

thm_2Ellist_2ELNTH__LGENLIST
(TMGJzX5ByWWHNbtNeAS97XJPRqNcggFZDHi)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ecombin_2ES$ to be $\lambda A.\lambda a : \iota.\lambda A.\lambda b : \iota.\lambda A.\lambda c : \iota.(\lambda V0f \in ((A_27c^{A_27b})^{A_27a}))$

Definition 3 We define $c_2Ecombin_2EC$ to be $\lambda A.\lambda a : \iota.\lambda A.\lambda b : \iota.\lambda A.\lambda c : \iota.(\lambda V0f \in ((A_27c^{A_27b})^{A_27a}))$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{1}$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{2}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{3}$$

Definition 4 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 5 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \tag{4}$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \tag{5}$$

Definition 6 We define c_2Ebool_2ET to be $(ap\ (ap\ (c_2Emin_2E_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

Definition 7 We define $c_2Ebool_2E_21$ to be $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A_27a})).(ap\ (ap\ (c_2Emin_2E_3D\ (2^{A_27a})))$

Definition 8 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num\ (c_2Enum_2ESUC_REP\ m))$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})ty_2Enum_2Enum) \quad (6)$$

Definition 9 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_2Earithmetic_2E_2B))$

Definition 10 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let $c_2Earithmetic_2E_2D : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2D \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})ty_2Enum_2Enum) \quad (7)$$

Let $ty_2Eone_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eone_2Eone \quad (8)$$

Definition 11 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p \Rightarrow p \Rightarrow Q)$ of type ι .

Definition 12 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21\ 2) (\lambda V2t \in 2.V0t))))$

Let $ty_2Esum_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Esum_2Esum\ A0\ A1) \quad (9)$$

Let $c_2Esum_2EABS_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Esum_2EABS_sum\ A_27a\ A_27b \in ((ty_2Esum_2Esum\ A_27a\ A_27b)^{((2^{A_27b})^{A_27a})^2}) \quad (10)$$

Definition 13 We define c_2Esum_2EINL to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0e \in A_27a.(ap (c_2Esum_2EABS_sum))$

Let $ty_2Eoption_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Eoption_2Eoption\ A0) \quad (11)$$

Let $c_2Eoption_2Eoption_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Eoption_2Eoption_ABS\ A_27a \in ((ty_2Eoption_2Eoption\ A_27a)^{(ty_2Esum_2Esum\ A_27a\ ty_2Eone_2Eone)}) \quad (12)$$

Definition 14 We define $c_2Eoption_2ESOME$ to be $\lambda A_27a : \iota.\lambda V0x \in A_27a.(ap (c_2Eoption_2Eoption_ABS))$

Definition 15 We define $c_2Ebool_2E_2F$ to be $(ap (c_2Ebool_2E_21\ 2) (\lambda V0t \in 2.V0t))$.

Definition 16 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) \text{ then } (the (\lambda x.x \in A \wedge P x))$ of type $\iota \Rightarrow \iota$.

Definition 17 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A_27a. (\lambda V2t2 \in A_27a. ($

Definition 18 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap\ V0P\ (ap\ (c_2Emin_2E_40$

Definition 19 We define c_2Eone_2Eone to be $(ap\ (c_2Emin_2E_40\ ty_2Eone_2Eone)\ (\lambda V0x \in ty_2Eone_2E$

Definition 20 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2. (ap\ (ap\ c_2Emin_2E_3D_3D_3E\ V0t)\ c_2Ebool_2E$

Definition 21 We define c_2Esum_2EINR to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0e \in A_27b. (ap\ (c_2Esum_2EABS$

Definition 22 We define $c_2Eoption_2ENONE$ to be $\lambda A_27a : \iota. (ap\ (c_2Eoption_2Eoption_ABS\ A_27a)\ ($

Definition 23 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in$

Definition 24 We define $c_2Ellist_2Elrep_ok$ to be $\lambda A_27a : \iota. (\lambda V0a0 \in ((ty_2Eoption_2Eoption\ A_27a)^{ty-$

Let $ty_2Ellist_2Ellist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. nonempty\ A0 \Rightarrow nonempty\ (ty_2Ellist_2Ellist\ A0) \quad (13)$$

Let $c_2Ellist_2Ellist_rep : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow c_2Ellist_2Ellist_rep\ A_27a \in \\ (((ty_2Eoption_2Eoption\ A_27a)^{ty_2Enum_2Enum})^{(ty_2Ellist_2Ellist\ A_27a)}) \quad (14)$$

Let $c_2Ellist_2Ellist_abs : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow c_2Ellist_2Ellist_abs\ A_27a \in \\ ((ty_2Ellist_2Ellist\ A_27a)^{((ty_2Eoption_2Eoption\ A_27a)^{ty_2Enum_2Enum})}) \quad (15)$$

Definition 25 We define $c_2Ellist_2ELCONS$ to be $\lambda A_27a : \iota. \lambda V0h \in A_27a. \lambda V1t \in (ty_2Ellist_2Ellist\ A$

Definition 26 We define $c_2Ellist_2ELNIL$ to be $\lambda A_27a : \iota. (ap\ (c_2Ellist_2Ellist_abs\ A_27a)\ (\lambda V0n \in ty$

Definition 27 We define $c_2Ecombin_2EK$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. (\lambda V0x \in A_27a. (\lambda V1y \in A_27b. V0x)$

Definition 28 We define $c_2Ecombin_2EI$ to be $\lambda A_27a : \iota. (ap\ (ap\ (c_2Ecombin_2ES\ A_27a\ (A_27a^{A_27a})\ A$

Let $c_2Ellist_2ELNTH : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow c_2Ellist_2ELNTH\ A_27a \in (((ty_2Eoption_2Eoption\ A_27a)^{ty_2Ellist_2Ellist\ A_27a})^{ty_2Enum_2Enum}) \quad (16)$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. nonempty\ A0 \Rightarrow \forall A1. nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \quad (17)$$

Definition 29 We define $c_2Ecombin_2Eo$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda A_27c : \iota. \lambda V0f \in (A_27b^{A_27c}). \lambda V1$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2ESND \\ A_27a\ A_27b \in (A_27b^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \end{aligned} \quad (18)$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EFST \\ A_27a\ A_27b \in (A_27a^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \end{aligned} \quad (19)$$

Definition 30 We define $c_2Epair_2EUNCURRY$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in ((A_27c^{A_27a})$

Let $c_2Eoption_2EOPTION_BIND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Eoption_2EOPTION_BIND \\ A_27a\ A_27b \in (((ty_2Eoption_2Eoption\ A_27a)^{(ty_2Eoption_2Eoption\ A_27a)^{A_27b}})^{(ty_2Eoption_2Eoption\ A_27b)}) \end{aligned} \quad (20)$$

Let $c_2Earithmetic_2EFUNPOW : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow c_2Earithmetic_2EFUNPOW\ A_27a \in \\ (((A_27a^{A_27a})^{ty_2Enum_2Enum})^{(A_27a^{A_27a})}) \end{aligned} \quad (21)$$

Let $c_2Eoption_2EOPTION_MAP : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Eoption_2EOPTION_MAP \\ A_27a\ A_27b \in (((ty_2Eoption_2Eoption\ A_27b)^{(ty_2Eoption_2Eoption\ A_27a)})^{(A_27b^{A_27a})}) \end{aligned} \quad (22)$$

Definition 31 We define $c_2Ellist_2ELUNFOLD$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in ((ty_2Eoption_2Eoption$

Let $c_2Ellist_2ELGENLIST : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow c_2Ellist_2ELGENLIST\ A_27a \in ((\\ (ty_2Ellist_2Ellist\ A_27a)^{(ty_2Eoption_2Eoption\ ty_2Enum_2Enum)})^{(A_27a^{ty_2Enum_2Enum})}) \end{aligned} \quad (23)$$

