thm_2Ellist_2ELNTH__LAPPEND (TMcoBTep1K5G7JcBaGU2xC3vLo21xBce4is)

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Definition 1 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.\mathbf{if}\ (\exists x \in A.p\ (ap\ P\ x))\ \mathbf{then}\ (the\ (\lambda x.x \in A \wedge p$
of type $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$
of type $\iota \Rightarrow \iota$.

Definition 3 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 4 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27a}$

Definition 5 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 6 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$
of type ι.

Definition 7 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Definition 8 *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.(a$

Let $ty\_2Ellist\_2Ellist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Ellist\_2Ellist\ A0) \quad (1)$$

Let $c\_2Ellist\_2ELAPPEND : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ellist\_2ELAPPEND\ A\_27a \in (((ty\_2Ellist\_2Ellist\ A\_27a)^{(ty\_2Ellist\_2Ellist\ A\_27a)})^{(ty\_2Ellist\_2Ellist\ A\_27a)}) \quad (2)$$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \quad (3)$$

Let $ty\_2Eone\_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Eone\_2Eone \quad (4)$$

Definition 9 *We define* c_2Eone_2Eone *to be* $(ap\ (\mathsf{c\_2Emin\_2E\_40}\ ty\_2Eone\_2Eone)\ (\lambda V0x \in ty\_2Eone\_2E$

Definition 10 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \mathsf{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \mathsf{c\_2Ebool\_2E}$

Let $ty\_2Esum\_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Esum\_2Esum\ A0\ A1) \tag{5}$$

Let $c\_2Esum\_2EABS\_\_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Esum\_2EABS\_\_sum\ A\_27a\ A\_27b \in ((ty\_2Esum\_2Esum\ A\_27a\ A\_27b)^{(((2^{A\_27b})^{A\_27a})^2)}) \tag{6}$$

Definition 11 *We define* c_2Esum_2EINR *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0e \in A\_27b.(ap\ (c\_2Esum\_2EABS$

Let $ty\_2Eoption\_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Eoption\_2Eoption\ A0) \tag{7}$$

Let $c\_2Eoption\_2Eoption\_\_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Eoption\_2Eoption\_\_ABS\ A\_27a \in ((ty\_2Eoption\_2Eoption\ A\_27a)^{(ty\_2Esum\_2Esum\ A\_27a\ ty\_2Eone\_2Eone)}) \tag{8}$$

Definition 12 *We define* c_2Eoption_2ENONE *to be* $\lambda A\_27a : \iota.(ap\ (c\_2Eoption\_2Eoption\_\_ABS\ A\_27a)\ ($

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \tag{9}$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \tag{10}$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \tag{11}$$

Definition 13 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num$

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \tag{12}$$

Definition 14 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Definition 15 *We define* c_2Earithmetic_2EZERO *to be* c_2Enum_2E0.

Let $c\_2Earithmetic\_2E\_2B : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{13}$$

Definition 16 *We define* c_2Earithmetic_2EBIT1 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

Definition 17 *We define* c_2Earithmetic_2ENUMERAL *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x.$

Let $c\_2Earithmetic\_2E\_2D : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{14}$$

Let $c\_2Ellist\_2Ellist\_\_rep : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ellist\_2Ellist\_\_rep\ A\_27a \in (((ty\_2Eoption\_2Eoption\ A\_27a)^{ty\_2Enum\_2Enum})^{(ty\_2Ellist\_2Ellist\ A\_27a)}) \tag{15}$$

Definition 18 *We define* c_2Esum_2EINL *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0e \in A\_27a.(ap\ (c\_2Esum\_2EABS$

Definition 19 *We define* c_2Eoption_2ESOME *to be* $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.(ap\ (c\_2Eoption\_2Eoption\_\_$

Let $c\_2Ellist\_2Ellist\_\_abs : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ellist\_2Ellist\_\_abs\ A\_27a \in ((ty\_2Ellist\_2Ellist\ A\_27a)^{((ty\_2Eoption\_2Eoption\ A\_27a)^{ty\_2Enum\_2Enum})}) \tag{16}$$

Definition 20 *We define* c_2Ellist_2ELCONS *to be* $\lambda A\_27a : \iota.\lambda V0h \in A\_27a.\lambda V1t \in (ty\_2Ellist\_2Ellist\ A$

Definition 21 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap$ (c_2Emin_2E_40

Definition 22 *We define* c_2Ellist_2ELNIL *to be* $\lambda A\_27a : \iota.(ap\ (c\_2Ellist\_2Ellist\_\_abs\ A\_27a)\ (\lambda V0n \in ty$

