

thm_2Ellist_2ELNTH_LMAP
(TMb7DcAtv5cL63wKgYLCeDexJwVZzoVZEiZ)

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Definition 1 We define `c_2Emin_2E_3D` to be $\lambda A. \lambda x \in A. \lambda y \in A. \text{inj_o } (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define `c_2Ebool_2E_2T` to be $(\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define `c_2Ebool_2E_21` to be $\lambda A. 27a : \iota. (\lambda V0P \in (2^{A-27a}). (\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^{A-27a}))))$

Definition 4 We define `c_2Ebool_2E_2F` to be $(\text{ap } (\text{c_2Ebool_2E_21 } 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define `c_2Emin_2E_3D_3D_3E` to be $\lambda P \in 2. \lambda Q \in 2. \text{inj_o } (p \Rightarrow q)$ of type ι .

Definition 6 We define `c_2Ebool_2E_27E` to be $(\lambda V0t \in 2. (\text{ap } (\text{ap } (\text{c_2Emin_2E_3D_3D_3E } V0t) (\text{c_2Ebool_2E_2F}))))$

Let `ty_2Enum_2Enum` : ι be given. Assume the following.

$$\text{nonempty } \text{ty_2Enum_2Enum} \tag{1}$$

Let `c_2Earithmetic_2E_2D` : ι be given. Assume the following.

$$\text{c_2Earithmetic_2E_2D} \in ((\text{ty_2Enum_2Enum}^{\text{ty_2Enum_2Enum}}) \text{ty_2Enum_2Enum}) \tag{2}$$

Definition 7 We define `c_2Ebool_2E_2F_5C` to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (\text{ap } (\text{c_2Ebool_2E_21 } 2) (\lambda V2t \in 2.V2t))))$

Definition 8 We define `c_2Emin_2E_40` to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p (\text{ap } P x)) \text{ then } (\text{the } (\lambda x. x \in A \wedge p x))$ of type $\iota \Rightarrow \iota$.

Definition 9 We define `c_2Ebool_2ECOND` to be $\lambda A. 27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A. 27a. (\lambda V2t2 \in A. 27a. (\text{ap } (\text{c_2Emin_2E_40 } (27a \wedge V2t2))))))$

Let `c_2Enum_2EZERO_REP` : ι be given. Assume the following.

$$\text{c_2Enum_2EZERO_REP} \in \text{omega} \tag{3}$$

Let `c_2Enum_2EABS_num` : ι be given. Assume the following.

$$\text{c_2Enum_2EABS_num} \in (\text{ty_2Enum_2Enum}^{\text{omega}}) \tag{4}$$

Definition 10 We define c_Enum_2E0 to be $(ap\ c_Enum_2EABS_num\ c_Enum_2EZERO_REP)$.

Definition 11 We define $c_Earithmetic_2EZERO$ to be c_Enum_2E0 .

Let $c_Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (5)$$

Let $c_Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_Enum_2ESUC_REP \in (\omega^{\omega}) \quad (6)$$

Definition 12 We define c_Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_Enum_2EABS_num$

$c_Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_Earithmetic_2E_2B \in ((ty_2Enum_2Enum)^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (7)$$

Definition 13 We define $c_Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ (ap\ c_Earithmetic$

Definition 14 We define $c_Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let $ty_2Eone_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eone_2Eone \quad (8)$$

Let $ty_2Esum_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Esum_2Esum\ A0\ A1) \quad (9)$$

Let $c_Esum_2EABS_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_Esum_2EABS_sum\ A_27a\ A_27b \in ((ty_2Esum_2Esum\ A_27a\ A_27b)^{((2^{A_27b})^{A_27a})^2}) \quad (10)$$

Definition 15 We define c_Esum_2EINL to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0e \in A_27a.(ap\ (c_Esum_2EABS$

Let $ty_2Eoption_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Eoption_2Eoption\ A0) \quad (11)$$

Let $c_Eoption_2Eoption_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_Eoption_2Eoption_ABS\ A_27a \in ((ty_2Eoption_2Eoption\ A_27a)^{(ty_2Esum_2Esum\ A_27a\ ty_2Eone_2Eone)}) \quad (12)$$

Definition 16 We define $c_Eoption_2ESOME$ to be $\lambda A_27a : \iota.\lambda V0x \in A_27a.(ap\ (c_Eoption_2Eoption$

Definition 17 We define $c_Ebool_2E_3F$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A-27a}). (ap\ V0P\ (ap\ (c_2Emin_2E_40$