Definition 32 We define $c_2Ellist_2ELHD$ to be $\lambda A_27a : \iota.\lambda V0ll \in (ty_2Ellist_2Ellist\ A_27a).(ap\ (ap\ (c_2$

Let $c_2Eoption_2Eoption_CASE : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Eoption_2Eoption_CASE \\ A_27a\ A_27b \in (((A_27b^{(A_27b^{A_27a})})^{A_27b})^{(ty_2Eoption_2Eoption\ A_27a)}) \end{aligned} \quad (24)$$

Definition 33 We define $c_2Ellist_2ELTL$ to be $\lambda A_27a : \iota.\lambda V0ll \in (ty_2Ellist_2Ellist\ A_27a).(ap\ (ap\ (ap$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod \\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \end{aligned} \quad (25)$$

Definition 34 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2E$

Definition 35 We define $c_2Ellist_2ELTL_HD$ to be $\lambda A_27a : \iota.\lambda V0ll \in (ty_2Ellist_2Ellist A_27a).(ap (ap$

Let $c_2Earithmetic_2EEVEN : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EEVEN \in (2^{ty_2Enum_2Enum}) \quad (26)$$

Let $c_2Earithmetic_2EODD : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EODD \in (2^{ty_2Enum_2Enum}) \quad (27)$$

Definition 36 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 37 We define $c_2Earithmetic_2E_3E$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 38 We define $c_2Earithmetic_2E_3E_3D$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Let $c_2Earithmetic_2EEXP : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EEXP \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (28)$$

Let $c_2Earithmetic_2E_2A : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2A \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (29)$$

Definition 39 We define $c_2Enumeral_2EiiSUC$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap c_2Enum_2ESUC (ap$

Definition 40 We define $c_2Enumeral_2EiZ$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Definition 41 We define $c_2Earithmetic_2EBIT2$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_2Earithmetic$

Definition 42 We define $c_2Earithmetic_2E_3C_3D$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Let $c_2Eoption_2EOPTION_JOIN : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow c_2Eoption_2EOPTION_JOIN A_27a \in \\ & ((ty_2Eoption_2Eoption A_27a)^{(ty_2Eoption_2Eoption (ty_2Eoption_2Eoption A_27a))}) \end{aligned} \quad (30)$$

Definition 43 We define $c_2Epair_2E_23_23$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda A_27d : \iota.\lambda V0f \in (A_27$

Definition 44 We define $c_2Epair_2Epair_CASE$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0p \in (ty_2Epair$

Definition 45 We define $c_2Eprim_rec_2EPRE$ to be $\lambda V0m \in ty_2Enum_2Enum.(ap (ap (ap (c_2Ebool_2E$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.((ap (ap c_2Earithmic_2E_2B V0m) c_2Enum_2E0) = V0m)) \quad (31)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.((ap (ap c_2Earithmic_2E_2B c_2Enum_2E0) V0m) = V0m) \wedge ((ap (ap c_2Earithmic_2E_2B V0m) c_2Enum_2E0) = V0m) \wedge ((ap (ap c_2Earithmic_2E_2B (ap c_2Enum_2ESUC V0m)) V1n) = (ap c_2Enum_2ESUC (ap (ap c_2Earithmic_2E_2B V0m) V1n))) \wedge ((ap (ap c_2Earithmic_2E_2B V0m) (ap c_2Enum_2ESUC V1n)) = (ap c_2Enum_2ESUC (ap (ap c_2Earithmic_2E_2B V0m) V1n))))))) \quad (32)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.((ap (ap c_2Earithmic_2E_2B V0m) V1n) = (ap (ap c_2Earithmic_2E_2B V1n) V0m)))) \quad (33)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(\forall V2p \in ty_2Enum_2Enum.((ap (ap c_2Earithmic_2E_2B V0m) (ap (ap c_2Earithmic_2E_2B V1n) V2p)) = (ap (ap c_2Earithmic_2E_2B (ap (ap c_2Earithmic_2E_2B V0m) V1n)) V2p)))))) \quad (34)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.((V0m = c_2Enum_2E0) \vee (\exists V1n \in ty_2Enum_2Enum.(V0m = (ap c_2Enum_2ESUC V1n)))))) \quad (35)$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum.((\neg(p (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) V0n))) \Leftrightarrow (V0n = c_2Enum_2E0))) \quad (36)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.((p (ap (ap c_2Eprim_rec_2E_3C (ap c_2Enum_2ESUC V0m)) (ap c_2Enum_2ESUC V1n))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C V0m) V1n)))))) \quad (37)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.((p (ap (ap c_2Eprim_rec_2E_3C V0m) V1n)) \Leftrightarrow (p (ap (ap c_2Earithmic_2E_3C_3D (ap c_2Enum_2ESUC V0m)) V1n)))))) \quad (38)$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum.(p (ap (ap c_2Earithmetic_2E_3C_3D c_2Enum_2E0) V0n))) \quad (39)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(\neg(p (ap (ap c_2Eprim_rec_2E_3C V0m) V1n))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D V1n) V0m)))))) \quad (40)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.(((ap (ap c_2Earithmetic_2E_2D c_2Enum_2E0) V0m) = c_2Enum_2E0) \wedge ((ap (ap c_2Earithmetic_2E_2D V0m) c_2Enum_2E0) = V0m)))) \quad (41)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.(((ap c_2Enum_2ESUC V0m) = (ap (ap c_2Earithmetic_2E_2B V0m) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))))) \quad (42)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.(((ap (ap c_2Earithmetic_2E_2D (ap c_2Enum_2ESUC V0m)) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) = V0m)))) \quad (43)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.(((ap c_2Eprim_rec_2EPRE V0m) = (ap (ap c_2Earithmetic_2E_2D V0m) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))))) \quad (44)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(((ap (ap c_2Earithmetic_2E_2A c_2Enum_2E0) V0m) = c_2Enum_2E0) \wedge (((ap (ap c_2Earithmetic_2E_2A V0m) c_2Enum_2E0) = c_2Enum_2E0) \wedge (((ap (ap c_2Earithmetic_2E_2A (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) V0m) = V0m) \wedge (((ap (ap c_2Earithmetic_2E_2A V0m) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) = V0m) \wedge ((ap (ap c_2Earithmetic_2E_2A (ap c_2Enum_2ESUC V0m)) V1n) = (ap (ap c_2Earithmetic_2E_2B (ap (ap c_2Earithmetic_2E_2A V0m) V1n)) V1n)) \wedge ((ap (ap c_2Earithmetic_2E_2A V0m) (ap c_2Enum_2ESUC V1n)) = (ap (ap c_2Earithmetic_2E_2B V0m) (ap (ap c_2Earithmetic_2E_2A V0m) V1n)))))))))) \quad (45)$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& \quad \forall V2p \in ty_2Enum_2Enum. (((p (ap (ap c_2Earithmetic_2E_3C_3D \\
& \quad V0m) V1n)) \wedge (p (ap (ap c_2Earithmetic_2E_3C_3D V1n) V2p))) \Rightarrow (p (\\
& \quad \quad ap (ap c_2Earithmetic_2E_3C_3D V0m) V2p)))))) \quad (46)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& \quad (V0m = V1n) \Leftrightarrow ((p (ap (ap c_2Earithmetic_2E_3C_3D V0m) V1n)) \wedge (p (\\
& \quad \quad ap (ap c_2Earithmetic_2E_3C_3D V1n) V0m)))))) \quad (47)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& \quad \forall V2p \in ty_2Enum_2Enum. ((p (ap (ap c_2Earithmetic_2E_3C_3D \\
& \quad (ap (ap c_2Earithmetic_2E_2B V0m) V1n)) (ap (ap c_2Earithmetic_2E_2B \\
& \quad V0m) V2p))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D V1n) V2p)))))) \quad (48)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& \quad (\neg(V0m = V1n)) \Leftrightarrow ((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Enum_2ESUC \\
& \quad V0m)) V1n)) \vee (p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Enum_2ESUC \\
& \quad V1n)) V0m)))))) \quad (49)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. ((ap c_2Enum_2ESUC V0n) = (ap (ap \\
& \quad c_2Earithmetic_2E_2B (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\
& \quad c_2Earithmetic_2EZERO))) V0n))) \quad (50)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0P \in (2^{ty_2Enum_2Enum}). (\forall V1a \in ty_2Enum_2Enum. \\
& \quad (\forall V2b \in ty_2Enum_2Enum. ((p (ap V0P (ap (ap c_2Earithmetic_2E_2D \\
& \quad V1a) V2b))) \Leftrightarrow (\forall V3d \in ty_2Enum_2Enum. (((V2b = (ap (ap c_2Earithmetic_2E_2B \\
& \quad V1a) V3d)) \Rightarrow (p (ap V0P c_2Enum_2E0))) \wedge ((V1a = (ap (ap c_2Earithmetic_2E_2B \\
& \quad V2b) V3d)) \Rightarrow (p (ap V0P V3d)))))))))) \quad (51)
\end{aligned}$$