Definition 23 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap$ (c_2Ebool_2E_21 2) $(\lambda V2t \in$

Definition 24 *We define* c_2Ellist_2Ellength__rel *to be* $\lambda A\_27a : \iota.(\lambda V0a0 \in (ty\_2Ellist\_2Ellist\ A\_27a).(\lambda V$

Definition 25 *We define* c_2Ellist_2ELFINITE *to be* $\lambda A\_27a : \iota.(\lambda V0a0 \in (ty\_2Ellist\_2Ellist\ A\_27a).(ap$ (c

Definition 26 *We define* c_2Ellist_2ELLENGTH *to be* $\lambda A\_27a : \iota.\lambda V0ll \in (ty\_2Ellist\_2Ellist\ A\_27a).(ap\ (a$

Let $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Elist\_2Elist\ A0) \tag{17}$$

Let $c\_2Elist\_2EEL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2EEL\ A\_27a \in ((A\_27a^{(ty\_2Elist\_2Elist\ A\_27a)})^{ty\_2Enum\_2Enum}) \tag{18}$$

Let $c\_2Ellist\_2ELTAKE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ellist\_2ELTAKE\ A\_27a \in (((ty\_2Eoption\_2Eoption\ (ty\_2Elist\_2Elist\ A\_27a))^{(ty\_2Ellist\_2Ellist\ A\_27a)})^{ty\_2Enum\_2Enum}) \tag{19}$$

Let $c\_2Ellist\_2ELNTH : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ellist\_2ELNTH\ A\_27a \in (((ty\_2Eoption\_2Eoption\ A\_27a)^{(ty\_2Ellist\_2Ellist\ A\_27a)})^{ty\_2Enum\_2Enum}) \tag{20}$$

Let $c\_2Earithmetic\_2EEVEN : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EEVEN \in (2^{ty\_2Enum\_2Enum}) \tag{21}$$

Let $c\_2Earithmetic\_2EODD : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EODD \in (2^{ty\_2Enum\_2Enum}) \tag{22}$$

Definition 27 *We define* c_2Eprim__rec_2E_3C *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enu$

Definition 28 *We define* c_2Earithmetic_2E_3E *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2En$

Definition 29 *We define* c_2Earithmetic_2E_3E_3D *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2$

Definition 30 *We define* c_2Eprim__rec_2EPRE *to be* $\lambda V0m \in ty_2Enum_2Enum.(ap\ (ap\ (ap\ ($c_2Ebool_2E

Let $c\_2Earithmetic\_2EEXP : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EEXP \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{23}$$

Let $c\_2Earithmetic\_2E\_2A : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2A \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{24}$$

Definition 31 *We define* c_2Enumeral_2EiZ *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x.$

Definition 32 *We define* c_2Earithmetic_2EBIT2 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

Definition 33 *We define* c_2Earithmetic_2E_3C_3D *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2$

Let $c\_2Eoption\_2Eoption\_\_CASE : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Eoption\_2Eoption\_\_CASE\ A\_27a\ A\_27b \in (((A\_27b^{(A\_27b^{A\_27a})})^{A\_27b})^{(ty\_2Eoption\_2Eoption\ A\_27a)}) \tag{25}$$

Let $c\_2Eoption\_2EOPTION\_\_MAP : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Eoption\_2EOPTION\_\_MAP\ A\_27a\ A\_27b \in (((ty\_2Eoption\_2Eoption\ A\_27b)^{(ty\_2Eoption\_2Eoption\ A\_27a)})^{(A\_27b^{A\_27a})}) \tag{26}$$

