thm_2Ellist_2ELNTH__ADD
(TMH4NQjKffWJ3bkaVQg3ZoXiGi4AiARA9ez)

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Definition 1 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$
of type $\iota \Rightarrow \iota.$

Definition 2 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$
of type $\iota \Rightarrow \iota.$

Definition 3 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap\ V0P\ (ap\ ($c_2Emin_2E_40$\ A$

Definition 4 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$
of type $\iota.$

Definition 5 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ ($c_2Emin_2E_3D$\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 6 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap\ (ap\ ($c_2Emin_2E_3D$\ (2^{A-27}$

Definition 7 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ ($c_2Ebool_2E_21$\ 2)\ (\lambda V2t \in 2$

Let $ty\_2Eoption\_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Eoption\_2Eoption\ A0) \qquad (1)$$

Let $ty\_2Ellist\_2Ellist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Ellist\_2Ellist\ A0) \qquad (2)$$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \qquad (3)$$

Let $c\_2Ellist\_2ELNTH : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ellist\_2ELNTH\ A\_27a \in (((ty\_2Eoption\_2Eoption$$
$$A\_27a)^{(ty\_2Ellist\_2Ellist\ A\_27a)})^{ty\_2Enum\_2Enum})$$
$$(4)$$

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \qquad (5)$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \qquad (6)$$

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Definition 8 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Definition 9 *We define* c_2Earithmetic_2EZERO *to be* c_2Enum_2E0.

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \qquad (7)$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \qquad (8)$$

Definition 10 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num$

Let $c\_2Earithmetic\_2E\_2B : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})$$
$$(9)$$

Definition 11 *We define* c_2Earithmetic_2EBIT1 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

Definition 12 *We define* c_2Earithmetic_2ENUMERAL *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x$.

Let $c\_2Earithmetic\_2E\_2D : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})$$
$$(10)$$

Let $c\_2Ellist\_2Ellist\_\_rep : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ellist\_2Ellist\_\_rep\ A\_27a \in$$
$$(((ty\_2Eoption\_2Eoption\ A\_27a)^{ty\_2Enum\_2Enum})^{(ty\_2Ellist\_2Ellist\ A\_27a)})$$
$$(11)$$

Let $ty\_2Eone\_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Eone\_2Eone \qquad (12)$$

Definition 13 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ ($c_2Ebool_2E_21$\ 2)\ (\lambda V2t \in$

Let $ty\_2Esum\_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Esum\_2Esum$$
$$A0\ A1)$$
$$(13)$$

Let $c\_2Esum\_2EABS\_\_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Esum\_2EABS\_\_sum$$
$$A\_27a\ A\_27b \in ((ty\_2Esum\_2Esum\ A\_27a\ A\_27b)^{(((2^{A\_27b})^{A\_27a})^2)})$$
$$(14)$$

Definition 14 *We define* c_2Esum_2EINL *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0e \in A\_27a.(ap\ (c_2Esum_2EABS

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Let $c\_2Eoption\_2Eoption\_\_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Eoption\_2Eoption\_\_ABS\ A\_27a \in$$
$$((ty\_2Eoption\_2Eoption\ A\_27a)^{(ty\_2Esum\_2Esum\ A\_27a\ ty\_2Eone\_2Eone)}) \quad (15)$$

Definition 15 *We define* c_2Eoption_2ESOME *to be* $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.(ap\ (c\_2Eoption\_2Eoption\_$

Definition 16 *We define* c_2Ebool_2EF *to be* $(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 17 *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

Let $c\_2Ellist\_2Ellist\_\_abs : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ellist\_2Ellist\_\_abs\ A\_27a \in$$
$$((ty\_2Ellist\_2Ellist\ A\_27a)^{((ty\_2Eoption\_2Eoption\ A\_27a)^{ty\_2Enum\_2Enum})}) \quad (16)$$

Definition 18 *We define* c_2Ellist_2ELCONS *to be* $\lambda A\_27a : \iota.\lambda V0h \in A\_27a.\lambda V1t \in (ty\_2Ellist\_2Ellist\ A\_$