Definition 18 We define c_2Eone_2Eone to be $(ap\ (c_2Emin_2E_40\ ty_2Eone_2Eone)\ (\lambda V0x \in ty_2Eone_2Eone$

Definition 19 We define c_2Esum_2EINR to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0e \in A_27b. (ap\ (c_2Esum_2EABS$

Definition 20 We define $c_2Eoption_2ENONE$ to be $\lambda A_27a : \iota. (ap\ (c_2Eoption_2Eoption_ABS\ A_27a)\ (a$

Definition 21 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in$

Definition 22 We define $c_2Ellist_2Elrep_ok$ to be $\lambda A_27a : \iota. (\lambda V0a0 \in ((ty_2Eoption_2Eoption\ A_27a)^{ty-}$

Let $ty_2Ellist_2Ellist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. nonempty\ A0 \Rightarrow nonempty\ (ty_2Ellist_2Ellist\ A0) \quad (13)$$

Let $c_2Ellist_2Ellist_rep : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a. nonempty\ A_27a \Rightarrow c_2Ellist_2Ellist_rep\ A_27a \in \\ (((ty_2Eoption_2Eoption\ A_27a)^{ty_2Enum_2Enum})^{(ty_2Ellist_2Ellist\ A_27a)}) \end{aligned} \quad (14)$$

Let $c_2Ellist_2Ellist_abs : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a. nonempty\ A_27a \Rightarrow c_2Ellist_2Ellist_abs\ A_27a \in \\ ((ty_2Ellist_2Ellist\ A_27a)^{(ty_2Eoption_2Eoption\ A_27a)^{ty_2Enum_2Enum}}) \end{aligned} \quad (15)$$

Definition 23 We define $c_2Ellist_2ELHD$ to be $\lambda A_27a : \iota. \lambda V0ll \in (ty_2Ellist_2Ellist\ A_27a). (ap\ (ap\ (c_2$

Let $c_2Eoption_2Eoption_CASE : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a. nonempty\ A_27a \Rightarrow \forall A_27b. nonempty\ A_27b \Rightarrow c_2Eoption_2Eoption_CASE \\ A_27a\ A_27b \in (((A_27b^{(A_27b^{A-27a})})^{A_27b})^{(ty_2Eoption_2Eoption\ A_27a)}) \end{aligned} \quad (16)$$

Definition 24 We define $c_2Ellist_2ELTL$ to be $\lambda A_27a : \iota. \lambda V0ll \in (ty_2Ellist_2Ellist\ A_27a). (ap\ (ap\ (ap$

Let $c_2Ellist_2ELNTH : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a. nonempty\ A_27a \Rightarrow c_2Ellist_2ELNTH\ A_27a \in (((ty_2Eoption_2Eoption \\ A_27a)^{(ty_2Ellist_2Ellist\ A_27a)})^{ty_2Enum_2Enum}) \end{aligned} \quad (17)$$

Definition 25 We define $c_2Ellist_2ELCONS$ to be $\lambda A_27a : \iota. \lambda V0h \in A_27a. \lambda V1t \in (ty_2Ellist_2Ellist\ A$

Definition 26 We define $c_2Ellist_2ELNIL$ to be $\lambda A_27a : \iota. (ap\ (c_2Ellist_2Ellist_abs\ A_27a)\ (\lambda V0n \in ty$

Let $c_2Ellist_2ELMAP : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a. nonempty\ A_27a \Rightarrow \forall A_27b. nonempty\ A_27b \Rightarrow c_2Ellist_2ELMAP \\ A_27a\ A_27b \in (((ty_2Ellist_2Ellist\ A_27b)^{(ty_2Ellist_2Ellist\ A_27a)})^{(A_27b^{A-27a})}) \end{aligned} \quad (18)$$

Let $c_2Eoption_2EOPTION_MAP : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Eoption_2EOPTION_MAP\ A_27a\ A_27b \in (((ty_2Eoption_2Eoption\ A_27b)^{(ty_2Eoption_2Eoption\ A_27a)})^{(A_27b^{A_27a})}) \quad (19)$$

Let $c_2Eoption_2EOPTION_JOIN : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Eoption_2EOPTION_JOIN\ A_27a \in ((ty_2Eoption_2Eoption\ A_27a)^{(ty_2Eoption_2Eoption\ (ty_2Eoption_2Eoption\ A_27a))}) \quad (20)$$