Let $c\_2Eoption\_2EIS\_\_SOME : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Eoption\_2EIS\_\_SOME\ A\_27a \in (2^{(ty\_2Eoption\_2Eoption\ A\_27a)}) \tag{27}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(((ap\ (ap\ c\_2Earithmetic\_2E\_2B\ \mathsf{c\_2Enum\_2E0})\ V0m) = V0m) \wedge (((ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ \mathsf{c\_2Enum\_2E0}) = V0m) \wedge (((ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V0m))\ V1n) = (ap\ \mathsf{c\_2Enum\_2ESUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ V1n))) \wedge ((ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V1n)) = (ap\ \mathsf{c\_2Enum\_2ESUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ V1n)))))))) \tag{28}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ V1n) = (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V1n)\ V0m)))) \tag{29}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.((V0m = \mathsf{c\_2Enum\_2E0}) \vee (\exists V1n \in ty\_2Enum\_2Enum.(V0m = (ap\ \mathsf{c\_2Enum\_2ESUC}\ V1n))))) \tag{30}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V0m))\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V1n))) \Leftrightarrow (p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V0m)\ V1n))))) \tag{31}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.((\neg(p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V0m)\ V1n))) \Leftrightarrow (p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V1n)\ V0m))))) \tag{32}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(((ap\ (ap\ c\_2Earithmetic\_2E\_2D\ \mathsf{c\_2Enum\_2E0})\ V0m) = \mathsf{c\_2Enum\_2E0}) \wedge ((ap\ (ap\ c\_2Earithmetic\_2E\_2D\ V0m)\ \mathsf{c\_2Enum\_2E0}) = V0m))) \tag{33}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\forall V1m \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2D\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V0n))\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V1m)) = (ap\ (ap\ c\_2Earithmetic\_2E\_2D\ V0n)\ V1m)))) \tag{34}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ \mathsf{c\_2Enum\_2E0})\ V0m) = \mathsf{c\_2Enum\_2E0})\wedge (((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0m)\ \mathsf{c\_2Enum\_2E0}) = \mathsf{c\_2Enum\_2E0})\wedge (((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO})))\ V0m) = V0m)\wedge (((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0m)\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO}))) = V0m)\wedge(((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V0m))\ V1n) = (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0m)\ V1n))\ V1n))\wedge((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0m)\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V1n)) = (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ (ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0m)\ V1n)))))))))) \tag{35}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(\forall V2p \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ V1n))\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ V2p)))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V1n)\ V2p)))))) \tag{36}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.((ap\ \mathsf{c\_2Enum\_2ESUC}\ V0n) = (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO})))\ V0n))) \tag{37}$$

Assume the following.

$$True \tag{38}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1)\Rightarrow(p\ V1t2))\Rightarrow(((p\ V1t2)\Rightarrow(p\ V0t1))\Rightarrow((p\ V0t1)\Leftrightarrow(p\ V1t2)))))) \tag{39}$$

Assume the following.

$$(\forall V0t \in 2.(False\Rightarrow(p\ V0t))) \tag{40}$$

Assume the following.

$$(\forall V0t \in 2.((p\ V0t)\vee(\neg(p\ V0t)))) \tag{41}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0t \in 2.((\forall V1x \in A\_27a.(p\ V0t))\Leftrightarrow(p\ V0t))) \tag{42}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\exists V1x \in A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \tag{43}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t)))))))) \tag{44}$$

Assume the following.

$$((\forall V0t \in 2.((\neg (\neg (p\ V0t))) \Leftrightarrow (p\ V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \tag{45}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(V0x = V0x)) \tag{46}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{47}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{48}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg (p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg (p\ V0t))))))) \tag{49}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).(\forall V1Q \in 2.(((\forall V2x \in A\_27a.(p\ (ap\ V0P\ V2x))) \wedge (p\ V1Q)) \Leftrightarrow (\forall V3x \in A\_27a.((p\ (ap\ V0P\ V3x)) \wedge (p\ V1Q))))))) \tag{50}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (2^{A\_27a}).((\forall V2x \in A\_27a.((p\ V0P) \vee (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow ((p\ V0P) \vee (\forall V3x \in A\_27a.(p\ (ap\ V1Q\ V3x))))))) \tag{51}$$

Assume the following.