Definition 19 *We define* c_2Eone_2Eone *to be* $(ap\ (c\_2Emin\_2E\_40\ ty\_2Eone\_2Eone)\ (\lambda V0x \in ty\_2Eone\_2$

Definition 20 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ c\_2Emin\_2E\_3D\_3D\_3E\ V0t)\ c\_2Ebool\_2E$

Definition 21 *We define* c_2Esum_2EINR *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0e \in A\_27b.(ap\ (c\_2Esum\_2EABS$

Definition 22 *We define* c_2Eoption_2ENONE *to be* $\lambda A\_27a : \iota.(ap\ (c\_2Eoption\_2Eoption\_\_ABS\ A\_27a)\ ($

Definition 23 *We define* c_2Ellist_2ELNIL *to be* $\lambda A\_27a : \iota.(ap\ (c\_2Ellist\_2Ellist\_\_abs\ A\_27a)\ (\lambda V0n \in ty\_$

Let $c\_2Ellist\_2ELDROP : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ellist\_2ELDROP\ A\_27a \in (((ty\_2Eoption\_2Eoption$$
$$(ty\_2Ellist\_2Ellist\ A\_27a))^{(ty\_2Ellist\_2Ellist\ A\_27a)})^{ty\_2Enum\_2Enum})$$
$$(17)$$

Let $c\_2Earithmetic\_2EEVEN : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EEVEN \in (2^{ty\_2Enum\_2Enum}) \quad (18)$$

Let $c\_2Earithmetic\_2EODD : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EODD \in (2^{ty\_2Enum\_2Enum}) \quad (19)$$

Definition 24 *We define* c_2Eprim__rec_2E_3C *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enu$

Definition 25 *We define* c_2Earithmetic_2E_3E *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2En$

Definition 26 *We define* c_2Earithmetic_2E_3E_3D *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2$

Definition 27 *We define* c_2Eprim__rec_2EPRE *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ (ap\ (ap\ (c\_2Ebool\_2E$

Let $c\_2Earithmetic\_2EEXP : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EEXP \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{20}$$

Let $c\_2Earithmetic\_2E\_2A : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2A \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{21}$$

Definition 28 *We define* c_2Enumeral_2EiiSUC *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap$ c_2Enum_2ESUC $(ap$

Definition 29 *We define* c_2Enumeral_2EiZ *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x.$

Definition 30 *We define* c_2Earithmetic_2EBIT2 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

Definition 31 *We define* c_2Earithmetic_2E_3C_3D *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2$

Let $c\_2Eoption\_2EOPTION\_\_BIND : \iota{\Rightarrow}\iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}\forall A\_27b.nonempty\ A\_27b{\Rightarrow}c\_2Eoption\_2EOPTION\_\_BIND$$
$$A\_27a\ A\_27b \in (((ty\_2Eoption\_2Eoption\ A\_27a)^{((ty\_2Eoption\_2Eoption\ A\_27a)^{A\_27b})})^{(ty\_2Eoption\_2Eoption\ A\_27b}) \tag{22}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)$$
$$\text{c\_2Enum\_2E0}) = V0m)) \tag{23}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.($$
$$(ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ V1n) = (ap\ (ap\ c\_2Earithmetic\_2E\_2B$$
$$V1n)\ V0m)))) \tag{24}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.($$
$$(ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ V1n) = (ap\ (ap\ c\_2Earithmetic\_2E\_2B$$
$$V1n)\ V0m)))) \tag{25}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.($$
$$\forall V2p \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)$$
$$(ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V1n)\ V2p)) = (ap\ (ap\ c\_2Earithmetic\_2E\_2B$$
$$(ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ V1n))\ V2p))))) \tag{26}$$

