b\ A_27a)\ V1f)\ V2s) = V3ll)))))) \\
& \tag{45}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0opt \in (ty_2Eoption_2Eoption \\
& A_27a).((V0opt = (c_2Eoption_2ENONE\ A_27a)) \vee (\exists V1x \in A_27a. \\
& (V0opt = (ap\ (c_2Eoption_2ESOME\ A_27a)\ V1x)))))) \\
& \tag{46}
\end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\ & (\forall V0v \in A.27b. (\forall V1f \in (A.27b^{A.27a}). ((ap\ (ap\ (ap\ (c.2Eoption.2Eoption_CASE \\ & A.27a\ A.27b)\ (c.2Eoption.2ENONE\ A.27a))\ V0v)\ V1f) = V0v))) \wedge (\forall V2x \in \\ & A.27a. (\forall V3v \in A.27b. (\forall V4f \in (A.27b^{A.27a}). ((ap\ (ap \\ & (ap\ (c.2Eoption.2Eoption_CASE\ A.27a\ A.27b)\ (ap\ (c.2Eoption.2ESOME \\ & A.27a)\ V2x))\ V3v)\ V4f) = (ap\ V4f\ V2x)))))) \end{aligned} \quad (47)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0x \in A.27a. (\forall V1y \in \\ & A.27a. (((ap\ (c.2Eoption.2ESOME\ A.27a)\ V0x) = (ap\ (c.2Eoption.2ESOME \\ & A.27a)\ V1y)) \Leftrightarrow (V0x = V1y)))) \end{aligned} \quad (48)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0x \in A.27a. (\neg((c.2Eoption.2ENONE \\ & A.27a) = (ap\ (c.2Eoption.2ESOME\ A.27a)\ V0x)))) \end{aligned} \quad (49)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\ & \forall V0f \in (A.27b^{A.27a}). (\forall V1x \in (ty.2Eoption.2Eoption \\ & A.27a). (\forall V2y \in A.27b. (((ap\ (ap\ (c.2Eoption.2EOPTION_MAP \\ & A.27a\ A.27b)\ V0f)\ V1x) = (ap\ (c.2Eoption.2ESOME\ A.27b)\ V2y)) \Leftrightarrow (\exists V3z \in \\ & A.27a. ((V1x = (ap\ (c.2Eoption.2ESOME\ A.27a)\ V3z)) \wedge (V2y = (ap\ V0f \\ & V3z)))))) \end{aligned} \quad (50)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\ & \forall V0f \in (A.27a^{A.27b}). (\forall V1x \in (ty.2Eoption.2Eoption \\ & A.27b). (((ap\ (ap\ (c.2Eoption.2EOPTION_MAP\ A.27b\ A.27a)\ V0f) \\ & V1x) = (c.2Eoption.2ENONE\ A.27a)) \Leftrightarrow (V1x = (c.2Eoption.2ENONE\ A.27b))) \wedge \\ & (((c.2Eoption.2ENONE\ A.27a) = (ap\ (ap\ (c.2Eoption.2EOPTION_MAP \\ & A.27b\ A.27a)\ V0f)\ V1x)) \Leftrightarrow (V1x = (c.2Eoption.2ENONE\ A.27b)))) \end{aligned} \quad (51)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\ & \forall V0x \in A.27a. (\forall V1y \in A.27b. (\forall V2a \in A.27a. (\forall V3b \in \\ & A.27b. (((ap\ (ap\ (c.2Epair.2E.2C\ A.27a\ A.27b)\ V0x)\ V1y) = (ap\ (ap \\ & (c.2Epair.2E.2C\ A.27a\ A.27b)\ V2a)\ V3b)) \Leftrightarrow ((V0x = V2a) \wedge (V1y = V3b)))))) \end{aligned} \quad (52)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\ & \forall V0x \in (ty.2Epair.2Eprod\ A.27a\ A.27b). (\exists V1q \in A.27a. \\ & (\exists V2r \in A.27b. (V0x = (ap\ (ap\ (c.2Epair.2E.2C\ A.27a\ A.27b) \\ & V1q)\ V2r)))) \end{aligned} \quad (53)$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\
& \quad nonempty\ A_27c \Rightarrow (\forall V0f \in ((A_27c^{A_27b})^{A_27a}). (\forall V1x \in \\
& \quad A_27a. (\forall V2y \in A_27b. ((ap\ (ap\ (c_2Epair_2EUNCURRY\ A_27a \\
& \quad A_27b\ A_27c)\ V0f)\ (ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27b)\ V1x)\ V2y))) = \\
& \quad (ap\ (ap\ V0f\ V1x)\ V2y))))))
\end{aligned} \tag{54}$$

Theorem 1

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\
& \quad nonempty\ A_27c \Rightarrow (\forall V0f \in (A_27b^{A_27a}). (\forall V1g \in ((ty_2Eoption_2Eoption \\
& \quad (ty_2Epair_2Eprod\ A_27c\ A_27a))^{A_27c}). (\forall V2s \in A_27c. (\\
& \quad (ap\ (ap\ (c_2Ellist_2ELMAP\ A_27a\ A_27b)\ V0f)\ (ap\ (ap\ (c_2Ellist_2ELUNFOLD \\
& \quad A_27a\ A_27c)\ V1g)\ V2s))) = (ap\ (ap\ (c_2Ellist_2ELUNFOLD\ A_27b\ A_27c) \\
& \quad (\lambda V3s \in A_27c. (ap\ (ap\ (c_2Eoption_2EOPTION_MAP\ (ty_2Epair_2Eprod \\
& \quad A_27c\ A_27a)\ (ty_2Epair_2Eprod\ A_27c\ A_27b))\ (ap\ (c_2Epair_2EUNCURRY \\
& \quad A_27c\ A_27a)\ (ty_2Epair_2Eprod\ A_27c\ A_27b)))\ (\lambda V4x \in A_27c. (\\
& \quad \lambda V5y \in A_27a. (ap\ (ap\ (c_2Epair_2E_2C\ A_27c\ A_27b)\ V4x)\ (ap\ V0f \\
& \quad V5y))))))\ (ap\ V1g\ V3s))))\ V2s))))))
\end{aligned}$$