Assume the following.

$$\text{True} \quad (52)$$

Assume the following.

$$\begin{aligned}
& (\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p \\
& \quad V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (53)
\end{aligned}$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p V0t))) \quad (54)$$

Assume the following.

$$(\forall V0t \in 2. ((p V0t) \vee (\neg(p V0t)))) \quad (55)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in A_27a.(p V0t)) \Leftrightarrow (p V0t))) \quad (56)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0t \in 2. ((\exists V1x \in A_27a.(p V0t)) \Leftrightarrow (p V0t))) \quad (57)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \wedge (p V1t2) \wedge (p V2t3)) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \wedge (p V2t3))))) \quad (58)$$

Assume the following.

$$(\forall V0t \in 2. (((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))) \quad (59)$$

Assume the following.

$$(\forall V0t \in 2. (((True \vee (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \vee True) \Leftrightarrow True) \wedge (((False \vee (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee False) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee (p V0t)) \Leftrightarrow (p V0t)))) \quad (60)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t))))) \quad (61)$$

Assume the following.

$$((\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (62)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (63)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (64)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\forall V0f \in (A_27b^{A_27a}). (\forall V1g \in (A_27b^{A_27a}). ((V0f = V1g) \Leftrightarrow (\forall V2x \in A_27a. ((ap\ V0f\ V2x) = (ap\ V1g\ V2x)))))) \quad (65)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg(p\ V0t)))))) \quad (66)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t1 \in A_27a. (\forall V1t2 \in A_27a. (((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ c_2Ebool_2ET)\ V0t1)\ V1t2) = V0t1) \wedge ((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ c_2Ebool_2EF)\ V0t1)\ V1t2) = V1t2)))) \quad (67)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (2^{A_27a}). ((\forall V2x \in A_27a. ((p\ V0P) \Rightarrow (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow ((p\ V0P) \Rightarrow (\forall V3x \in A_27a. (p\ (ap\ V1Q\ V3x))))))) \quad (68)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p\ V0A) \vee (p\ V1B) \vee (p\ V2C)) \Leftrightarrow (((p\ V0A) \vee (p\ V1B)) \vee (p\ V2C)))))) \quad (69)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p\ V0A) \wedge (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A) \vee \neg(p\ V1B)))) \wedge (((\neg(p\ V0A) \vee (p\ V1B)) \Leftrightarrow ((\neg(p\ V0A) \wedge \neg(p\ V1B)))))) \quad (70)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((p\ V0A) \Rightarrow (p\ V1B)) \Leftrightarrow ((\neg(p\ V0A) \vee (p\ V1B)))))) \quad (71)$$

Assume the following.

$$(\forall V0t \in 2. (((p\ V0t) \Rightarrow False) \Leftrightarrow ((p\ V0t) \Leftrightarrow False))) \quad (72)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \quad (73)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Leftrightarrow (p V1t2)) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \vee ((\neg(p V0t1)) \wedge (\neg(p V1t2)))))))) \quad (74)$$

Assume the following.

$$2.(((\forall V0x \in 2.(\forall V1x_{.27} \in 2.(\forall V2y \in 2.(\forall V3y_{.27} \in 2.(((p V0x) \Leftrightarrow (p V1x_{.27})) \wedge ((p V1x_{.27}) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_{.27})))))) \Rightarrow ((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_{.27}) \Rightarrow (p V3y_{.27})))))) \Rightarrow \quad (75)$$

Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow (\forall V0P \in 2.(\forall V1Q \in 2.(\forall V2x \in A_{.27a}.(\forall V3x_{.27} \in A_{.27a}.(\forall V4y \in A_{.27a}.(\forall V5y_{.27} \in A_{.27a}.(((p V0P) \Leftrightarrow (p V1Q)) \wedge ((p V1Q) \Rightarrow (V2x = V3x_{.27})) \wedge ((\neg(p V1Q)) \Rightarrow (V4y = V5y_{.27})))) \Rightarrow ((ap (ap (ap (c_{.2Ebool_{.2ECOND}} A_{.27a}) V0P) V2x) V4y) = (ap (ap (ap (c_{.2Ebool_{.2ECOND}} A_{.27a}) V1Q) V3x_{.27} V5y_{.27})))))))))) \quad (76)$$

Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow (\forall V0a \in A_{.27a}.(\exists V1x \in A_{.27a}.(V1x = V0a))) \quad (77)$$

Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow (\forall V0P \in (2^{A_{.27a}}).(\forall V1a \in A_{.27a}.((\exists V2x \in A_{.27a}.((V2x = V1a) \wedge (p (ap V0P V2x)))) \Leftrightarrow (p (ap V0P V1a)))))) \quad (78)$$

Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow ((\forall V0t1 \in A_{.27a}.(\forall V1t2 \in A_{.27a}.((ap (ap (ap (c_{.2Ebool_{.2ECOND}} A_{.27a}) c_{.2Ebool_{.2ET}} V0t1) V1t2) = V0t1))) \wedge (\forall V2t1 \in A_{.27a}.(\forall V3t2 \in A_{.27a}.((ap (ap (ap (c_{.2Ebool_{.2ECOND}} A_{.27a}) c_{.2Ebool_{.2EF}} V2t1) V3t2) = V3t2)))))) \quad (79)$$

Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow \forall A_{.27b}.nonempty A_{.27b} \Rightarrow \forall A_{.27c}.nonempty A_{.27c} \Rightarrow (\forall V0f \in (A_{.27b}^{A_{.27a}}).(\forall V1g \in (A_{.27a}^{A_{.27c}}).(\forall V2x \in A_{.27c}.((ap (ap (ap (c_{.2Ecombin_{.2Eo}} A_{.27c} A_{.27b} A_{.27a}) V0f) V1g) V2x) = (ap V0f (ap V1g V2x)))))) \quad (80)$$

Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow \forall A_{.27b}.nonempty A_{.27b} \Rightarrow \forall A_{.27c}.nonempty A_{.27c} \Rightarrow (\forall V0f \in (A_{.27c}^{A_{.27a}}).(\forall V1g \in (A_{.27a}^{A_{.27b}}).((ap (ap (c_{.2Ecombin_{.2Eo}} A_{.27b} A_{.27c} A_{.27a}) (\lambda V2x \in A_{.27a}.(ap V0f V2x))) V1g) = (\lambda V3x \in A_{.27b}.(ap V0f (ap V1g V3x)))))) \quad (81)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow & ((\forall V0a \in (ty_2Ellist_2Ellist\ A.27a).((ap\ (c_2Ellist_2Ellist_abs\ A.27a)\ (ap\ (c_2Ellist_2Ellist_rep\ A.27a)\ V0a)) = V0a)) \wedge (\forall V1r \in ((ty_2Eoption_2Eoption\ A.27a)^{ty_2Enum_2Enum}). \\ & ((p\ (ap\ (c_2Ellist_2Elrep_ok\ A.27a)\ V1r)) \Leftrightarrow ((ap\ (c_2Ellist_2Ellist_rep\ A.27a)\ (ap\ (c_2Ellist_2Ellist_abs\ A.27a)\ V1r)) = V1r)))) \end{aligned} \quad (82)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow & (\forall V0h \in A.27a.(\forall V1t \in (ty_2Ellist_2Ellist\ A.27a).((ap\ (c_2Ellist_2Ellist_rep\ A.27a)\ (ap\ (ap\ (c_2Ellist_2ELCONS\ A.27a)\ V0h)\ V1t)) = (\lambda V2n \in ty_2Enum_2Enum. \\ & (ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ (ty_2Eoption_2Eoption\ A.27a))\ (ap\ (ap\ (c_2Emin_2E_3D\ ty_2Enum_2Enum)\ V2n)\ c_2Enum_2E0))\ (ap\ (c_2Eoption_2ESOME\ A.27a)\ V0h))\ (ap\ (ap\ (c_2Ellist_2Ellist_rep\ A.27a)\ V1t)\ (ap\ (ap\ c_2Earithmetic_2E_2D\ V2n)\ (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT1\ c_2Earithmetic_2EZERO)))))))))) \end{aligned} \quad (83)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow & ((ap\ (c_2Ellist_2Ellist_rep\ A.27a)\ (c_2Ellist_2ELNIL\ A.27a)) = (\lambda V0n \in ty_2Enum_2Enum.(c_2Eoption_2ENONE\ A.27a))) \end{aligned} \quad (84)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow & (\forall V0ll \in (ty_2Ellist_2Ellist\ A.27a).((ap\ (c_2Ellist_2ELTL\ A.27a)\ V0ll) = (ap\ (ap\ (c_2Eoption_2EOPTION_MAP\ (ty_2Epair_2Eprod\ (ty_2Ellist_2Ellist\ A.27a)\ A.27a)\ (ty_2Ellist_2Ellist\ A.27a))\ (c_2Epair_2EFST\ (ty_2Ellist_2Ellist\ A.27a)\ A.27a))\ (ap\ (c_2Ellist_2ELTL_HD\ A.27a)\ V0ll)))) \end{aligned} \quad (85)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow & (\forall V0ll1 \in (ty_2Ellist_2Ellist\ A.27a).(\forall V1ll2 \in (ty_2Ellist_2Ellist\ A.27a).(((ap\ (c_2Ellist_2ELTL_HD\ A.27a)\ V0ll1) = (ap\ (c_2Ellist_2ELTL_HD\ A.27a)\ V1ll2)) \Rightarrow (V0ll1 = V1ll2)))) \end{aligned} \quad (86)$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow ((\forall V0l \in (ty_2Ellist_2Ellist \\
& A.27a).(ap\ (ap\ (c_2Ellist_2ELNTH\ A.27a)\ c_2Enum_2E0)\ V0l) = \\
& (ap\ (c_2Ellist_2ELHD\ A.27a)\ V0l)) \wedge (\forall V1n \in ty_2Enum_2Enum. \\
& (\forall V2l \in (ty_2Ellist_2Ellist\ A.27a).(ap\ (ap\ (c_2Ellist_2ELNTH \\
& A.27a)\ (ap\ c_2Enum_2ESUC\ V1n))\ V2l) = (ap\ (c_2Eoption_2EOPTION_JOIN \\
& A.27a)\ (ap\ (ap\ (c_2Eoption_2EOPTION_MAP\ (ty_2Ellist_2Ellist \\
& A.27a)\ (ty_2Eoption_2Eoption\ A.27a))\ (ap\ (c_2Ellist_2ELNTH\ A.27a) \\
& V1n))\ (ap\ (c_2Ellist_2ELTL\ A.27a)\ V2l))))))
\end{aligned} \tag{87}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0l1 \in (ty_2Ellist_2Ellist \\
& A.27a).(\forall V1l2 \in (ty_2Ellist_2Ellist\ A.27a).(V0l1 = \\
& V1l2) \Leftrightarrow (\forall V2n \in ty_2Enum_2Enum.(ap\ (ap\ (c_2Ellist_2ELNTH \\
& A.27a)\ V2n)\ V0l1) = (ap\ (ap\ (c_2Ellist_2ELNTH\ A.27a)\ V2n)\ V1l2))))))
\end{aligned} \tag{88}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\
& \forall V0f \in ((ty_2Eoption_2Eoption\ (ty_2Epair_2Eprod\ A.27b \\
& A.27a))^{A.27b}).(\forall V1x \in A.27b.(ap\ (c_2Ellist_2ELTL_HD \\
& A.27a)\ (ap\ (ap\ (c_2Ellist_2ELUNFOLD\ A.27a\ A.27b)\ V0f)\ V1x)) = (ap \\
& (ap\ (c_2Eoption_2EOPTION_MAP\ (ty_2Epair_2Eprod\ A.27b\ A.27a) \\
& (ty_2Epair_2Eprod\ (ty_2Ellist_2Ellist\ A.27a)\ A.27a))\ (ap\ (ap \\
& (c_2Epair_2E.23.23\ A.27b\ A.27a)\ (ty_2Ellist_2Ellist\ A.27a)\ A.27a) \\
& (ap\ (c_2Ellist_2ELUNFOLD\ A.27a\ A.27b)\ V0f))\ (c_2Ecombin_2EI\ A.27a))) \\
& (ap\ V0f\ V1x))))
\end{aligned} \tag{89}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\
& \forall V0f \in ((ty_2Eoption_2Eoption\ (ty_2Epair_2Eprod\ A.27b \\
& A.27a))^{A.27b}).(\forall V1x \in A.27b.(\forall V2n \in ty_2Enum_2Enum. \\
& (((ap\ (ap\ (c_2Ellist_2ELNTH\ A.27a)\ c_2Enum_2E0)\ (ap\ (ap\ (c_2Ellist_2ELUNFOLD \\
& A.27a\ A.27b)\ V0f)\ V1x)) = (ap\ (ap\ (c_2Eoption_2EOPTION_MAP\ (ty_2Epair_2Eprod \\
& A.27b\ A.27a)\ A.27a)\ (c_2Epair_2ESND\ A.27b\ A.27a))\ (ap\ V0f\ V1x)))) \wedge \\
& ((ap\ (ap\ (c_2Ellist_2ELNTH\ A.27a)\ (ap\ c_2Enum_2ESUC\ V2n))\ (ap\ (\\
& ap\ (c_2Ellist_2ELUNFOLD\ A.27a\ A.27b)\ V0f)\ V1x)) = (ap\ (ap\ (ap\ (c_2Eoption_2Eoption_CASE \\
& (ty_2Epair_2Eprod\ A.27b\ A.27a)\ (ty_2Eoption_2Eoption\ A.27a)) \\
& (ap\ V0f\ V1x))\ (c_2Eoption_2ENONE\ A.27a))\ (\lambda V3v \in (ty_2Epair_2Eprod \\
& A.27b\ A.27a).(ap\ (ap\ (c_2Epair_2Epair_CASE\ (ty_2Eoption_2Eoption \\
& A.27a)\ A.27b\ A.27a)\ V3v)\ (\lambda V4tx \in A.27b.(\lambda V5hx \in A.27a.(ap \\
& (ap\ (c_2Ellist_2ELNTH\ A.27a)\ V2n)\ (ap\ (ap\ (c_2Ellist_2ELUNFOLD \\
& A.27a\ A.27b)\ V0f)\ V4tx))))))))))
\end{aligned} \tag{90}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow ((\forall V0f \in (A.27a^{ty.2Enum.2Enum}). \\
& ((ap (ap (c.2Ellist.2ELGENLIST\ A.27a)\ V0f)\ (c.2Eoption.2ENONE \\
& ty.2Enum.2Enum))) = (ap (ap (c.2Ellist.2ELUNFOLD\ A.27a\ ty.2Enum.2Enum) \\
& (\lambda V1n \in ty.2Enum.2Enum.(ap (c.2Eoption.2ESOME\ (ty.2Epair.2Eprod \\
& ty.2Enum.2Enum\ A.27a))\ (ap (ap (c.2Epair.2E.2C\ ty.2Enum.2Enum \\
& A.27a)\ (ap (ap\ c.2Earithmetic.2E.2B\ V1n)\ (ap\ c.2Earithmetic.2ENUMERAL \\
& (ap\ c.2Earithmetic.2EBIT1\ c.2Earithmetic.2EZERO))))\ (ap\ V0f \\
& V1n))))\ c.2Enum.2E0)) \wedge (\forall V2f \in (A.27a^{ty.2Enum.2Enum}). \\
& (\forall V3lim \in ty.2Enum.2Enum.((ap (ap (c.2Ellist.2ELGENLIST \\
& A.27a)\ V2f)\ (ap (c.2Eoption.2ESOME\ ty.2Enum.2Enum)\ V3lim)) = (\\
ap (ap (c.2Ellist.2ELUNFOLD\ A.27a\ ty.2Enum.2Enum) (\lambda V4n \in ty.2Enum.2Enum. \\
& (ap (ap (ap (c.2Ebool.2ECOND\ (ty.2Eoption.2Eoption\ (ty.2Epair.2Eprod \\
& ty.2Enum.2Enum\ A.27a)))\ (ap (ap\ c.2Eprim.2rec.2E.3C\ V4n)\ V3lim)) \\
& (ap (c.2Eoption.2ESOME\ (ty.2Epair.2Eprod\ ty.2Enum.2Enum\ A.27a)) \\
& (ap (ap (c.2Epair.2E.2C\ ty.2Enum.2Enum\ A.27a)\ (ap (ap\ c.2Earithmetic.2E.2B \\
& V4n)\ (ap\ c.2Earithmetic.2ENUMERAL\ (ap\ c.2Earithmetic.2EBIT1 \\
& c.2Earithmetic.2EZERO))))\ (ap\ V2f\ V4n))))\ (c.2Eoption.2ENONE \\
& (ty.2Epair.2Eprod\ ty.2Enum.2Enum\ A.27a))))\ c.2Enum.2E0)))))) \\
& \hspace{10em} (91)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0l \in (ty.2Ellist.2Ellist \\
& A.27a)).((ap (c.2Ellist.2ELTL.2HD\ A.27a)\ V0l) = (ap (ap (c.2Eoption.2EOPTION.2BIND \\
& (ty.2Epair.2Eprod\ (ty.2Ellist.2Ellist\ A.27a)\ A.27a)\ A.27a)\ (\\
ap (c.2Ellist.2ELHD\ A.27a)\ V0l)) (\lambda V1h \in A.27a.(ap (ap (c.2Eoption.2EOPTION.2BIND \\
& (ty.2Epair.2Eprod\ (ty.2Ellist.2Ellist\ A.27a)\ A.27a)\ (ty.2Ellist.2Ellist \\
& A.27a))\ (ap (c.2Ellist.2ELTL\ A.27a)\ V0l)) (\lambda V2t \in (ty.2Ellist.2Ellist \\
& A.27a).(ap (c.2Eoption.2ESOME\ (ty.2Epair.2Eprod\ (ty.2Ellist.2Ellist \\
& A.27a)\ A.27a))\ (ap (ap (c.2Epair.2E.2C\ (ty.2Ellist.2Ellist\ A.27a) \\
& A.27a)\ V2t)\ V1h)))))) \\
& \hspace{10em} (92)
\end{aligned}$$