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Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.($$
$$((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ \textsf{c\_2Enum\_2E0})\ V0m) = \textsf{c\_2Enum\_2E0}) \wedge$$
$$(((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0m)\ \textsf{c\_2Enum\_2E0}) = \textsf{c\_2Enum\_2E0}) \wedge$$
$$(((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ (ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}$$
$$(ap\ \textsf{c\_2Earithmetic\_2EBIT1}\ \textsf{c\_2Earithmetic\_2EZERO})))\ V0m) = V0m) \wedge$$
$$(((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0m)\ (ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}$$
$$(ap\ \textsf{c\_2Earithmetic\_2EBIT1}\ \textsf{c\_2Earithmetic\_2EZERO}))) = V0m) \wedge ($$
$$((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ (ap\ \textsf{c\_2Enum\_2ESUC}\ V0m))\ V1n) = (ap$$
$$(ap\ c\_2Earithmetic\_2E\_2B\ (ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0m)\ V1n))$$
$$V1n)) \wedge ((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0m)\ (ap\ \textsf{c\_2Enum\_2ESUC}\ V1n)) =$$
$$(ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ (ap\ (ap\ c\_2Earithmetic\_2E\_2A$$
$$V0m)\ V1n))))))))))$$
$$\tag{27}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.($$
$$\forall V2p \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \textsf{c\_2Earithmetic\_2E\_3C\_3D}$$
$$(ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ V1n))\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B$$
$$V0m)\ V2p))) \Leftrightarrow (p\ (ap\ (ap\ \textsf{c\_2Earithmetic\_2E\_3C\_3D}\ V1n)\ V2p)))))))$$
$$\tag{28}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.($$
$$(\neg(V0m = V1n)) \Leftrightarrow ((p\ (ap\ (ap\ \textsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ \textsf{c\_2Enum\_2ESUC}$$
$$V0m))\ V1n)) \vee (p\ (ap\ (ap\ \textsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ \textsf{c\_2Enum\_2ESUC}$$
$$V1n))\ V0m))))))$$
$$\tag{29}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.((ap\ \textsf{c\_2Enum\_2ESUC}\ V0n) = (ap\ (ap$$
$$c\_2Earithmetic\_2E\_2B\ (ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \textsf{c\_2Earithmetic\_2EBIT1}$$
$$\textsf{c\_2Earithmetic\_2EZERO}))) \ V0n)))$$
$$\tag{30}$$

Assume the following.
$$True \tag{31}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in$$
$$A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t)))$$
$$\tag{32}$$

Assume the following.

$$(\forall V0t \in 2.(((True \vee (p\ V0t)) \Leftrightarrow True) \wedge ((((p\ V0t) \vee True) \Leftrightarrow True) \wedge$$
$$(((False \vee (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \vee False) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \vee$$
$$(p\ V0t)) \Leftrightarrow (p\ V0t)))))))$$
$$\tag{33}$$

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Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p\ V0t)))\Leftrightarrow(p\ V0t)))\wedge(((\neg True)\Leftrightarrow False)\wedge \\ ((\neg False)\Leftrightarrow True)))$$

(34)

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0x \in A\_27a.((V0x = V0x)\Leftrightarrow \\ True))$$

(35)

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0x \in A\_27a.(\forall V1y \in \\ A\_27a.((V0x = V1y)\Leftrightarrow(V1y = V0x))))$$

(36)

Assume the following.

$$(\forall V0t \in 2.(((True\Leftrightarrow(p\ V0t))\Leftrightarrow(p\ V0t))\wedge((((p\ V0t)\Leftrightarrow True)\Leftrightarrow \\ (p\ V0t))\wedge(((False\Leftrightarrow(p\ V0t))\Leftrightarrow(\neg(p\ V0t)))\wedge(((p\ V0t)\Leftrightarrow False)\Leftrightarrow(\neg(\\ p\ V0t)))))))$$

(37)

Assume the following.

$$(\forall V0t \in 2.(((p\ V0t)\Rightarrow False)\Leftrightarrow((p\ V0t)\Leftrightarrow False)))$$

(38)

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0l \in (ty\_2Ellist\_2Ellist \\ A\_27a).((V0l = (\mathsf{c\_2Ellist\_2ELNIL}\ A\_27a))\vee(\exists V1h \in A\_27a. \\ (\exists V2t \in (ty\_2Ellist\_2Ellist\ A\_27a).(V0l = (ap\ (ap\ (\mathsf{c\_2Ellist\_2ELCONS} \\ A\_27a)\ V1h)\ V2t))))))$$

(39)