$$(\forall V0t \in 2.(((p\ V0t) \Rightarrow False) \Leftrightarrow ((p\ V0t) \Leftrightarrow False))) \tag{52}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \tag{53}$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x\_27 \in 2.(\forall V2y \in 2.(\forall V3y\_27 \in 2.((((p\ V0x) \Leftrightarrow (p\ V1x\_27)) \wedge ((p\ V1x\_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y\_27)))) \Rightarrow (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x\_27) \Rightarrow (p\ V3y\_27)))))))) \tag{54}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in 2.(\forall V2x \in A\_27a.(\forall V3x\_27 \in A\_27a.(\forall V4y \in A\_27a.(\forall V5y\_27 \in A\_27a.((((p\ V0P) \Leftrightarrow (p\ V1Q)) \wedge (((p\ V1Q) \Rightarrow (V2x = V3x\_27)) \wedge ((\neg(p\ V1Q)) \Rightarrow (V4y = V5y\_27)))) \Rightarrow ((ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ V0P)\ V2x)\ V4y) = (ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ V1Q)\ V3x\_27)\ V5y\_27))))))))) \tag{55}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0a \in A\_27a.(\exists V1x \in A\_27a.(V1x = V0a))) \tag{56}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow ((\forall V0t1 \in A\_27a.(\forall V1t2 \in A\_27a.((ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ \mathsf{c\_2Ebool\_2ET})\ V0t1)\ V1t2) = V0t1))) \wedge (\forall V2t1 \in A\_27a.(\forall V3t2 \in A\_27a.((ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ \mathsf{c\_2Ebool\_2EF})\ V2t1)\ V3t2) = V3t2)))) \tag{57}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c.nonempty\ A\_27c \Rightarrow ((\forall V0n \in ty\_2Enum\_2Enum.((ap\ (ap\ (c\_2Ellist\_2ELNTH\ A\_27a)\ V0n)\ (\mathsf{c\_2Ellist\_2ELNIL}\ A\_27a)) = (\mathsf{c\_2Eoption\_2ENONE}\ A\_27a))) \wedge ((\forall V1h \in A\_27b.(\forall V2t \in (ty\_2Ellist\_2Ellist\ A\_27b).((ap\ (ap\ (c\_2Ellist\_2ELNTH\ A\_27b)\ \mathsf{c\_2Enum\_2E0})\ (ap\ (ap\ (\mathsf{c\_2Ellist\_2ELCONS}\ A\_27b)\ V1h)\ V2t)) = (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27b)\ V1h)))) \wedge (\forall V3n \in ty\_2Enum\_2Enum.(\forall V4h \in A\_27c.(\forall V5t \in (ty\_2Ellist\_2Ellist\ A\_27c).((ap\ (ap\ (c\_2Ellist\_2ELNTH\ A\_27c)\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V3n))\ (ap\ (ap\ (\mathsf{c\_2Ellist\_2ELCONS}\ A\_27c)\ V4h)\ V5t)) = (ap\ (ap\ (c\_2Ellist\_2ELNTH\ A\_27c)\ V3n)\ V5t))))))) \tag{58}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0n \in ty\_2Enum\_2Enum.(\forall V1ll \in (ty\_2Ellist\_2Ellist\ A\_27a).(((ap\ (ap\ (c\_2Ellist\_2ELTAKE\ A\_27a)\ V0n)\ V1ll) = (\mathsf{c\_2Eoption\_2ENONE}\ (ty\_2Elist\_2Elist\ A\_27a))) \Rightarrow ((ap\ (ap\ (c\_2Ellist\_2ELNTH\ A\_27a)\ V0n)\ V1ll) = (\mathsf{c\_2Eoption\_2ENONE}\ A\_27a))))) \tag{59}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow ((\forall V0x \in (ty\_2Ellist\_2Ellist\ A\_27a).((ap\ (ap\ (c\_2Ellist\_2ELAPPEND\ A\_27a)\ (\mathsf{c\_2Ellist\_2ELNIL}\ A\_27a))\ V0x) = V0x)) \land (\forall V1h \in A\_27a.(\forall V2t \in (ty\_2Ellist\_2Ellist\ A\_27a).(\forall V3x \in (ty\_2Ellist\_2Ellist\ A\_27a).((ap\ (ap\ (c\_2Ellist\_2ELAPPEND\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Ellist\_2ELCONS}\ A\_27a)\ V1h)\ V2t))\ V3x) = (ap\ (ap\ (\mathsf{c\_2Ellist\_2ELCONS}\ A\_27a)\ V1h)\ (ap\ (ap\ (c\_2Ellist\_2ELAPPEND\ A\_27a)\ V2t)\ V3x))))))) \tag{60}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0n \in ty\_2Enum\_2Enum.