Assume the following.

$$True \quad (21)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (22)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg(p\ V0t)))))) \quad (23)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (24)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg(p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p\ V0t)))))) \quad (25)$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1y \in 2.(\forall V2z \in 2.(\forall V3w \in 2.(((p\ V0x) \Rightarrow (p\ V1y)) \wedge ((p\ V2z) \Rightarrow (p\ V3w))) \Rightarrow (((p\ V0x) \wedge (p\ V2z)) \Rightarrow ((p\ V1y) \wedge (p\ V3w)))))) \quad (26)$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1y \in 2.(\forall V2z \in 2.(\forall V3w \in 2.(((p\ V0x) \Rightarrow (p\ V1y)) \wedge ((p\ V2z) \Rightarrow (p\ V3w))) \Rightarrow (((p\ V0x) \vee (p\ V2z)) \Rightarrow ((p\ V1y) \vee (p\ V3w)))))) \quad (27)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}).(\forall V1Q \in (2^{A_27a}).((\forall V2x \in A_27a.((p\ (ap\ V0P\ V2x)) \Rightarrow (p\ (ap\ V1Q\ V2x)))) \Rightarrow ((\exists V3x \in A_27a.(p\ (ap\ V0P\ V3x))) \Rightarrow (\exists V4x \in A_27a.(p\ (ap\ V1Q\ V4x)))))) \quad (28)$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.\text{nonempty } A_27a \Rightarrow ((\forall V0a \in (ty_2Ellist_2Ellist \\
& A_27a).((ap (c_2Ellist_2Ellist_abs A_27a) (ap (c_2Ellist_2Ellist_rep \\
& A_27a) V0a)) = V0a)) \wedge (\forall V1r \in ((ty_2Eoption_2Eoption A_27a)^{ty_2Enum_2Enum}). \\
& ((p (ap (c_2Ellist_2Elrep_ok A_27a) V1r)) \Leftrightarrow ((ap (c_2Ellist_2Ellist_rep \\
& A_27a) (ap (c_2Ellist_2Ellist_abs A_27a) V1r)) = V1r))))
\end{aligned} \tag{29}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0h \in A_27a. (\forall V1t \in \\
& (ty_2Ellist_2Ellist A_27a). (((ap (c_2Ellist_2ELHD A_27a) (ap \\
& (ap (c_2Ellist_2ELCONS A_27a) V0h) V1t)) = (ap (c_2Eoption_2ESOME \\
& A_27a) V0h)) \wedge ((ap (c_2Ellist_2ELTL A_27a) (ap (ap (c_2Ellist_2ELCONS \\
& A_27a) V0h) V1t)) = (ap (c_2Eoption_2ESOME (ty_2Ellist_2Ellist \\
& A_27a)) V1t))))))
\end{aligned} \tag{30}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0l \in (ty_2Ellist_2Ellist \\
& A_27a). ((V0l = (c_2Ellist_2ELNIL A_27a)) \vee (\exists V1h \in A_27a. \\
& (\exists V2t \in (ty_2Ellist_2Ellist A_27a). (V0l = (ap (ap (c_2Ellist_2ELCONS \\
& A_27a) V1h) V2t))))))
\end{aligned} \tag{31}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.\text{nonempty } A_27a \Rightarrow ((\forall V0ll \in (ty_2Ellist_2Ellist \\
& A_27a). ((ap (ap (c_2Ellist_2ELNTH A_27a) c_2Enum_2E0) V0ll) = \\
& (ap (c_2Ellist_2ELHD A_27a) V0ll)) \wedge (\forall V1n \in ty_2Enum_2Enum. \\
& (\forall V2ll \in (ty_2Ellist_2Ellist A_27a). ((ap (ap (c_2Ellist_2ELNTH \\
& A_27a) (ap c_2Enum_2ESUC V1n)) V2ll) = (ap (c_2Eoption_2EOPTION_JOIN \\
& A_27a) (ap (ap (c_2Eoption_2EOPTION_MAP (ty_2Ellist_2Ellist \\
& A_27a) (ty_2Eoption_2Eoption A_27a)) (ap (c_2Ellist_2ELNTH A_27a) \\
& V1n)) (ap (c_2Ellist_2ELTL A_27a) V2ll))))))
\end{aligned} \tag{32}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.\text{nonempty } A_27a \Rightarrow \forall A_27b.\text{nonempty } A_27b \Rightarrow (\\
& (\forall V0f \in (A_27b^{A_27a}). ((ap (ap (c_2Ellist_2ELMAP A_27a A_27b) \\
& V0f) (c_2Ellist_2ELNIL A_27a)) = (c_2Ellist_2ELNIL A_27b))) \wedge \\
& (\forall V1f \in (A_27b^{A_27a}). (\forall V2h \in A_27a. (\forall V3t \in \\
& (ty_2Ellist_2Ellist A_27a). ((ap (ap (c_2Ellist_2ELMAP A_27a \\
& A_27b) V1f) (ap (ap (c_2Ellist_2ELCONS A_27a) V2h) V3t)) = (ap (ap \\
& (c_2Ellist_2ELCONS A_27b) (ap V1f V2h)) (ap (ap (c_2Ellist_2ELMAP \\
& A_27a A_27b) V1f) V3t))))))
\end{aligned} \tag{33}$$