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow\forall A\_27b.nonempty\ A\_27b\Rightarrow\forall A\_27c. \\ nonempty\ A\_27c\Rightarrow((\forall V0n \in ty\_2Enum\_2Enum.((ap\ (ap\ (c\_2Ellist\_2ELNTH \\ A\_27a)\ V0n)\ (\mathsf{c\_2Ellist\_2ELNIL}\ A\_27a)) = (\mathsf{c\_2Eoption\_2ENONE}\ A\_27a)))\wedge \\ ((\forall V1h \in A\_27b.(\forall V2t \in (ty\_2Ellist\_2Ellist\ A\_27b). \\ ((ap\ (ap\ (\mathsf{c\_2Ellist\_2ELNTH}\ A\_27b)\ \mathsf{c\_2Enum\_2E0})\ (ap\ (ap\ (\mathsf{c\_2Ellist\_2ELCONS} \\ A\_27b)\ V1h)\ V2t)) = (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27b)\ V1h))))\wedge(\forall V3n \in \\ ty\_2Enum\_2Enum.(\forall V4h \in A\_27c.(\forall V5t \in (ty\_2Ellist\_2Ellist \\ A\_27c).((ap\ (ap\ (c\_2Ellist\_2ELNTH\ A\_27c)\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V3n)) \\ (ap\ (ap\ (\mathsf{c\_2Ellist\_2ELCONS}\ A\_27c)\ V4h)\ V5t)) = (ap\ (ap\ (c\_2Ellist\_2ELNTH \\ A\_27c)\ V3n)\ V5t)))))))))$$

(40)

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Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c.$$
$$nonempty\ A\_27c \Rightarrow ((\forall V0ll \in (ty\_2Ellist\_2Ellist\ A\_27a).($$
$$(ap\ (ap\ (c\_2Ellist\_2ELDROP\ A\_27a)\ \mathsf{c\_2Enum\_2E0})\ V0ll) = (ap\ (\mathsf{c\_2Eoption\_2ESOME}$$
$$(ty\_2Ellist\_2Ellist\ A\_27a))\ V0ll))) \wedge ((\forall V1n \in ty\_2Enum\_2Enum.$$
$$((ap\ (ap\ (c\_2Ellist\_2ELDROP\ A\_27b)\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V1n))\ (\mathsf{c\_2Ellist\_2ELNIL}$$
$$A\_27b)) = (\mathsf{c\_2Eoption\_2ENONE}\ (ty\_2Ellist\_2Ellist\ A\_27b)))) \wedge$$
$$(\forall V2n \in ty\_2Enum\_2Enum.(\forall V3h \in A\_27c.(\forall V4t \in$$
$$(ty\_2Ellist\_2Ellist\ A\_27c).((ap\ (ap\ (c\_2Ellist\_2ELDROP\ A\_27c)$$
$$(ap\ \mathsf{c\_2Enum\_2ESUC}\ V2n))\ (ap\ (ap\ (\mathsf{c\_2Ellist\_2ELCONS}\ A\_27c)\ V3h)$$
$$V4t)) = (ap\ (ap\ (c\_2Ellist\_2ELDROP\ A\_27c)\ V2n)\ V4t)))))))$$

$$(41)$$

Assume the following.

$$(\forall V0P \in (2^{ty\_2Enum\_2Enum}).(((p\ (ap\ V0P\ \mathsf{c\_2Enum\_2E0})) \wedge$$
$$(\forall V1n \in ty\_2Enum\_2Enum.((p\ (ap\ V0P\ V1n)) \Rightarrow (p\ (ap\ V0P\ (ap\ \mathsf{c\_2Enum\_2ESUC}$$
$$V1n)))))) \Rightarrow (\forall V2n \in ty\_2Enum\_2Enum.(p\ (ap\ V0P\ V2n)))))$$

$$(42)$$

Assume the following.

