# thm_2Ellist_2ELNTH__HD__LDROP (TMd6NNuBNT8hxmVG1fXFfawCHpFgeRUcTz6)

October 26, 2020

**Definition 1** *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota{\Rightarrow}\iota$.

**Definition 2** *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \tag{1}$$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{2}$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \tag{3}$$

**Definition 3** *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Let $ty\_2Eoption\_2Eoption : \iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0{\Rightarrow}nonempty\ (ty\_2Eoption\_2Eoption\ A0) \tag{4}$$

Let $ty\_2Ellist\_2Ellist : \iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0{\Rightarrow}nonempty\ (ty\_2Ellist\_2Ellist\ A0) \tag{5}$$

Let $c\_2Ellist\_2Ellist\_\_rep : \iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}c\_2Ellist\_2Ellist\_\_rep\ A\_27a \in (((ty\_2Eoption\_2Eoption\ A\_27a)^{ty\_2Enum\_2Enum})^{(ty\_2Ellist\_2Ellist\ A\_27a)}) \tag{6}$$

**Definition 4** *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

**Definition 5** *We define* c_2Ellist_2ELHD *to be* $\lambda A\_27a : \iota.\lambda V0ll \in (ty\_2Ellist\_2Ellist\ A\_27a).(ap\ (ap\ (c\_2E$

Let $c\_2Ellist\_2ELNTH : \iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}c\_2Ellist\_2ELNTH\ A\_27a \in (((ty\_2Eoption\_2Eoption\ A\_27a)^{(ty\_2Ellist\_2Ellist\ A\_27a)})^{ty\_2Enum\_2Enum}) \quad (7)$$

**Definition 6** *We define* c_2Earithmetic_2EZERO *to be* c_2Enum_2E0*.*

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \quad (8)$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \quad (9)$$

**Definition 7** *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num\ ($

Let $c\_2Earithmetic\_2E\_2B : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (10)$$

**Definition 8** *We define* c_2Earithmetic_2EBIT1 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic\_$

**Definition 9** *We define* c_2Earithmetic_2ENUMERAL *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x$*.*

Let $c\_2Ellist\_2Ellist\_\_abs : \iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}c\_2Ellist\_2Ellist\_\_abs\ A\_27a \in ((ty\_2Ellist\_2Ellist\ A\_27a)^{((ty\_2Eoption\_2Eoption\ A\_27a)^{ty\_2Enum\_2Enum})}) \quad (11)$$

Let $ty\_2Eone\_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Eone\_2Eone \quad (12)$$

**Definition 10** *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P{\Rightarrow}p\ Q)$ *of type* $\iota$*.*

**Definition 11** *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in$

Let $ty\_2Esum\_2Esum : \iota{\Rightarrow}\iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0{\Rightarrow}\forall A1.nonempty\ A1{\Rightarrow}nonempty\ (ty\_2Esum\_2Esum\ A0\ A1) \quad (13)$$

Let $c\_2Esum\_2EABS\_\_sum : \iota{\Rightarrow}\iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}\forall A\_27b.nonempty\ A\_27b{\Rightarrow}c\_2Esum\_2EABS\_\_sum\ A\_27a\ A\_27b \in ((ty\_2Esum\_2Esum\ A\_27a\ A\_27b)^{(((2^{A\_27b})^{A\_27a})^{2})}) \quad (14)$$

**Definition 12** *We define* c_2Esum_2EINL *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0e \in A\_27a.(ap\ (c\_2Esum\_2EABS$

Let $c\_2Eoption\_2Eoption\_\_ABS : \iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}c\_2Eoption\_2Eoption\_\_ABS\ A\_27a \in ((ty\_2Eoption\_2Eoption\ A\_27a)^{(ty\_2Esum\_2Esum\ A\_27a\ ty\_2Eone\_2Eone)}) \quad (15)$$

**Definition 13** *We define* c_2Eoption_2ESOME *to be* $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.(ap\ (c\_2Eoption\_2Eoption\_\_$

**Definition 14** *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^{A}.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$
*of type* $\iota{\Rightarrow}\iota$.