(\forall V1l1 \in (ty\_2Ellist\_2Ellist\ A\_27a).(\forall V2l2 \in (ty\_2Ellist\_2Ellist\ A\_27a).((p\ (ap\ (c\_2Eoption\_2EIS\_\_SOME\ (ty\_2Elist\_2Elist\ A\_27a))\ (ap\ (ap\ (c\_2Ellist\_2ELTAKE\ A\_27a)\ V0n)\ V1l1))) \Rightarrow ((ap\ (ap\ (c\_2Ellist\_2ELTAKE\ A\_27a)\ V0n)\ (ap\ (ap\ (c\_2Ellist\_2ELAPPEND\ A\_27a)\ V1l1)\ V2l2)) = (ap\ (ap\ (c\_2Ellist\_2ELTAKE\ A\_27a)\ V0n)\ V1l1)))))) \tag{61}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (((ap\ (\mathsf{c\_2Ellist\_2ELLENGTH}\ A\_27a)\ (\mathsf{c\_2Ellist\_2ELNIL}\ A\_27a)) = (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ ty\_2Enum\_2Enum)\ \mathsf{c\_2Enum\_2E0})) \land (\forall V0h \in A\_27b.(\forall V1t \in (ty\_2Ellist\_2Ellist\ A\_27b).((ap\ (\mathsf{c\_2Ellist\_2ELLENGTH}\ A\_27b)\ (ap\ (ap\ (\mathsf{c\_2Ellist\_2ELCONS}\ A\_27b)\ V0h)\ V1t)) = (ap\ (ap\ (c\_2Eoption\_2EOPTION\_\_MAP\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)\ \mathsf{c\_2Enum\_2ESUC})\ (ap\ (\mathsf{c\_2Ellist\_2ELLENGTH}\ A\_27b)\ V1t)))))) \tag{62}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0ll \in (ty\_2Ellist\_2Ellist\ A\_27a).((p\ (ap\ (\mathsf{c\_2Ellist\_2ELFINITE}\ A\_27a)\ V0ll)) \Rightarrow (\exists V1n \in ty\_2Enum\_2Enum.((ap\ (\mathsf{c\_2Ellist\_2ELLENGTH}\ A\_27a)\ V0ll) = (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ ty\_2Enum\_2Enum)\ V1n))))) \tag{63}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0ll \in (ty\_2Ellist\_2Ellist\ A\_27a).((p\ (ap\ (\mathsf{c\_2Ellist\_2ELFINITE}\ A\_27a)\ V0ll)) \Leftrightarrow (\exists V1n \in ty\_2Enum\_2Enum.((ap\ (\mathsf{c\_2Ellist\_2ELLENGTH}\ A\_27a)\ V0ll) = (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ ty\_2Enum\_2Enum)\ V1n))))) \tag{64}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{(ty\_2Ellist\_2Ellist\ A\_27a)}). (((p\ (ap\ V0P\ (\mathsf{c\_2Ellist\_2ELNIL}\ A\_27a))) \wedge (\forall V1h \in A\_27a.(\forall V2t \in (ty\_2Ellist\_2Ellist\ A\_27a).(((p\ (ap\ (\mathsf{c\_2Ellist\_2ELFINITE}\ A\_27a)\ V2t)) \wedge (p\ (ap\ V0P\ V2t))) \Rightarrow (p\ (ap\ V0P\ (ap\ (ap\ (\mathsf{c\_2Ellist\_2ELCONS}\ A\_27a)\ V1h)\ V2t))))))) \Rightarrow (\forall V3a0 \in (ty\_2Ellist\_2Ellist\ A\_27a). ((p\ (ap\ (\mathsf{c\_2Ellist\_2ELFINITE}\ A\_27a)\ V3a0)) \Rightarrow (p\ (ap\ V0P\ V3a0))))))) \tag{65}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0n \in ty\_2Enum\_2Enum.(\forall V1ll \in (ty\_2Ellist\_2Ellist\ A\_27a).(\forall V2m \in ty\_2Enum\_2Enum. (\forall V3l \in (ty\_2Elist\_2Elist\ A\_27a).((((ap\ (ap\ (c\_2Ellist\_2ELTAKE\ A\_27a)\ V0n)\ V1ll) = (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ (ty\_2Elist\_2Elist\ A\_27a))\ V3l)) \wedge (p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V2m)\ V0n))) \Rightarrow ((ap\ (ap\ (c\_2Ellist\_2ELNTH\ A\_27a)\ V2m)\ V1ll) = (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ (ap\ (ap\ (c\_2Elist\_2EEL\ A\_27a)\ V2m)\ V3l)))))))) \tag{66}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0ll \in (ty\_2Ellist\_2Ellist\ A\_27a).((p\ (ap\ (\mathsf{c\_2Ellist\_2ELFINITE}\ A\_27a)\ V0ll)) \Leftrightarrow (\exists V1n \in ty\_2Enum\_2Enum.((ap\ (ap\ (c\_2Ellist\_2ELNTH\ A\_27a)\ V1n)\ V0ll) = (\mathsf{c\_2Eoption\_2ENONE}\ A\_27a))))) \tag{67}$$