Assume the following.

$$\begin{aligned}
& (\forall V0P \in (2^{ty_2Enum_2Enum}).(((p (ap V0P c_2Enum_2E0)) \wedge \\
& (\forall V1n \in ty_2Enum_2Enum.((p (ap V0P V1n)) \Rightarrow (p (ap V0P (ap c_2Enum_2ESUC \\
& V1n)))))) \Rightarrow (\forall V2n \in ty_2Enum_2Enum.(p (ap V0P V2n))))))
\end{aligned} \tag{34}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow (\\
& (\forall V0v \in A_27b.(\forall V1f \in (A_27b^{A_27a}).((ap (ap (ap (c_2Eoption_2Eoption_CASE \\
& A_27a A_27b) (c_2Eoption_2ENONE A_27a)) V0v) V1f) = V0v))) \wedge (\forall V2x \in \\
& A_27a.(\forall V3v \in A_27b.(\forall V4f \in (A_27b^{A_27a}).((ap (ap \\
& (ap (c_2Eoption_2Eoption_CASE A_27a A_27b) (ap (c_2Eoption_2ESOME \\
& A_27a) V2x)) V3v) V4f) = (ap V4f V2x))))))
\end{aligned} \tag{35}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in \\
& A_27a.(((ap (c_2Eoption_2ESOME A_27a) V0x) = (ap (c_2Eoption_2ESOME \\
& A_27a) V1y)) \Leftrightarrow (V0x = V1y))))
\end{aligned} \tag{36}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow (\\
& (\forall V0f \in (A_27b^{A_27a}).(\forall V1x \in A_27a.((ap (ap (c_2Eoption_2EOPTION_MAP \\
& A_27a A_27b) V0f) (ap (c_2Eoption_2ESOME A_27a) V1x)) = (ap (c_2Eoption_2ESOME \\
& A_27b) (ap V0f V1x)))))) \wedge (\forall V2f \in (A_27b^{A_27a}).((ap (ap (c_2Eoption_2EOPTION_MAP \\
& A_27a A_27b) V2f) (c_2Eoption_2ENONE A_27a)) = (c_2Eoption_2ENONE \\
& A_27b))))
\end{aligned} \tag{37}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty A_27a \Rightarrow (((ap (c_2Eoption_2EOPTION_JOIN \\
& A_27a) (c_2Eoption_2ENONE (ty_2Eoption_2Eoption A_27a))) = (\\
& c_2Eoption_2ENONE A_27a)) \wedge (\forall V0x \in (ty_2Eoption_2Eoption \\
& A_27a).((ap (c_2Eoption_2EOPTION_JOIN A_27a) (ap (c_2Eoption_2ESOME \\
& (ty_2Eoption_2Eoption A_27a) V0x)) = V0x)))
\end{aligned} \tag{38}$$

Theorem 1

$$\begin{aligned}
& \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow (\\
& \forall V0n \in ty_2Enum_2Enum.(\forall V1f \in (A_27b^{A_27a}).(\forall V2l \in \\
& (ty_2Ellist_2Ellist A_27a).((ap (ap (c_2Ellist_2ELNTH A_27b) \\
& V0n) (ap (ap (c_2Ellist_2ELMAP A_27a A_27b) V1f) V2l)) = (ap (ap (\\
& c_2Eoption_2EOPTION_MAP A_27a A_27b) V1f) (ap (ap (c_2Ellist_2ELNTH \\
& A_27a) V0n) V2l))))))
\end{aligned}$$