**Definition 15** *We define* c_2Eone_2Eone *to be* $(ap\ (\text{c\_2Emin\_2E\_40}\ ty\_2Eone\_2Eone)\ (\lambda V0x \in ty\_2Eone\_2$

**Definition 16** *We define* c_2Ebool_2EF *to be* $(ap\ (\text{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

**Definition 17** *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \text{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \text{c\_2Ebool\_2E}$

**Definition 18** *We define* c_2Esum_2EINR *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0e \in A\_27b.(ap\ (c\_2Esum\_2EABS$

**Definition 19** *We define* c_2Eoption_2ENONE *to be* $\lambda A\_27a : \iota.(ap\ (c\_2Eoption\_2Eoption\_\_ABS\ A\_27a)\ ($

Let $c\_2Eoption\_2Eoption\_\_CASE : \iota{\Rightarrow}\iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}\forall A\_27b.nonempty\ A\_27b{\Rightarrow}c\_2Eoption\_2Eoption\_\_CASE\ A\_27a\ A\_27b \in (((A\_27b^{(A\_27b^{A\_27a})})^{A\_27b})^{(ty\_2Eoption\_2Eoption\ A\_27a)}) \quad (16)$$

**Definition 20** *We define* c_2Ellist_2ELTL *to be* $\lambda A\_27a : \iota.\lambda V0ll \in (ty\_2Ellist\_2Ellist\ A\_27a).(ap\ (ap\ (ap$

Let $c\_2Earithmetic\_2EFUNPOW : \iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}c\_2Earithmetic\_2EFUNPOW\ A\_27a \in (((A\_27a^{A\_27a})^{ty\_2Enum\_2Enum})^{(A\_27a^{A\_27a})}) \quad (17)$$

Let $c\_2Ellist\_2ELDROP : \iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}c\_2Ellist\_2ELDROP\ A\_27a \in (((ty\_2Eoption\_2Eoption\ (ty\_2Ellist\_2Ellist\ A\_27a))^{(ty\_2Ellist\_2Ellist\ A\_27a)})^{ty\_2Enum\_2Enum}) \quad (18)$$

**Definition 21** *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ (\text{c\_2Emin\_2E\_40}$

**Definition 22** *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\text{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in$

Let $c\_2Eoption\_2EOPTION\_\_MAP : \iota{\Rightarrow}\iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}\forall A\_27b.nonempty\ A\_27b{\Rightarrow}c\_2Eoption\_2EOPTION\_\_MAP\ A\_27a\ A\_27b \in (((ty\_2Eoption\_2Eoption\ A\_27b)^{(ty\_2Eoption\_2Eoption\ A\_27a)})^{(A\_27b^{A\_27a})}) \quad (19)$$

Let $c\_2Eoption\_2EOPTION\_\_JOIN : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Eoption\_2EOPTION\_\_JOIN\ A\_27a \in ((ty\_2Eoption\_2Eoption\ A\_27a)^{(ty\_2Eoption\_2Eoption\ (ty\_2Eoption\_2Eoption\ A\_27a))}) \tag{20}$$

Let $c\_2Eoption\_2EOPTION\_\_BIND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Eoption\_2EOPTION\_\_BIND\ A\_27a\ A\_27b \in (((ty\_2Eoption\_2Eoption\ A\_27a)^{((ty\_2Eoption\_2Eoption\ A\_27a)^{A\_27b})})^{(ty\_2Eoption\_2Eoption\ A\_27} \tag{21}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow ((\forall V0f \in (A\_27a^{A\_27a}).(\forall V1x \in A\_27a.((ap\ (ap\ (ap\ (c\_2Earithmetic\_2EFUNPOW\ A\_27a)\ V0f)\ \mathsf{c\_2Enum\_2E0})\ V1x) = V1x))) \wedge (\forall V2f \in (A\_27a^{A\_27a}).(\forall V3n \in ty\_2Enum\_2Enum.(\forall V4x \in A\_27a.((ap\ (ap\ (ap\ (c\_2Earithmetic\_2EFUNPOW\ A\_27a)\ V2f)\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V3n))\ V4x) = (ap\ (ap\ (ap\ (c\_2Earithmetic\_2EFUNPOW\ A\_27a)\ V2f)\ V3n)\ (ap\ V2f\ V4x)))))) \tag{22}$$

Assume the following.

$$True \tag{23}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \tag{24}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{25}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{26}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow ((\forall V0ll \in (ty\_2Ellist\_2Ellist\ A\_27a).((ap\ (ap\ (c\_2Ellist\_2ELNTH\ A\_27a)\ \mathsf{c\_2Enum\_2E0})\ V0ll) = (ap\ (\mathsf{c\_2Ellist\_2ELHD}\ A\_27a)\ V0ll))) \wedge (\forall V1n \in ty\_2Enum\_2Enum.(\forall V2ll \in (ty\_2Ellist\_2Ellist\ A\_27a).((ap\ (ap\ (c\_2Ellist\_2ELNTH\ A\_27a)\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V1n))\ V2ll) = (ap\ (c\_2Eoption\_2EOPTION\_\_JOIN\ A\_27a)\ (ap\ (ap\ (c\_2Eoption\_2EOPTION\_\_MAP\ (ty\_2Ellist\_2Ellist\ A\_27a)\ (ty\_2Eoption\_2Eoption\ A\_27a))\ (ap\ (c\_2Ellist\_2ELNTH\ A\_27a)\ V1n))\ (ap\ (\mathsf{c\_2Ellist\_2ELTL}\ A\_27a)\ V2ll))))))) \tag{27}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0n \in ty\_2Enum\_2Enum.( \forall V1ll \in (ty\_2Ellist\_2Ellist\ A\_27a).((ap\ (ap\ (c\_2Ellist\_2ELDROP\ A\_27a)\ V0n)\ V1ll) = (ap\ (ap\ (ap\ (c\_2Earithmetic\_2EFUNPOW\ (ty\_2Eoption\_2Eoption\ (ty\_2Ellist\_2Ellist\ A\_27a)))\ (\lambda V2m \in (ty\_2Eoption\_2Eoption\ (ty\_2Ellist\_2Ellist\ A\_27a)).(ap\ (ap\ (c\_2Eoption\_2EOPTION\_\_BIND\ (ty\_2Ellist\_2Ellist\ A\_27a)\ (ty\_2Ellist\_2Ellist\ A\_27a))\ V2m)\ (\mathsf{c\_2Ellist\_2ELTL}\ A\_27a))))\ V0n)\ (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ (ty\_2Ellist\_2Ellist\ A\_27a))\ V1ll)))))) \tag{28}$$