Assume the following.

((∀V0n ∈ ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2B
c_2Enum_2E0) V0n) = V0n))∧((∀V1n ∈ ty_2Enum_2Enum.((ap
(ap c_2Earithmetic_2E_2B V1n) c_2Enum_2E0) = V1n))∧((∀V2n ∈
ty_2Enum_2Enum.(∀V3m ∈ ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2B
(ap c_2Earithmetic_2ENUMERAL V2n)) (ap c_2Earithmetic_2ENUMERAL
V3m)) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Enumeral_2EiZ (ap
(ap c_2Earithmetic_2E_2B V2n) V3m))))))∧((∀V4n ∈ ty_2Enum_2Enum.
((ap (ap c_2Earithmetic_2E_2A c_2Enum_2E0) V4n) = c_2Enum_2E0))∧
((∀V5n ∈ ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2A
V5n) c_2Enum_2E0) = c_2Enum_2E0))∧((∀V6n ∈ ty_2Enum_2Enum.
(∀V7m ∈ ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2A (
ap c_2Earithmetic_2ENUMERAL V6n)) (ap c_2Earithmetic_2ENUMERAL
V7m)) = (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2E_2A
V6n) V7m)))))∧((∀V8n ∈ ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2D
c_2Enum_2E0) V8n) = c_2Enum_2E0))∧((∀V9n ∈ ty_2Enum_2Enum.
((ap (ap c_2Earithmetic_2E_2D V9n) c_2Enum_2E0) = V9n))∧((∀V10n ∈
ty_2Enum_2Enum.(∀V11m ∈ ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2D
(ap c_2Earithmetic_2ENUMERAL V10n)) (ap c_2Earithmetic_2ENUMERAL
V11m)) = (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2E_2D
V10n) V11m)))))∧((∀V12n ∈ ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2EEXP
c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1
V12n))) = c_2Enum_2E0))∧((∀V13n ∈ ty_2Enum_2Enum.((ap
(ap c_2Earithmetic_2EEXP c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL
(ap c_2Earithmetic_2EBIT2 V13n))) = c_2Enum_2E0))∧((∀V14n ∈
ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2EEXP V14n) c_2Enum_2E0) =
(ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))))∧
((∀V15n ∈ ty_2Enum_2Enum.(∀V16m ∈ ty_2Enum_2Enum.
((ap (ap c_2Earithmetic_2EEXP (ap c_2Earithmetic_2ENUMERAL V15n))
(ap c_2Earithmetic_2ENUMERAL V16m)) = (ap c_2Earithmetic_2ENUMERAL
(ap (ap c_2Earithmetic_2EEXP V15n) V16m)))))∧(((ap c_2Enum_2ESUC
c_2Enum_2E0) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1
c_2Earithmetic_2EZERO)))∧((∀V17n ∈ ty_2Enum_2Enum.(
(ap c_2Enum_2ESUC (ap c_2Earithmetic_2ENUMERAL V17n)) = (ap c_2Earithmetic_2ENUMERAL
(ap c_2Enum_2ESUC V17n))))∧(((ap c_2Eprim__rec_2EPRE c_2Enum_2E0) =
c_2Enum_2E0)∧((∀V18n ∈ ty_2Enum_2Enum.((ap c_2Eprim__rec_2EPRE
(ap c_2Earithmetic_2ENUMERAL V18n)) = (ap c_2Earithmetic_2ENUMERAL
(ap c_2Eprim__rec_2EPRE V18n))))∧((∀V19n ∈ ty_2Enum_2Enum.
(((ap c_2Earithmetic_2ENUMERAL V19n) = c_2Enum_2E0)⇔(V19n = c_2Earithmetic_2EZERO)))∧
((∀V20n ∈ ty_2Enum_2Enum.((c_2Enum_2E0 = (ap c_2Earithmetic_2ENUMERAL
V20n))⇔(V20n = c_2Earithmetic_2EZERO)))∧((∀V21n ∈ ty_2Enum_2Enum.
(∀V22m ∈ ty_2Enum_2Enum.(((ap c_2Earithmetic_2ENUMERAL
V21n) = (ap c_2Earithmetic_2ENUMERAL V22m))⇔(V21n = V22m))))∧
((∀V23n ∈ ty_2Enum_2Enum.((p (ap (ap c_2Eprim__rec_2E_3C
V23n) c_2Enum_2E0))⇔False))∧((∀V24n ∈ ty_2Enum_2Enum.
((p (ap (ap c_2Eprim__rec_2E_3C c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL
V24n)))⇔(p (ap (ap c_2Eprim__rec_2E_3C c_2Earithmetic_2EZERO)
V24n))))∧((∀V25n ∈ ty_2Enum_2Enum.(∀V26m ∈ ty_2Enum_2Enum.
((p (ap (ap c_2Eprim__rec_2E_3C (ap c_2Earithmetic_2ENUMERAL
V25n)) (ap c_2Earithmetic_2ENUMERAL V26m)))⇔(p (ap (ap c_2Eprim__rec_2E_3C
V25n) V26m)))))∧((∀V27n ∈ ty_2Enum_2Enum.((p (ap (ap c_2Earithmetic_2E_3E
c_2Enum_2E0) V27n))⇔False))∧((∀V28n ∈ ty_2Enum_2Enum.
((p (ap (ap c_2Earithmetic_2E_3E (ap c_2Earithmetic_2ENUMERAL
V28n)) c_2Enum_2E0))⇔(p (ap (ap c_2Eprim__rec_2E_3C c_2Earithmetic_2EZERO)
V28n))))∧((∀V29n ∈ ty_2Enum_2Enum.(∀V30m ∈ ty_2Enum_2Enum.
((p (ap (ap c_2Earithmetic_2E_3E (ap c_2Earithmetic_2ENUMERAL
V29n)) (ap c_2Earithmetic_2ENUMERAL V30m)))⇔(p (ap (ap c_2Eprim__rec_2E_3C
V30m) V29n)))))∧((∀V31n ∈ ty_2Enum_2Enum.((p (ap (ap c_2Earithmetic_2E_3C_3D
c_2Enum_2E0) V31n))⇔True))∧((∀V32n ∈ ty_2Enum_2Enum.
((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2ENUMERAL