Assume the following.

$$(\forall V0n \in ty.2Enum.2Enum.(\neg((ap\ c.2Enum.2ESUC\ V0n) = c.2Enum.2E0))) \hspace{10em} (93)$$

Assume the following.

$$\begin{aligned}
& (\forall V0P \in (2^{ty.2Enum.2Enum}).((p (ap\ V0P\ c.2Enum.2E0)) \wedge \\
& (\forall V1n \in ty.2Enum.2Enum.((p (ap\ V0P\ V1n)) \Rightarrow (p (ap\ V0P\ (ap\ c.2Enum.2ESUC \\
& V1n)))))) \Rightarrow (\forall V2n \in ty.2Enum.2Enum.(p (ap\ V0P\ V2n)))))) \\
& \hspace{10em} (94)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2B \\
& \quad c_2Enum_2E0) V0n) = V0n)) \wedge ((\forall V1n \in ty_2Enum_2Enum.((ap \\
& \quad (ap c_2Earithmetic_2E_2B V1n) c_2Enum_2E0) = V1n)) \wedge ((\forall V2n \in \\
& \quad ty_2Enum_2Enum.(\forall V3m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2B \\
& \quad (ap c_2Earithmetic_2ENUMERAL V2n)) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V3m)) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Enumeral_2EiZ (ap \\
& \quad (ap c_2Earithmetic_2E_2B V2n) V3m)))))) \wedge ((\forall V4n \in ty_2Enum_2Enum. \\
& \quad ((ap (ap c_2Earithmetic_2E_2A c_2Enum_2E0) V4n) = c_2Enum_2E0)) \wedge \\
& \quad ((\forall V5n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2A \\
& \quad V5n) c_2Enum_2E0) = c_2Enum_2E0)) \wedge ((\forall V6n \in ty_2Enum_2Enum. \\
& \quad (\forall V7m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2A (\\
& \quad ap c_2Earithmetic_2ENUMERAL V6n)) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V7m)) = (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2E_2A \\
& \quad V6n) V7m)))))) \wedge ((\forall V8n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2D \\
& \quad c_2Enum_2E0) V8n) = c_2Enum_2E0)) \wedge ((\forall V9n \in ty_2Enum_2Enum. \\
& \quad ((ap (ap c_2Earithmetic_2E_2D V9n) c_2Enum_2E0) = V9n)) \wedge ((\forall V10n \in \\
& \quad ty_2Enum_2Enum.(\forall V11m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2D \\
& \quad (ap c_2Earithmetic_2ENUMERAL V10n)) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V11m)) = (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2E_2D \\
& \quad V10n) V11m)))))) \wedge ((\forall V12n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2EEXP \\
& \quad c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\
& \quad V12n))) = c_2Enum_2E0)) \wedge ((\forall V13n \in ty_2Enum_2Enum.((ap \\
& \quad (ap c_2Earithmetic_2EEXP c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap c_2Earithmetic_2EBIT2 V13n))) = c_2Enum_2E0)) \wedge ((\forall V14n \in \\
& \quad ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2EEXP V14n) c_2Enum_2E0) = \\
& \quad (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))))) \wedge \\
& \quad ((\forall V15n \in ty_2Enum_2Enum.(\forall V16m \in ty_2Enum_2Enum. \\
& \quad ((ap (ap c_2Earithmetic_2EEXP (ap c_2Earithmetic_2ENUMERAL V15n)) \\
& \quad (ap c_2Earithmetic_2ENUMERAL V16m)) = (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap (ap c_2Earithmetic_2EEXP V15n) V16m)))))) \wedge ((ap c_2Enum_2ESUC \\
& \quad c_2Enum_2E0) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\
& \quad c_2Earithmetic_2EZERO))) \wedge ((\forall V17n \in ty_2Enum_2Enum. (\\
& \quad (ap c_2Enum_2ESUC (ap c_2Earithmetic_2ENUMERAL V17n)) = (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap c_2Enum_2ESUC V17n)))))) \wedge ((ap c_2Eprim_rec_2EPRE c_2Enum_2E0) = \\
& \quad c_2Enum_2E0) \wedge ((\forall V18n \in ty_2Enum_2Enum.((ap c_2Eprim_rec_2EPRE \\
& \quad (ap c_2Earithmetic_2ENUMERAL V18n)) = (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap c_2Eprim_rec_2EPRE V18n)))))) \wedge ((\forall V19n \in ty_2Enum_2Enum. \\
& \quad (((ap c_2Earithmetic_2ENUMERAL V19n) = c_2Enum_2E0) \Leftrightarrow (V19n = c_2Earithmetic_2EZERO))) \wedge \\
& \quad ((\forall V20n \in ty_2Enum_2Enum.((c_2Enum_2E0 = (ap c_2Earithmetic_2ENUMERAL \\
& \quad V20n)) \Leftrightarrow (V20n = c_2Earithmetic_2EZERO))) \wedge ((\forall V21n \in ty_2Enum_2Enum. \\
& \quad (\forall V22m \in ty_2Enum_2Enum.(((ap c_2Earithmetic_2ENUMERAL \\
& \quad V21n) = (ap c_2Earithmetic_2ENUMERAL V22m)) \Leftrightarrow (V21n = V22m)))))) \wedge \\
& \quad ((\forall V23n \in ty_2Enum_2Enum.((p (ap (ap c_2Eprim_rec_2E_3C \\
& \quad V23n) c_2Enum_2E0)) \Leftrightarrow False)) \wedge ((\forall V24n \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V24n))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmetic_2EZERO) \\
& \quad V24n)))))) \wedge ((\forall V25n \in ty_2Enum_2Enum.(\forall V26m \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Eprim_rec_2E_3C (ap c_2Earithmetic_2ENUMERAL \\
& \quad V25n)) (ap c_2Earithmetic_2ENUMERAL V26m))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C \\
& \quad V25n) V26m)))))) \wedge ((\forall V27n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmetic_2E_3E \\
& \quad c_2Enum_2E0) V27n)) \Leftrightarrow False)) \wedge ((\forall V28n \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Earithmetic_2E_3E (ap c_2Earithmetic_2ENUMERAL \\
& \quad V28n)) c_2Enum_2E0)) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmetic_2EZERO) \\
& \quad V28n)))))) \wedge ((\forall V29n \in ty_2Enum_2Enum.(\forall V30m \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Earithmetic_2E_3E (ap c_2Earithmetic_2ENUMERAL \\
& \quad V29n)) (ap c_2Earithmetic_2ENUMERAL V30m))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C \\