$((\forall V0n \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2B$
c_2Enum_2E0) $V0n) = V0n))\wedge((\forall V1n \in ty\_2Enum\_2Enum.((ap$
$(ap\ c\_2Earithmetic\_2E\_2B\ V1n)$ c_2Enum_2E0) $= V1n))\wedge((\forall V2n \in$
$ty\_2Enum\_2Enum.(\forall V3m \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2B$
$(ap$ c_2Earithmetic_2ENUMERAL $V2n))\ (ap$ c_2Earithmetic_2ENUMERAL
$V3m)) = (ap$ c_2Earithmetic_2ENUMERAL $(ap$ c_2Enumeral_2EiZ $(ap$
$(ap\ c\_2Earithmetic\_2E\_2B\ V2n)\ V3m))))))\wedge((\forall V4n \in ty\_2Enum\_2Enum.$
$((ap\ (ap\ c\_2Earithmetic\_2E\_2A$ c_2Enum_2E0) $V4n) =$ c_2Enum_2E0)$)\wedge$
$((\forall V5n \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2A$
$V5n)$ c_2Enum_2E0) $=$ c_2Enum_2E0)$)\wedge((\forall V6n \in ty\_2Enum\_2Enum.$
$(\forall V7m \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ ($
ap c_2Earithmetic_2ENUMERAL $V6n))\ (ap$ c_2Earithmetic_2ENUMERAL
$V7m)) = (ap$ c_2Earithmetic_2ENUMERAL $(ap\ (ap\ c\_2Earithmetic\_2E\_2A$
$V6n)\ V7m)))))\wedge((\forall V8n \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2D$
c_2Enum_2E0) $V8n) =$ c_2Enum_2E0)$)\wedge((\forall V9n \in ty\_2Enum\_2Enum.$
$((ap\ (ap\ c\_2Earithmetic\_2E\_2D\ V9n)$ c_2Enum_2E0) $= V9n))\wedge((\forall V10n \in$
$ty\_2Enum\_2Enum.(\forall V11m \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2D$
$(ap$ c_2Earithmetic_2ENUMERAL $V10n))\ (ap$ c_2Earithmetic_2ENUMERAL
$V11m)) = (ap$ c_2Earithmetic_2ENUMERAL $(ap\ (ap\ c\_2Earithmetic\_2E\_2D$
$V10n)\ V11m)))))\wedge((\forall V12n \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2EEXP$
c_2Enum_2E0) $(ap$ c_2Earithmetic_2ENUMERAL $(ap$ c_2Earithmetic_2EBIT1
$V12n))) =$ c_2Enum_2E0)$)\wedge((\forall V13n \in ty\_2Enum\_2Enum.((ap$
$(ap\ c\_2Earithmetic\_2EEXP$ c_2Enum_2E0) $(ap$ c_2Earithmetic_2ENUMERAL
$(ap$ c_2Earithmetic_2EBIT2 $V13n))) =$ c_2Enum_2E0)$)\wedge((\forall V14n \in$
$ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2EEXP\ V14n)$ c_2Enum_2E0) $=$
$(ap$ c_2Earithmetic_2ENUMERAL $(ap$ c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))$)\wedge$
$((\forall V15n \in ty\_2Enum\_2Enum.(\forall V16m \in ty\_2Enum\_2Enum.$
$((ap\ (ap\ c\_2Earithmetic\_2EEXP\ (ap$ c_2Earithmetic_2ENUMERAL $V15n))$
$(ap$ c_2Earithmetic_2ENUMERAL $V16m)) = (ap$ c_2Earithmetic_2ENUMERAL
$(ap\ (ap\ c\_2Earithmetic\_2EEXP\ V15n)\ V16m)))))\wedge(((ap$ c_2Enum_2ESUC
c_2Enum_2E0) $= (ap$ c_2Earithmetic_2ENUMERAL $(ap$ c_2Earithmetic_2EBIT1
c_2Earithmetic_2EZERO)))$)\wedge((\forall V17n \in ty\_2Enum\_2Enum.($
$(ap$ c_2Enum_2ESUC $(ap$ c_2Earithmetic_2ENUMERAL $V17n)) = (ap$ c_2Earithmetic_2ENUMERAL
$(ap$ c_2Enum_2ESUC $V17n))))\wedge(((ap$ c_2Eprim__rec_2EPRE c_2Enum_2E0) $=$
c_2Enum_2E0)$)\wedge((\forall V18n \in ty\_2Enum\_2Enum.((ap$ c_2Eprim__rec_2EPRE
$(ap$ c_2Earithmetic_2ENUMERAL $V18n)) = (ap$ c_2Earithmetic_2ENUMERAL
$(ap$ c_2Eprim__rec_2EPRE $V18n))))\wedge((\forall V19n \in ty\_2Enum\_2Enum.$
$(((ap$ c_2Earithmetic_2ENUMERAL $V19n) =$ c_2Enum_2E0)$\Leftrightarrow(V19n =$ c_2Earithmetic_2EZERO)))$\wedge$
$((\forall V20n \in ty\_2Enum\_2Enum.(($c_2Enum_2E0 $= (ap$ c_2Earithmetic_2ENUMERAL
$V20n))\Leftrightarrow(V20n =$ c_2Earithmetic_2EZERO)))$\wedge((\forall V21n \in ty\_2Enum\_2Enum.$
$(\forall V22m \in ty\_2Enum\_2Enum.(((ap$ c_2Earithmetic_2ENUMERAL
$V21n) = (ap$ c_2Earithmetic_2ENUMERAL $V22m))\Leftrightarrow(V21n = V22m))))\wedge$
$((\forall V23n \in ty\_2Enum\_2Enum.((p\ (ap\ (ap$ c_2Eprim__rec_2E_3C
$V23n)$ c_2Enum_2E0))$\Leftrightarrow False))\wedge((\forall V24n \in ty\_2Enum\_2Enum.$
$((p\ (ap\ (ap$ c_2Eprim__rec_2E_3C c_2Enum_2E0) $(ap$ c_2Earithmetic_2ENUMERAL
$V24n)))\Leftrightarrow(p\ (ap\ (ap$ c_2Eprim__rec_2E_3C c_2Earithmetic_2EZERO)
$V24n))))\wedge((\forall V25n \in ty\_2Enum\_2Enum.(\forall V26m \in ty\_2Enum\_2Enum.$
$((p\ (ap\ (ap$ c_2Eprim__rec_2E_3C $(ap$ c_2Earithmetic_2ENUMERAL
$V25n))\ (ap$ c_2Earithmetic_2ENUMERAL $V26m)))\Leftrightarrow(p\ (ap\ (ap$ c_2Eprim__rec_2E_3C
$V25n)\ V26m)))))\wedge((\forall V27n \in ty\_2Enum\_2Enum.((p\ (ap\ (ap$ c_2Earithmetic_2E_3E
c_2Enum_2E0) $V27n))\Leftrightarrow False))\wedge((\forall V28n \in ty\_2Enum\_2Enum.$
$((p\ (ap\ (ap$ c_2Earithmetic_2E_3E $(ap$ c_2Earithmetic_2ENUMERAL
$V28n))$ c_2Enum_2E0))$\Leftrightarrow(p\ (ap\ (ap$ c_2Eprim__rec_2E_3C c_2Earithmetic_2EZERO)
$V28n))))\wedge((\forall V29n \in ty\_2Enum\_2Enum.(\forall V30m \in ty\_2Enum\_2Enum.$
$((p\ (ap\ (ap$ c_2Earithmetic_2E_3E $(ap$ c_2Earithmetic_2ENUMERAL
$V29n))\ (ap$ c_2Earithmetic_2ENUMERAL $V30m)))\Leftrightarrow(p\ (ap\ (ap$ c_2Eprim__rec_2E_3C
$V30m)\ V29n)))))\wedge((\forall V31n \in ty\_2Enum\_2Enum.((p\ (ap\ (ap$ c_2Earithmetic_2E_3C_3D
c_2Enum_2E0) $V31n))\Leftrightarrow True))\wedge((\forall V32n \in ty\_2Enum\_2Enum.$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\forall V1m \in ty\_2Enum\_2Enum.($$