Assume the following.

$$(\forall V0P \in (2^{ty\_2Enum\_2Enum}).(((p\ (ap\ V0P\ \mathsf{c\_2Enum\_2E0})) \wedge (\forall V1n \in ty\_2Enum\_2Enum.((p\ (ap\ V0P\ V1n)) \Rightarrow (p\ (ap\ V0P\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V1n)))))) \Rightarrow (\forall V2n \in ty\_2Enum\_2Enum.(p\ (ap\ V0P\ V2n))))) \tag{29}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0opt \in (ty\_2Eoption\_2Eoption\ A\_27a).((V0opt = (\mathsf{c\_2Eoption\_2ENONE}\ A\_27a)) \vee (\exists V1x \in A\_27a.(V0opt = (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V1x))))) \tag{30}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ((\forall V0f \in (A\_27b^{A\_27a}).(\forall V1x \in A\_27a.((ap\ (ap\ (c\_2Eoption\_2EOPTION\_\_MAP\ A\_27a\ A\_27b)\ V0f)\ (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V1x)) = (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27b)\ (ap\ V0f\ V1x))))) \wedge (\forall V2f \in (A\_27b^{A\_27a}).((ap\ (ap\ (c\_2Eoption\_2EOPTION\_\_MAP\ A\_27a\ A\_27b)\ V2f)\ (\mathsf{c\_2Eoption\_2ENONE}\ A\_27a)) = (\mathsf{c\_2Eoption\_2ENONE}\ A\_27b)))) \tag{31}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (((ap\ (c\_2Eoption\_2EOPTION\_\_JOIN\ A\_27a)\ (\mathsf{c\_2Eoption\_2ENONE}\ (ty\_2Eoption\_2Eoption\ A\_27a))) = (\mathsf{c\_2Eoption\_2ENONE}\ A\_27a)) \wedge (\forall V0x \in (ty\_2Eoption\_2Eoption\ A\_27a).((ap\ (c\_2Eoption\_2EOPTION\_\_JOIN\ A\_27a)\ (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ (ty\_2Eoption\_2Eoption\ A\_27a))\ V0x)) = V0x))) \tag{32}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ((\forall V0f \in ((ty\_2Eoption\_2Eoption\ A\_27a)^{A\_27b}).((ap\ (ap\ (c\_2Eoption\_2EOPTION\_\_BIND\ A\_27a\ A\_27b)\ (\mathsf{c\_2Eoption\_2ENONE}\ A\_27b))\ V0f) = (\mathsf{c\_2Eoption\_2ENONE}\ A\_27a))) \wedge (\forall V1x \in A\_27b.(\forall V2f \in ((ty\_2Eoption\_2Eoption\ A\_27a)^{A\_27b}).((ap\ (ap\ (c\_2Eoption\_2EOPTION\_\_BIND\ A\_27a\ A\_27b)\ (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27b)\ V1x))\ V2f) = (ap\ V2f\ V1x))))) \tag{33}$$

**Theorem 1**

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0n \in ty\_2Enum\_2Enum.( \forall V1ll \in (ty\_2Ellist\_2Ellist\ A\_27a).((ap\ (ap\ (c\_2Ellist\_2ELNTH\ A\_27a)\ V0n)\ V1ll) = (ap\ (ap\ (c\_2Eoption\_2EOPTION\_\_BIND\ A\_27a\ (ty\_2Ellist\_2Ellist\ A\_27a))\ (ap\ (ap\ (c\_2Ellist\_2ELDROP\ A\_27a)\ V0n)\ V1ll))\ (\mathsf{c\_2Ellist\_2ELHD}\ A\_27a)))))$$