& \quad V30m) V29n)))))) \wedge ((\forall V31n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmetic_2E_3C_3D \\
& \quad c_2Enum_2E0) V31n)) \Leftrightarrow True)) \wedge ((\forall V32n \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2ENUMERAL
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\\
& ((ap\ c_2Enumeral_2EiZ\ (ap\ (ap\ c_2Earithmetic_2E_2B\ c_2Earithmetic_2EZERO) \\
& V0n)) = V0n) \wedge (((ap\ c_2Enumeral_2EiZ\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& V0n)\ c_2Earithmetic_2EZERO)) = V0n) \wedge (((ap\ c_2Enumeral_2EiZ\ (\\
& ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT1\ V0n))\ (\\
& ap\ c_2Earithmetic_2EBIT1\ V1m))) = (ap\ c_2Earithmetic_2EBIT2\ (\\
& ap\ c_2Enumeral_2EiZ\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge \\
& (((ap\ c_2Enumeral_2EiZ\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT1 \\
& V0n))\ (ap\ c_2Earithmetic_2EBIT2\ V1m))) = (ap\ c_2Earithmetic_2EBIT1 \\
& (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge (\\
& ((ap\ c_2Enumeral_2EiZ\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT2 \\
& V0n))\ (ap\ c_2Earithmetic_2EBIT1\ V1m))) = (ap\ c_2Earithmetic_2EBIT1 \\
& (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge (\\
& ((ap\ c_2Enumeral_2EiZ\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT2 \\
& V0n))\ (ap\ c_2Earithmetic_2EBIT2\ V1m))) = (ap\ c_2Earithmetic_2EBIT2 \\
& (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge (\\
& ((ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ c_2Earithmetic_2EZERO) \\
& V0n)) = (ap\ c_2Enum_2ESUC\ V0n)) \wedge (((ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& V0n)\ c_2Earithmetic_2EZERO)) = (ap\ c_2Enum_2ESUC\ V0n)) \wedge (((ap \\
& c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT1 \\
& V0n))\ (ap\ c_2Earithmetic_2EBIT1\ V1m))) = (ap\ c_2Earithmetic_2EBIT1 \\
& (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge (\\
& ((ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT1 \\
& V0n))\ (ap\ c_2Earithmetic_2EBIT2\ V1m))) = (ap\ c_2Earithmetic_2EBIT2 \\
& (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge (\\
& ((ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT2 \\
& V0n))\ (ap\ c_2Earithmetic_2EBIT1\ V1m))) = (ap\ c_2Earithmetic_2EBIT2 \\
& (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge (\\
& ((ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT2 \\
& V0n))\ (ap\ c_2Earithmetic_2EBIT2\ V1m))) = (ap\ c_2Earithmetic_2EBIT1 \\
& (ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge \\
& (((ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ c_2Earithmetic_2EZERO) \\
& V0n)) = (ap\ c_2Enumeral_2EiiSUC\ V0n)) \wedge (((ap\ c_2Enumeral_2EiiSUC \\
& (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ c_2Earithmetic_2EZERO)) = (\\
& ap\ c_2Enumeral_2EiiSUC\ V0n)) \wedge (((ap\ c_2Enumeral_2EiiSUC\ (ap\ (\\
& ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT1\ V0n))\ (ap\ c_2Earithmetic_2EBIT1 \\
& V1m))) = (ap\ c_2Earithmetic_2EBIT2\ (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& V0n)\ V1m)))) \wedge (((ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& (ap\ c_2Earithmetic_2EBIT1\ V0n))\ (ap\ c_2Earithmetic_2EBIT2\ V1m))) = \\
& (ap\ c_2Earithmetic_2EBIT1\ (ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& V0n)\ V1m)))) \wedge (((ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& (ap\ c_2Earithmetic_2EBIT2\ V0n))\ (ap\ c_2Earithmetic_2EBIT1\ V1m))) = \\
& (ap\ c_2Earithmetic_2EBIT1\ (ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& V0n)\ V1m)))) \wedge (((ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& (ap\ c_2Earithmetic_2EBIT2\ V0n))\ (ap\ c_2Earithmetic_2EBIT2\ V1m))) = \\
& (ap\ c_2Earithmetic_2EBIT2\ (ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& V0n)\ V1m))))))))))))))))))))))))))))))))))
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\\
& ((p (ap (ap c_2Earithmic_2E_3C_3D c_2Earithmic_2EZERO) V0n)) \Leftrightarrow \\
& True) \wedge (((p (ap (ap c_2Earithmic_2E_3C_3D (ap c_2Earithmic_2EBIT1 \\
& V0n)) c_2Earithmic_2EZERO)) \Leftrightarrow False) \wedge (((p (ap (ap c_2Earithmic_2E_3C_3D \\
& (ap c_2Earithmic_2EBIT2 V0n)) c_2Earithmic_2EZERO)) \Leftrightarrow False) \wedge \\
& (((p (ap (ap c_2Earithmic_2E_3C_3D (ap c_2Earithmic_2EBIT1 \\
& V0n)) (ap c_2Earithmic_2EBIT1 V1m))) \Leftrightarrow (p (ap (ap c_2Earithmic_2E_3C_3D \\
& V0n) V1m))) \wedge (((p (ap (ap c_2Earithmic_2E_3C_3D (ap c_2Earithmic_2EBIT1 \\
& V0n)) (ap c_2Earithmic_2EBIT2 V1m))) \Leftrightarrow (p (ap (ap c_2Earithmic_2E_3C_3D \\
& V0n) V1m))) \wedge (((p (ap (ap c_2Earithmic_2E_3C_3D (ap c_2Earithmic_2EBIT2 \\
& V0n)) (ap c_2Earithmic_2EBIT1 V1m))) \Leftrightarrow (\neg (p (ap (ap c_2Earithmic_2E_3C_3D \\
& V1m) V0n)))) \wedge ((p (ap (ap c_2Earithmic_2E_3C_3D (ap c_2Earithmic_2EBIT2 \\
& V0n)) (ap c_2Earithmic_2EBIT2 V1m))) \Leftrightarrow (p (ap (ap c_2Earithmic_2E_3C_3D \\
& V0n) V1m))))))))))
\end{aligned} \tag{97}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty A_27a \Rightarrow (\forall V0opt \in (ty_2Eoption_2Eoption \\
& A_27a). ((V0opt = (c_2Eoption_2ENONE A_27a)) \vee (\exists V1x \in A_27a. \\
& (V0opt = (ap (c_2Eoption_2ESOME A_27a) V1x))))))
\end{aligned} \tag{98}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow (\\
& (\forall V0v \in A_27b. (\forall V1f \in (A_27b^{A_27a}). ((ap (ap (ap (c_2Eoption_2Eoption_CASE \\