$$((ap\ \mathsf{c\_2Enumeral\_2EiZ}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ \mathsf{c\_2Earithmetic\_2EZERO})$$
$$V0n)) = V0n) \wedge (((ap\ \mathsf{c\_2Enumeral\_2EiZ}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B$$
$$V0n)\ \mathsf{c\_2Earithmetic\_2EZERO})) = V0n) \wedge (((ap\ \mathsf{c\_2Enumeral\_2EiZ}\ ($$
$$ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V0n))\ ($$
$$ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V1m))) = (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ ($$
$$ap\ \mathsf{c\_2Enumeral\_2EiZ}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ V1m)))) \wedge$$
$$(((ap\ \mathsf{c\_2Enumeral\_2EiZ}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}$$
$$V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V1m))) = (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}$$
$$(ap\ \mathsf{c\_2Enum\_2ESUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ V1m)))) \wedge ($$
$$((ap\ \mathsf{c\_2Enumeral\_2EiZ}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}$$
$$V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V1m))) = (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}$$
$$(ap\ \mathsf{c\_2Enum\_2ESUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ V1m)))) \wedge ($$
$$((ap\ \mathsf{c\_2Enumeral\_2EiZ}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}$$
$$V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V1m))) = (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}$$
$$(ap\ \mathsf{c\_2Enum\_2ESUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ V1m)))) \wedge ($$
$$((ap\ \mathsf{c\_2Enum\_2ESUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ \mathsf{c\_2Earithmetic\_2EZERO})$$
$$V0n)) = (ap\ \mathsf{c\_2Enum\_2ESUC}\ V0n)) \wedge (((ap\ \mathsf{c\_2Enum\_2ESUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B$$
$$V0n)\ \mathsf{c\_2Earithmetic\_2EZERO})) = (ap\ \mathsf{c\_2Enum\_2ESUC}\ V0n)) \wedge (((ap$$
$$\mathsf{c\_2Enum\_2ESUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}$$
$$V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V1m))) = (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}$$
$$(ap\ \mathsf{c\_2Enum\_2ESUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ V1m)))) \wedge ($$
$$((ap\ \mathsf{c\_2Enum\_2ESUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}$$
$$V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V1m))) = (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}$$
$$(ap\ \mathsf{c\_2Enum\_2ESUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ V1m)))) \wedge ($$
$$((ap\ \mathsf{c\_2Enum\_2ESUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}$$
$$V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V1m))) = (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}$$
$$(ap\ \mathsf{c\_2Enum\_2ESUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ V1m)))) \wedge ($$
$$((ap\ \mathsf{c\_2Enum\_2ESUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}$$
$$V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V1m))) = (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}$$
$$(ap\ \mathsf{c\_2Enumeral\_2EiiSUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ V1m)))) \wedge$$
$$(((ap\ \mathsf{c\_2Enumeral\_2EiiSUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ \mathsf{c\_2Earithmetic\_2EZERO})$$
$$V0n)) = (ap\ \mathsf{c\_2Enumeral\_2EiiSUC}\ V0n)) \wedge (((ap\ \mathsf{c\_2Enumeral\_2EiiSUC}$$
$$(ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ \mathsf{c\_2Earithmetic\_2EZERO})) = ($$
$$ap\ \mathsf{c\_2Enumeral\_2EiiSUC}\ V0n)) \wedge (((ap\ \mathsf{c\_2Enumeral\_2EiiSUC}\ (ap\ ($$
$$ap\ c\_2Earithmetic\_2E\_2B\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}$$
$$V1m))) = (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B$$
$$V0n)\ V1m)))) \wedge (((ap\ \mathsf{c\_2Enumeral\_2EiiSUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B$$
$$(ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V1m))) =$$
$$(ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ (ap\ \mathsf{c\_2Enumeral\_2EiiSUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B$$
$$V0n)\ V1m)))) \wedge (((ap\ \mathsf{c\_2Enumeral\_2EiiSUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B$$
$$(ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V1m))) =$$
$$(ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ (ap\ \mathsf{c\_2Enumeral\_2EiiSUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B$$
$$V0n)\ V1m)))) \wedge ((ap\ \mathsf{c\_2Enumeral\_2EiiSUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B$$
$$(ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V1m))) =$$
$$(ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ (ap\ \mathsf{c\_2Enumeral\_2EiiSUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B$$
$$V0n)\ V1m))))))))))))))))))))))))$$