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\forall V1m \in ty\_2Enum\_2Enum.(((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ \mathsf{c\_2Earithmetic\_2EZERO})\ V0n))\Leftrightarrow True)\wedge(((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V0n))\ \mathsf{c\_2Earithmetic\_2EZERO}))\Leftrightarrow False)\wedge(((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V0n))\ \mathsf{c\_2Earithmetic\_2EZERO}))\Leftrightarrow False)\wedge (((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V1m)))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V0n)\ V1m)))\wedge(((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V1m)))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V0n)\ V1m)))\wedge(((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V1m)))\Leftrightarrow(\neg(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V1m)\ V0n))))\wedge((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V1m)))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V0n)\ V1m))))))))))) \tag{69}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0opt \in (ty\_2Eoption\_2Eoption\ A\_27a).((V0opt = (\mathsf{c\_2Eoption\_2ENONE}\ A\_27a))\vee(\exists V1x \in A\_27a. (V0opt = (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V1x))))) \tag{70}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow\forall A\_27b.nonempty\ A\_27b\Rightarrow((\forall V0v \in A\_27b.(\forall V1f \in (A\_27b^{A\_27a}).((ap\ (ap\ (ap\ (c\_2Eoption\_2Eoption\_\_CASE\ A\_27a\ A\_27b)\ (\mathsf{c\_2Eoption\_2ENONE}\ A\_27a))\ V0v)\ V1f) = V0v)))\wedge(\forall V2x \in A\_27a.(\forall V3v \in A\_27b.(\forall V4f \in (A\_27b^{A\_27a}).((ap\ (ap\ (ap\ (c\_2Eoption\_2Eoption\_\_CASE\ A\_27a\ A\_27b)\ (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V2x))\ V3v)\ V4f) = (ap\ V4f\ V2x)))))) \tag{71}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0x \in A\_27a.(\forall V1y \in A\_27a.(((ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V0x) = (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V1y))\Leftrightarrow(V0x = V1y)))) \tag{72}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0x \in A\_27a.(\neg((\mathsf{c\_2Eoption\_2ENONE}\ A\_27a) = (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V0x)))) \tag{73}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0x \in A\_27a.(\neg((ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V0x) = (\mathsf{c\_2Eoption\_2ENONE}\ A\_27a)))) \tag{74}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ((\forall V0f \in (A\_27b^{A\_27a}).(\forall V1x \in A\_27a.((ap\ (ap\ (c\_2Eoption\_2EOPTION\_\_MAP\ A\_27a\ A\_27b)\ V0f)\ (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V1x)) = (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27b)\ (ap\ V0f\ V1x)))))\wedge(\forall V2f \in (A\_27b^{A\_27a}).((ap\ (ap\ (c\_2Eoption\_2EOPTION\_\_MAP\ A\_27a\ A\_27b)\ V2f)\ (\mathsf{c\_2Eoption\_2ENONE}\ A\_27a)) = (\mathsf{c\_2Eoption\_2ENONE}\ A\_27b)))) \tag{75}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow ((\forall V0x \in A\_27a.((p\ (ap\ (c\_2Eoption\_2EIS\_\_SOME\ A\_27a)\ (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V0x))) \Leftrightarrow True)) \wedge ((p\ (ap\ (c\_2Eoption\_2EIS\_\_SOME\ A\_27a)\ (\mathsf{c\_2Eoption\_2ENONE}\ A\_27a))) \Leftrightarrow False)) \tag{76}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0opt \in (ty\_2Eoption\_2Eoption\ A\_27a).((\exists V1x \in A\_27a.(V0opt = (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V1x))) \vee (V0opt = (\mathsf{c\_2Eoption\_2ENONE}\ A\_27a)))) \tag{77}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\neg(p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V0n)\ \mathsf{c\_2Enum\_2E0})))) \tag{78}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ \mathsf{c\_2Enum\_2E0})\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V0n)))) \tag{79}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \tag{80}$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \tag{81}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow (((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False))))) \tag{82}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p\ V0A)) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False))))) \tag{83}$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p\ V0A))\Rightarrow False)\Rightarrow(((p\ V0A)\Rightarrow False)\Rightarrow False))) \tag{84}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow((p\ V1q)\Leftrightarrow(p\ V2r)))\Leftrightarrow(((p\ V0p)\vee((p\ V1q)\vee(p\ V2r)))\wedge(((p\ V0p)\vee((\neg(p\ V2r))\vee(\neg(p\ V1q))))\wedge(((p\ V1q)\vee((\neg(p\ V2r))\vee(\neg(p\ V0p))))\wedge((p\ V2r)\vee ((\neg(p\ V1q))\vee(\neg(p\ V0p))))))))))) \tag{85}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow((p\ V1q)\wedge(p\ V2r)))\Leftrightarrow(((p\ V0p)\vee((\neg(p\ V1q))\vee(\neg(p\ V2r))))\wedge(((p\ V1q)\vee (\neg(p\ V0p)))\wedge((p\ V2r)\vee(\neg(p\ V0p))))))))) \tag{86}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow((p\ V1q)\vee(p\ V2r)))\Leftrightarrow(((p\ V0p)\vee(\neg(p\ V1q)))\wedge(((p\ V0p)\vee(\neg(p\ V2r)))\wedge ((p\ V1q)\vee((p\ V2r)\vee(\neg(p\ V0p)))))))))) \tag{87}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow((p\ V1q)\Rightarrow(p\ V2r)))\Leftrightarrow(((p\ V0p)\vee(p\ V1q))\wedge(((p\ V0p)\vee(\neg(p\ V2r)))\wedge((\neg(p\ V1q))\vee((p\ V2r)\vee(\neg(p\ V0p)))))))))) \tag{88}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p\ V0p)\Leftrightarrow(\neg(p\ V1q)))\Leftrightarrow(((p\ V0p)\vee (p\ V1q))\wedge((\neg(p\ V1q))\vee(\neg(p\ V0p))))))) \tag{89}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p)\Rightarrow(p\ V1q)))\Rightarrow(p\ V0p)))) \tag{90}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p)\Rightarrow(p\ V1q)))\Rightarrow(\neg(p\ V1q))))) \tag{91}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p)\vee(p\ V1q)))\Rightarrow(\neg(p\ V0p))))) \tag{92}$$

