

thm_2Ellist_2ELNTH_THM (TMYQmHkBj- Jav9LVqcMjSwcCPyrGtGF8qdLB)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2))) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x)$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a})))$

Definition 4 We define $c_2Ebool_2E_2F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p \Rightarrow q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_27E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2F))$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{1}$$

Let $c_2Earithmetic_2E_2D : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2D \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \tag{2}$$

Definition 7 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t)))$

Definition 8 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A \wedge p))$ of type $\iota \Rightarrow \iota$.

Definition 9 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.(ap$

Definition 10 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t)))$

Definition 11 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap V0P (ap (c_2Emin_2E_40$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{3}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{4}$$

Definition 12 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 13 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \tag{5}$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \tag{6}$$

Definition 14 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num\ m)$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \tag{7}$$

Definition 15 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ (ap\ c_2Earithmetic_2E_2B\ n))\ 0)$

Definition 16 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let $ty_2Eone_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eone_2Eone \tag{8}$$

Let $ty_2Esum_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Esum_2Esum\ A0\ A1) \tag{9}$$

Let $c_2Esum_2EABS_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Esum_2EABS_sum\ A_27a\ A_27b \in ((ty_2Esum_2Esum\ A_27a\ A_27b)^{((2^{A_27b})^{A_27a})^2}) \tag{10}$$

Definition 17 We define c_2Esum_2EINL to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0e \in A_27a.(ap\ (c_2Esum_2EABS_sum\ A_27a\ A_27b)\ e)$

Let $ty_2Eoption_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Eoption_2Eoption\ A0) \tag{11}$$

Let $c_2Eoption_2Eoption_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Eoption_2Eoption_ABS\ A_27a \in ((ty_2Eoption_2Eoption\ A_27a)^{ty_2Esum_2Esum\ A_27a\ ty_2Eone_2Eone}) \tag{12}$$

Definition 18 We define $c_2Eoption_2ESOME$ to be $\lambda A_27a : \iota. \lambda V0x \in A_27a. (ap (c_2Eoption_2Eoption_2ABS A_27a) (V0x))$

Definition 19 We define c_2Eone_2Eone to be $(ap (c_2Emin_2E40 ty_2Eone_2Eone) (\lambda V0x \in ty_2Eone_2Eone. V0x))$

Definition 20 We define c_2Esum_2EINR to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0e \in A_27b. (ap (c_2Esum_2EABS A_27a) (V0e))$

Definition 21 We define $c_2Eoption_2ENONE$ to be $\lambda A_27a : \iota. (ap (c_2Eoption_2Eoption_2ABS A_27a) (V0))$

Definition 22 We define $c_2Ellist_2Elrep_ok$ to be $\lambda A_27a : \iota. (\lambda V0a0 \in ((ty_2Eoption_2Eoption A_27a)^{ty_2Eoption_2Eoption}))$

Let $ty_2Ellist_2Ellist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. nonempty A0 \Rightarrow nonempty (ty_2Ellist_2Ellist A0) \quad (13)$$

Let $c_2Ellist_2Ellist_abs : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow c_2Ellist_2Ellist_abs A_27a \in ((ty_2Ellist_2Ellist A_27a)^{(ty_2Eoption_2Eoption A_27a)^{ty_2Enum_2Enum}}) \quad (14)$$

Definition 23 We define $c_2Ellist_2ELNIL$ to be $\lambda A_27a : \iota. (ap (c_2Ellist_2Ellist_abs A_27a) (\lambda V0n \in ty_2Ellist_2Ellist A_27a. V0n))$

Let $c_2Ellist_2Ellist_rep : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow c_2Ellist_2Ellist_rep A_27a \in (((ty_2Eoption_2Eoption A_27a)^{ty_2Enum_2Enum})^{(ty_2Ellist_2Ellist A_27a)}) \quad (15)$$

Definition 24 We define $c_2Ellist_2ELCONS$ to be $\lambda A_27a : \iota. \lambda V0h \in A_27a. \lambda V1t \in (ty_2Ellist_2Ellist A_27a). (ap (ap (c_2Ellist_2Ellist A_27a) (V1t)) (V0h))$

Definition 25 We define $c_2Ellist_2ELHD$ to be $\lambda A_27a : \iota. \lambda V0ll \in (ty_2Ellist_2Ellist A_27a). (ap (ap (c_2Ellist_2Ellist A_27a) (V0ll)) (V0ll))$

Let $c_2Eoption_2Eoption_CASE : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow \forall A_27b. nonempty A_27b \Rightarrow c_2Eoption_2Eoption_CASE A_27a A_27b \in (((A_27b^{(A_27b^{A_27a})})^{A_27b})^{(ty_2Eoption_2Eoption A_27a)}) \quad (16)$$

Definition 26 We define $c_2Ellist_2ELTL$ to be $\lambda A_27a : \iota. \lambda V0ll \in (ty_2Ellist_2Ellist A_27a). (ap (ap (ap (c_2Ellist_2Ellist A_27a) (V0ll)) (V0ll)) (V0ll))$