& A_27a A_27b) (c_2Eoption_2ENONE A_27a)) V0v) V1f) = V0v))) \wedge (\forall V2x \in \\
& A_27a. (\forall V3v \in A_27b. (\forall V4f \in (A_27b^{A_27a}). ((ap (ap \\
& (ap (c_2Eoption_2Eoption_CASE A_27a A_27b) (ap (c_2Eoption_2ESOME \\
& A_27a) V2x)) V3v) V4f) = (ap V4f V2x))))))
\end{aligned} \tag{99}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in \\
& A_27a. (((ap (c_2Eoption_2ESOME A_27a) V0x) = (ap (c_2Eoption_2ESOME \\
& A_27a) V1y)) \Leftrightarrow (V0x = V1y))))
\end{aligned} \tag{100}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a. (\neg ((c_2Eoption_2ENONE \\
& A_27a) = (ap (c_2Eoption_2ESOME A_27a) V0x))))
\end{aligned} \tag{101}$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\ & (\forall V0f \in (A.27b^{A.27a}).(\forall V1x \in A.27a.((ap\ (ap\ (c.2Eoption.2EOPTION_MAP \\ & A.27a\ A.27b)\ V0f)\ (ap\ (c.2Eoption.2ESOME\ A.27a)\ V1x)) = (ap\ (c.2Eoption.2ESOME \\ & A.27b)\ (ap\ V0f\ V1x)))))) \wedge (\forall V2f \in (A.27b^{A.27a}).((ap\ (ap\ (c.2Eoption.2EOPTION_MAP \\ & A.27a\ A.27b)\ V2f)\ (c.2Eoption.2ENONE\ A.27a)) = (c.2Eoption.2ENONE \\ & A.27b)))))) \end{aligned} \tag{102}$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow (((ap\ (c.2Eoption.2EOPTION_JOIN \\ & A.27a)\ (c.2Eoption.2ENONE\ (ty.2Eoption.2Eoption\ A.27a))) = (\\ & c.2Eoption.2ENONE\ A.27a)) \wedge (\forall V0x \in (ty.2Eoption.2Eoption \\ & A.27a).((ap\ (c.2Eoption.2EOPTION_JOIN\ A.27a)\ (ap\ (c.2Eoption.2ESOME \\ & (ty.2Eoption.2Eoption\ A.27a))\ V0x)) = V0x))) \end{aligned} \tag{103}$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\ & \forall V0f \in (A.27b^{A.27a}).(\forall V1x \in (ty.2Eoption.2Eoption \\ & A.27a).(\forall V2y \in A.27b.(((ap\ (ap\ (c.2Eoption.2EOPTION_MAP \\ & A.27a\ A.27b)\ V0f)\ V1x) = (ap\ (c.2Eoption.2ESOME\ A.27b)\ V2y))) \Leftrightarrow (\exists V3z \in \\ & A.27a.((V1x = (ap\ (c.2Eoption.2ESOME\ A.27a)\ V3z)) \wedge (V2y = (ap\ V0f \\ & V3z))))))))) \end{aligned} \tag{104}$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\ & \forall V0f \in (A.27a^{A.27b}).(\forall V1x \in (ty.2Eoption.2Eoption \\ & A.27b).((((ap\ (ap\ (c.2Eoption.2EOPTION_MAP\ A.27b\ A.27a)\ V0f) \\ & V1x) = (c.2Eoption.2ENONE\ A.27a)) \Leftrightarrow (V1x = (c.2Eoption.2ENONE\ A.27b)))) \wedge \\ & (((c.2Eoption.2ENONE\ A.27a) = (ap\ (ap\ (c.2Eoption.2EOPTION_MAP \\ & A.27b\ A.27a)\ V0f)\ V1x)) \Leftrightarrow (V1x = (c.2Eoption.2ENONE\ A.27b)))))) \end{aligned} \tag{105}$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow \forall A.27c. \\ & nonempty\ A.27c \Rightarrow (\forall V0f \in (A.27b^{A.27c}).(\forall V1g \in (A.27c^{A.27a}). \\ & (\forall V2x \in (ty.2Eoption.2Eoption\ A.27a).((ap\ (ap\ (c.2Eoption.2EOPTION_MAP \\ & A.27c\ A.27b)\ V0f)\ (ap\ (ap\ (c.2Eoption.2EOPTION_MAP\ A.27a\ A.27c) \\ & V1g)\ V2x)) = (ap\ (ap\ (c.2Eoption.2EOPTION_MAP\ A.27a\ A.27b)\ (ap \\ & (ap\ (c.2Ecombin.2Eo\ A.27a\ A.27b\ A.27c)\ V0f)\ V1g))\ V2x)))))) \end{aligned} \tag{106}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\
& (\forall V0f \in ((ty_2Eoption_2Eoption\ A.27a)^{A.27b}).((ap\ (ap\ (\\
& c_2Eoption_2EOPTION_BIND\ A.27a\ A.27b)\ (c_2Eoption_2ENONE\ A.27b)) \\
& V0f) = (c_2Eoption_2ENONE\ A.27a))) \wedge (\forall V1x \in A.27b.(\forall V2f \in \\
& ((ty_2Eoption_2Eoption\ A.27a)^{A.27b}).((ap\ (ap\ (c_2Eoption_2EOPTION_BIND \\
& A.27a\ A.27b)\ (ap\ (c_2Eoption_2ESOME\ A.27b)\ V1x))\ V2f) = (ap\ V2f\ V1x)))))) \\
& (107)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\
& \forall V0x \in A.27a.(\forall V1y \in A.27b.(\forall V2a \in A.27a.(\forall V3b \in \\
& A.27b.(((ap\ (ap\ (c_2Epair_2E_2C\ A.27a\ A.27b)\ V0x)\ V1y) = (ap\ (ap \\
& (c_2Epair_2E_2C\ A.27a\ A.27b)\ V2a)\ V3b)) \Leftrightarrow ((V0x = V2a) \wedge (V1y = V3b)))))) \\
& (108)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\
& \forall V0x \in A.27a.(\forall V1y \in A.27b.((ap\ (c_2Epair_2EFST\ A.27a \\
& A.27b)\ (ap\ (ap\ (c_2Epair_2E_2C\ A.27a\ A.27b)\ V0x)\ V1y)) = V0x))) \\
& (109)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\
& \forall V0x \in A.27a.(\forall V1y \in A.27b.((ap\ (c_2Epair_2ESND\ A.27a \\
& A.27b)\ (ap\ (ap\ (c_2Epair_2E_2C\ A.27a\ A.27b)\ V0x)\ V1y)) = V1y))) \\
& (110)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\
& \forall V0P \in (2^{(ty_2Epair_2Eprod\ A.27a\ A.27b)}).((\exists V1p \in \\
& (ty_2Epair_2Eprod\ A.27a\ A.27b).(p\ (ap\ V0P\ V1p))) \Leftrightarrow (\exists V2p_1 \in \\
& A.27a.(\exists V3p_2 \in A.27b.(p\ (ap\ V0P\ (ap\ (ap\ (c_2Epair_2E_2C \\
& A.27a\ A.27b)\ V2p_1)\ V3p_2)))))) \\
& (111)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow \forall A.27c. \\
& nonempty\ A.27c \Rightarrow \forall A.27d.nonempty\ A.27d \Rightarrow (\forall V0p \in (ty_2Epair_2Eprod \\
& A.27a\ A.27b).(\forall V1f \in (A.27c^{A.27a}).(\forall V2g \in (A.27d^{A.27b}). \\
& ((ap\ (c_2Epair_2EFST\ A.27c\ A.27d)\ (ap\ (ap\ (ap\ (c_2Epair_2E_23.23 \\
& A.27a\ A.27b\ A.27c\ A.27d)\ V1f)\ V2g)\ V0p)) = (ap\ V1f\ (ap\ (c_2Epair_2EFST \\
& A.27a\ A.27b)\ V0p)))))) \\
& (112)
\end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow \forall A.27c. \\ & \quad nonempty\ A.27c \Rightarrow (\forall V0x \in A.27b. (\forall V1y \in A.27c. (\forall V2f \in \\ & \quad ((A.27a^{A.27c})^{A.27b}). ((ap\ (ap\ (c.2Epair.2Epair_CASE\ A.27a\ A.27b \\ & \quad A.27c)\ (ap\ (ap\ (c.2Epair.2E.2C\ A.27b\ A.27c)\ V0x)\ V1y))\ V2f) = (ap \\ & \quad (ap\ V2f\ V0x)\ V1y)))))) \end{aligned} \quad (113)$$