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$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p)\vee(p\ V1q)))\Rightarrow(\neg(p\ V1q))))) \tag{93}$$

Assume the following.

$$(\forall V0p \in 2.((\neg(\neg(p\ V0p)))\Rightarrow(p\ V0p))) \tag{94}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0n \in ty\_2Enum\_2Enum.(\forall V1l1 \in (ty\_2Ellist\_2Ellist\ A\_27a).(\forall V2l2 \in (ty\_2Ellist\_2Ellist\ A\_27a).((ap\ (ap\ (c\_2Ellist\_2ELNTH\ A\_27a)\ V0n)\ (ap\ (ap\ (c\_2Ellist\_2ELAPPEND\ A\_27a)\ V1l1)\ V2l2)) = (ap\ (ap\ (ap\ (c\_2Eoption\_2Eoption\_\_CASE\ ty\_2Enum\_2Enum\ (ty\_2Eoption\_2Eoption\ A\_27a))\ (ap\ (\mathsf{c\_2Ellist\_2ELLENGTH}\ A\_27a)\ V1l1))\ (ap\ (ap\ (c\_2Ellist\_2ELNTH\ A\_27a)\ V0n)\ V1l1))\ (\lambda V3m \in ty\_2Enum\_2Enum.\ (ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ (ty\_2Eoption\_2Eoption\ A\_27a))\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V0n)\ V3m))\ (ap\ (ap\ (c\_2Ellist\_2ELNTH\ A\_27a)\ V0n)\ V1l1))\ (ap\ (ap\ (c\_2Ellist\_2ELNTH\ A\_27a)\ (ap\ (ap\ c\_2Earithmetic\_2E\_2D\ V0n)\ V3m))\ V2l2))))))))$$