$$(44)$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\forall V1m \in ty\_2Enum\_2Enum.($$
$$((p\ (ap\ (ap\ \textsf{c\_2Earithmetic\_2E\_3C\_3D}\ \textsf{c\_2Earithmetic\_2EZERO})\ V0n)) \Leftrightarrow$$
$$True) \wedge (((p\ (ap\ (ap\ \textsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ \textsf{c\_2Earithmetic\_2EBIT1}$$
$$V0n))\ \textsf{c\_2Earithmetic\_2EZERO})) \Leftrightarrow False) \wedge (((p\ (ap\ (ap\ \textsf{c\_2Earithmetic\_2E\_3C\_3D}$$
$$(ap\ \textsf{c\_2Earithmetic\_2EBIT2}\ V0n))\ \textsf{c\_2Earithmetic\_2EZERO})) \Leftrightarrow False) \wedge$$
$$(((p\ (ap\ (ap\ \textsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ \textsf{c\_2Earithmetic\_2EBIT1}$$
$$V0n))\ (ap\ \textsf{c\_2Earithmetic\_2EBIT1}\ V1m))) \Leftrightarrow (p\ (ap\ (ap\ \textsf{c\_2Earithmetic\_2E\_3C\_3D}$$
$$V0n)\ V1m))) \wedge (((p\ (ap\ (ap\ \textsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ \textsf{c\_2Earithmetic\_2EBIT1}$$
$$V0n))\ (ap\ \textsf{c\_2Earithmetic\_2EBIT2}\ V1m))) \Leftrightarrow (p\ (ap\ (ap\ \textsf{c\_2Earithmetic\_2E\_3C\_3D}$$
$$V0n)\ V1m))) \wedge (((p\ (ap\ (ap\ \textsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ \textsf{c\_2Earithmetic\_2EBIT2}$$
$$V0n))\ (ap\ \textsf{c\_2Earithmetic\_2EBIT1}\ V1m))) \Leftrightarrow (\neg(p\ (ap\ (ap\ \textsf{c\_2Earithmetic\_2E\_3C\_3D}$$
$$V1m)\ V0n)))) \wedge ((p\ (ap\ (ap\ \textsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ \textsf{c\_2Earithmetic\_2EBIT2}$$
$$V0n))\ (ap\ \textsf{c\_2Earithmetic\_2EBIT2}\ V1m))) \Leftrightarrow (p\ (ap\ (ap\ \textsf{c\_2Earithmetic\_2E\_3C\_3D}$$
$$V0n)\ V1m)))))))))))$$