Assume the following.

$$\begin{aligned} & (((ap\ c.2Eprim_rec.2EPRE\ c.2Enum.2E0) = c.2Enum.2E0) \wedge (\forall V0m \in \\ & \quad ty.2Enum.2Enum. ((ap\ c.2Eprim_rec.2EPRE\ (ap\ c.2Enum.2ESUC\ V0m)) = \\ & \quad V0m))) \end{aligned} \quad (114)$$

Assume the following.

$$(\forall V0n \in ty.2Enum.2Enum. (\neg(p\ (ap\ (ap\ c.2Eprim_rec.2E.3C\ V0n)\ V0n)))) \quad (115)$$

Assume the following.

$$(\forall V0n \in ty.2Enum.2Enum. (\neg(p\ (ap\ (ap\ c.2Eprim_rec.2E.3C\ V0n)\ c.2Enum.2E0)))) \quad (116)$$

Assume the following.

$$\begin{aligned} & (\forall V0n \in ty.2Enum.2Enum. (p\ (ap\ (ap\ c.2Eprim_rec.2E.3C\ c.2Enum.2E0) \\ & \quad (ap\ c.2Enum.2ESUC\ V0n)))) \end{aligned} \quad (117)$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \quad (118)$$

Assume the following.

$$(\forall V0A \in 2. ((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \quad (119)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\ & \quad (((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))))) \end{aligned} \quad (120)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(\neg(p\ V0A)) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\ & \quad ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))))) \end{aligned} \quad (121)$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p\ V0A)) \Rightarrow False) \Rightarrow (((p\ V0A) \Rightarrow False) \Rightarrow False))) \quad (122)$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \vee 0p) \Leftrightarrow (\\
& (p \vee 1q) \Leftrightarrow (p \vee 2r))) \Leftrightarrow (((p \vee 0p) \vee ((p \vee 1q) \vee (p \vee 2r))) \wedge (((p \vee 0p) \vee ((\neg \\
& p \vee 2r)) \vee (\neg(p \vee 1q)))) \wedge (((p \vee 1q) \vee ((\neg(p \vee 2r)) \vee (\neg(p \vee 0p)))) \wedge ((p \vee 2r) \vee \\
& ((\neg(p \vee 1q)) \vee (\neg(p \vee 0p))))))))))
\end{aligned} \tag{123}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \vee 0p) \Leftrightarrow (\\
& (p \vee 1q) \wedge (p \vee 2r))) \Leftrightarrow (((p \vee 0p) \vee ((\neg(p \vee 1q)) \vee (\neg(p \vee 2r)))) \wedge (((p \vee 1q) \vee \\
& (\neg(p \vee 0p))) \wedge ((p \vee 2r) \vee (\neg(p \vee 0p))))))
\end{aligned} \tag{124}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \vee 0p) \Leftrightarrow (\\
& (p \vee 1q) \vee (p \vee 2r))) \Leftrightarrow (((p \vee 0p) \vee (\neg(p \vee 1q))) \wedge (((p \vee 0p) \vee (\neg(p \vee 2r))) \wedge \\
& ((p \vee 1q) \vee ((p \vee 2r) \vee (\neg(p \vee 0p))))))))
\end{aligned} \tag{125}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \vee 0p) \Leftrightarrow (\\
& (p \vee 1q) \Rightarrow (p \vee 2r))) \Leftrightarrow (((p \vee 0p) \vee (p \vee 1q)) \wedge (((p \vee 0p) \vee (\neg(p \vee 2r))) \wedge ((\\
& \neg(p \vee 1q)) \vee ((p \vee 2r) \vee (\neg(p \vee 0p))))))
\end{aligned} \tag{126}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (((p \vee 0p) \Leftrightarrow (\neg(p \vee 1q))) \Leftrightarrow (((p \vee 0p) \vee \\
& (p \vee 1q)) \wedge ((\neg(p \vee 1q)) \vee (\neg(p \vee 0p))))))
\end{aligned} \tag{127}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \vee 0p) \Rightarrow (p \vee 1q))) \Rightarrow (p \vee 0p))) \tag{128}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \vee 0p) \Rightarrow (p \vee 1q))) \Rightarrow (\neg(p \vee 1q)))) \tag{129}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \vee 0p) \vee (p \vee 1q))) \Rightarrow (\neg(p \vee 0p)))) \tag{130}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \vee 0p) \vee (p \vee 1q))) \Rightarrow (\neg(p \vee 1q)))) \tag{131}$$

Assume the following.

$$(\forall V0p \in 2. ((\neg(\neg(p \vee 0p))) \Rightarrow (p \vee 0p))) \tag{132}$$

Theorem 1

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0n \in ty_2Enum_2Enum.(\\ & \quad \forall V1f \in (A_27a^{ty_2Enum_2Enum}).(\forall V2lim \in (ty_2Eoption_2Eoption \\ & \quad ty_2Enum_2Enum).((ap\ (ap\ (c_2Ellist_2ELNTH\ A_27a)\ V0n)\ (ap\ (ap \\ & (c_2Ellist_2ELGENLIST\ A_27a)\ V1f)\ V2lim)) = (ap\ (ap\ (ap\ (c_2Eoption_2Eoption_CASE \\ & \quad ty_2Enum_2Enum\ (ty_2Eoption_2Eoption\ A_27a))\ V2lim)\ (ap\ (c_2Eoption_2ESOME \\ & \quad A_27a)\ (ap\ V1f\ V0n)))\ (\lambda V3lim0 \in ty_2Enum_2Enum.(ap\ (ap\ (ap\ (\\ & \quad c_2Ebool_2ECOND\ (ty_2Eoption_2Eoption\ A_27a))\ (ap\ (ap\ c_2Eprim_rec_2E_3C \\ & \quad V0n)\ V3lim0))\ (ap\ (c_2Eoption_2ESOME\ A_27a)\ (ap\ V1f\ V0n)))\ (c_2Eoption_2ENONE \\ & \quad A_27a))))))))) \end{aligned}$$