Let $c_2Ellist_2ELNTH : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow c_2Ellist_2ELNTH A_27a \in (((ty_2Eoption_2Eoption A_27a)^{(ty_2Ellist_2Ellist A_27a)})^{ty_2Enum_2Enum}) \quad (17)$$

Let $c_2Eoption_2EOPTION_MAP : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow \forall A_27b. nonempty A_27b \Rightarrow c_2Eoption_2EOPTION_MAP A_27a A_27b \in (((ty_2Eoption_2Eoption A_27b)^{(ty_2Eoption_2Eoption A_27a)})^{(A_27b^{A_27a})}) \quad (18)$$

Let $c_2Eoption_2EOPTION_JOIN : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Eoption_2EOPTION_JOIN\ A_27a \in ((ty_2Eoption_2Eoption\ A_27a)^{(ty_2Eoption_2Eoption\ (ty_2Eoption_2Eoption\ A_27a))}) \quad (19)$$

Assume the following.

$$True \quad (20)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (21)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((p\ V0t) \Rightarrow False) \Leftrightarrow \neg(p\ V0t)))))) \quad (22)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (23)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg(p\ V0t)))))) \quad (24)$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1y \in 2.(\forall V2z \in 2.(\forall V3w \in 2.(((p\ V0x) \Rightarrow (p\ V1y)) \wedge ((p\ V2z) \Rightarrow (p\ V3w)) \Rightarrow (((p\ V0x) \wedge (p\ V2z)) \Rightarrow ((p\ V1y) \wedge (p\ V3w)))))))) \quad (25)$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1y \in 2.(\forall V2z \in 2.(\forall V3w \in 2.(((p\ V0x) \Rightarrow (p\ V1y)) \wedge ((p\ V2z) \Rightarrow (p\ V3w)) \Rightarrow (((p\ V0x) \vee (p\ V2z)) \Rightarrow ((p\ V1y) \vee (p\ V3w)))))))) \quad (26)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}).(\forall V1Q \in (2^{A_27a}).((\forall V2x \in A_27a.((p\ (ap\ V0P\ V2x)) \Rightarrow (p\ (ap\ V1Q\ V2x)))) \Rightarrow ((\exists V3x \in A_27a.(p\ (ap\ V0P\ V3x))) \Rightarrow (\exists V4x \in A_27a.(p\ (ap\ V1Q\ V4x))))))) \quad (27)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow & ((\forall V0a \in (ty_2Ellist_2Ellist \\ & A_27a).((ap\ (c_2Ellist_2Ellist_abs\ A_27a)\ (ap\ (c_2Ellist_2Ellist_rep \\ & A_27a)\ V0a)) = V0a)) \wedge (\forall V1r \in ((ty_2Eoption_2Eoption\ A_27a)^{ty_2Enum_2Enum}). \\ & ((p\ (ap\ (c_2Ellist_2Elrep_ok\ A_27a)\ V1r)) \Leftrightarrow ((ap\ (c_2Ellist_2Ellist_rep \\ & A_27a)\ (ap\ (c_2Ellist_2Ellist_abs\ A_27a)\ V1r)) = V1r)))) \end{aligned} \quad (28)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow & (\forall V0h \in A_27a.(\forall V1t \in \\ & (ty_2Ellist_2Ellist\ A_27a).(((ap\ (c_2Ellist_2ELHD\ A_27a)\ (ap \\ & (ap\ (c_2Ellist_2ELCONS\ A_27a)\ V0h)\ V1t)) = (ap\ (c_2Eoption_2ESOME \\ & A_27a)\ V0h)) \wedge ((ap\ (c_2Ellist_2ELTL\ A_27a)\ (ap\ (ap\ (c_2Ellist_2ELCONS \\ & A_27a)\ V0h)\ V1t)) = (ap\ (c_2Eoption_2ESOME\ (ty_2Ellist_2Ellist \\ & A_27a))\ V1t)))))) \end{aligned} \quad (29)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow & ((\forall V0ll \in (ty_2Ellist_2Ellist \\ & A_27a).((ap\ (ap\ (c_2Ellist_2ELNTH\ A_27a)\ c_2Enum_2E0)\ V0ll) = \\ & (ap\ (c_2Ellist_2ELHD\ A_27a)\ V0ll))) \wedge (\forall V1n \in ty_2Enum_2Enum. \\ & (\forall V2ll \in (ty_2Ellist_2Ellist\ A_27a).((ap\ (ap\ (c_2Ellist_2ELNTH \\ & A_27a)\ (ap\ c_2Enum_2ESUC\ V1n))\ V2ll) = (ap\ (c_2Eoption_2EOPTION_JOIN \\ & A_27a)\ (ap\ (ap\ (c_2Eoption_2EOPTION_MAP\ (ty_2Ellist_2Ellist \\ & A_27a)\ (ty_2Eoption_2Eoption\ A_27a))\ (ap\ (c_2Ellist_2ELNTH\ A_27a)\ \\ & V1n))\ (ap\ (c_2Ellist_2ELTL\ A_27a)\ V2ll)))))) \end{aligned} \quad (30)$$