$$(45)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ($$
$$(\forall V0f \in ((ty\_2Eoption\_2Eoption\ A\_27a)^{A\_27b}).((ap\ (ap\ ($$
$$c\_2Eoption\_2EOPTION\_\_BIND\ A\_27a\ A\_27b)\ (\textsf{c\_2Eoption\_2ENONE}\ A\_27b))$$
$$V0f) = (\textsf{c\_2Eoption\_2ENONE}\ A\_27a))) \wedge (\forall V1x \in A\_27b.(\forall V2f \in$$
$$((ty\_2Eoption\_2Eoption\ A\_27a)^{A\_27b}).((ap\ (ap\ (c\_2Eoption\_2EOPTION\_\_BIND$$
$$A\_27a\ A\_27b)\ (ap\ (\textsf{c\_2Eoption\_2ESOME}\ A\_27b)\ V1x))\ V2f) = (ap\ V2f\ V1x)))))$$

$$(46)$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0a \in ty\_2Enum\_2Enum.($$
$$\forall V1b \in ty\_2Enum\_2Enum.(\forall V2ll \in (ty\_2Ellist\_2Ellist$$
$$A\_27a).((ap\ (ap\ (c\_2Ellist\_2ELNTH\ A\_27a)\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B$$
$$V0a)\ V1b))\ V2ll) = (ap\ (ap\ (c\_2Eoption\_2EOPTION\_\_BIND\ A\_27a\ (ty\_2Ellist\_2Ellist$$
$$A\_27a))\ (ap\ (ap\ (c\_2Ellist\_2ELDROP\ A\_27a)\ V0a)\ V2ll))\ (ap\ (c\_2Ellist\_2ELNTH$$
$$A\_27a)\ V1b))))))$$