Assume the following.

$$\begin{aligned} (\forall V0P \in (2^{ty_2Enum_2Enum}).(((p\ (ap\ V0P\ c_2Enum_2E0)) \wedge \\ (\forall V1n \in ty_2Enum_2Enum.((p\ (ap\ V0P\ V1n)) \Rightarrow (p\ (ap\ V0P\ (ap\ c_2Enum_2ESUC \\ V1n)))))) \Rightarrow (\forall V2n \in ty_2Enum_2Enum.(p\ (ap\ V0P\ V2n)))))) \end{aligned} \quad (31)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow & (\\ (\forall V0v \in A_27b.(\forall V1f \in (A_27b^{A_27a}).((ap\ (ap\ (ap\ (c_2Eoption_2Eoption_CASE \\ A_27a\ A_27b)\ (c_2Eoption_2ENONE\ A_27a))\ V0v)\ V1f) = V0v))) \wedge (\forall V2x \in \\ A_27a.(\forall V3v \in A_27b.(\forall V4f \in (A_27b^{A_27a}).((ap\ (ap \\ (ap\ (c_2Eoption_2Eoption_CASE\ A_27a\ A_27b)\ (ap\ (c_2Eoption_2ESOME \\ A_27a)\ V2x))\ V3v)\ V4f) = (ap\ V4f\ V2x)))))) \end{aligned} \quad (32)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow & (\forall V0x \in A_27a.(\forall V1y \in \\ & A_27a.(((ap\ (c_2Eoption_2ESOME\ A_27a)\ V0x) = (ap\ (c_2Eoption_2ESOME \\ & A_27a)\ V1y)) \Leftrightarrow (V0x = V1y)))) \end{aligned} \quad (33)$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& (\forall V0f \in (A_27b^{A_27a}).(\forall V1x \in A_27a.((ap\ (ap\ (c_2Eoption_2EOPTION_MAP \\
& A_27a\ A_27b)\ V0f)\ (ap\ (c_2Eoption_2ESOME\ A_27a)\ V1x)) = (ap\ (c_2Eoption_2ESOME \\
& A_27b)\ (ap\ V0f\ V1x)))))) \wedge (\forall V2f \in (A_27b^{A_27a}).((ap\ (ap\ (c_2Eoption_2EOPTION_MAP \\
& A_27a\ A_27b)\ V2f)\ (c_2Eoption_2ENONE\ A_27a)) = (c_2Eoption_2ENONE \\
& A_27b))))))
\end{aligned} \tag{34}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (((ap\ (c_2Eoption_2EOPTION_JOIN \\
& A_27a)\ (c_2Eoption_2ENONE\ (ty_2Eoption_2Eoption\ A_27a))) = (\\
& c_2Eoption_2ENONE\ A_27a)) \wedge (\forall V0x \in (ty_2Eoption_2Eoption \\
& A_27a).((ap\ (c_2Eoption_2EOPTION_JOIN\ A_27a)\ (ap\ (c_2Eoption_2ESOME \\
& (ty_2Eoption_2Eoption\ A_27a)\ V0x)) = V0x))))
\end{aligned} \tag{35}$$

Theorem 1

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\
& nonempty\ A_27c \Rightarrow ((\forall V0n \in ty_2Enum_2Enum.((ap\ (ap\ (c_2Ellist_2ELNTH \\
& A_27a)\ V0n)\ (c_2Ellist_2ELNIL\ A_27a)) = (c_2Eoption_2ENONE\ A_27a))) \wedge \\
& ((\forall V1h \in A_27b.(\forall V2t \in (ty_2Ellist_2Ellist\ A_27b). \\
& ((ap\ (ap\ (c_2Ellist_2ELNTH\ A_27b)\ c_2Enum_2E0)\ (ap\ (ap\ (c_2Ellist_2ELCONS \\
& A_27b)\ V1h)\ V2t))) = (ap\ (c_2Eoption_2ESOME\ A_27b)\ V1h)))))) \wedge (\forall V3n \in \\
& ty_2Enum_2Enum.(\forall V4h \in A_27c.(\forall V5t \in (ty_2Ellist_2Ellist \\
& A_27c).((ap\ (ap\ (c_2Ellist_2ELNTH\ A_27c)\ (ap\ c_2Enum_2ESUC\ V3n)) \\
& (ap\ (ap\ (c_2Ellist_2ELCONS\ A_27c)\ V4h)\ V5t)) = (ap\ (ap\ (c_2Ellist_2ELNTH \\
& A_27c)\ V3n)\ V5t)))))))))
\end{aligned}$$