

thm_2Ellist_2ELPREFIX__APPEND (TMJmy- hEbyH8Cr2HNyvQaGzwYXamSoDcDGK6)

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Definition 1 We define `c_2Emin_2E_40` to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p \text{ (ap } P \ x)) \text{ then (the } (\lambda x. x \in A \wedge p \text{ of type } \iota \Rightarrow \iota).$

Let `c_2Enum_2EZERO__REP` : ι be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \text{omega} \quad (1)$$

Let `ty_2Enum_2Enum` : ι be given. Assume the following.

$$\text{nonempty } ty_2Enum_2Enum \quad (2)$$

Let `c_2Enum_2EABS__num` : ι be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\text{omega}}) \quad (3)$$

Definition 2 We define `c_2Emin_2E_3D` to be $\lambda A. \lambda x \in A. \lambda y \in A. \text{inj_o } (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define `c_2Enum_2E0` to be $(\text{ap } c_2Enum_2EABS_num \ c_2Enum_2EZERO_REP)$.

Definition 4 We define `c_2Earithmic_2EZERO` to be `c_2Enum_2E0`.

Let `c_2Enum_2EREP__num` : ι be given. Assume the following.

$$c_2Enum_2EREP_num \in (\text{omega}^{ty_2Enum_2Enum}) \quad (4)$$

Let `c_2Enum_2ESUC__REP` : ι be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\text{omega}^{\text{omega}}) \quad (5)$$

Definition 5 We define `c_2Ebool_2ET` to be $(\text{ap } (\text{ap } (c_2Emin_2E_3D \ (2^2)) \ (\lambda V0x \in 2. V0x)) \ (\lambda V1x \in 2. V1x))$

Definition 6 We define `c_2Ebool_2E_21` to be $\lambda A. \lambda a : \iota. (\lambda V0P \in (2^{A-27a}). (\text{ap } (\text{ap } (c_2Emin_2E_3D \ (2^{A-27a})))$

Definition 7 We define `c_2Enum_2ESUC` to be $\lambda V0m \in ty_2Enum_2Enum. (\text{ap } c_2Enum_2EABS_num \ (c_2Enum_2ESUC_REP \ m))$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (6)$$

Definition 8 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_2Earithmetic_2E_2B))$

Definition 9 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let $c_2Earithmetic_2E_2D : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2D \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (7)$$

Let $ty_2Eoption_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Eoption_2Eoption A0) \quad (8)$$

Let $ty_2Ellist_2Ellist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Ellist_2Ellist A0) \quad (9)$$

Let $c_2Ellist_2Ellist_rep : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow c_2Ellist_2Ellist_rep A_27a \in \\ & (((ty_2Eoption_2Eoption A_27a)^{ty_2Enum_2Enum})^{(ty_2Ellist_2Ellist A_27a)}) \end{aligned} \quad (10)$$

Let $ty_2Eone_2Eone : \iota$ be given. Assume the following.

$$nonempty ty_2Eone_2Eone \quad (11)$$

Definition 10 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 11 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2)) (\lambda V2t \in 2)))$

Let $ty_2Esum_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Esum_2Esum A0 A1) \quad (12)$$

Let $c_2Esum_2EABS_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Esum_2EABS_sum \\ & A_27a A_27b \in ((ty_2Esum_2Esum A_27a A_27b)^{((2^{A_27b})^{A_27a})^2}) \end{aligned} \quad (13)$$

Definition 12 We define c_2Esum_2EINL to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0e \in A_27a.(ap (c_2Esum_2EABS_sum))$

Let $c_2Eoption_2Eoption_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Eoption_2Eoption_ABS A_27a \in ((ty_2Eoption_2Eoption A_27a)^{(ty_2Esum_2Esum A_27a ty_2Eone_2Eone)}) \quad (14)$$

Definition 13 We define $c_2Eoption_2ESOME$ to be $\lambda A_27a : \iota. \lambda V0x \in A_27a. (ap (c_2Eoption_2Eoption_2ABS A_27a) V0x)$

Definition 14 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21_2) (\lambda V0t \in 2.V0t))$.

Definition 15 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A_27a. (\lambda V2t2 \in A_27a. (c_2Ebool_2E_21_2 V1t1 V2t2))))$

Let $c_2Ellist_2Ellist_abs : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow c_2Ellist_2Ellist_abs A_27a \in ((ty_2Ellist_2Ellist A_27a)^{(ty_2Eoption_2Eoption A_27a)^{ty_2Enum_2Enum}}) \quad (15)$$

Definition 16 We define $c_2Ellist_2ELCONS$ to be $\lambda A_27a : \iota. \lambda V0h \in A_27a. \lambda V1t \in (ty_2Ellist_2Ellist A_27a) V0h. (c_2Ellist_2Ellist_abs A_27a V1t)$

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. nonempty A0 \Rightarrow nonempty (ty_2Elist_2Elist A0) \quad (16)$$

Let $c_2Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow c_2Elist_2ECONS A_27a \in (((ty_2Elist_2Elist A_27a)^{(ty_2Elist_2Elist A_27a)})^{A_27a}) \quad (17)$$

Definition 17 We define c_2Eone_2Eone to be $(ap (c_2Emin_2E_40 ty_2Eone_2Eone) (\lambda V0x \in ty_2Eone_2Eone V0x))$

Definition 18 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2. (ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_7E))$

Definition 19 We define c_2Esum_2EINR to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0e \in A_27b. (ap (c_2Esum_2EABS A_27a A_27b) V0e)$

Definition 20 We define $c_2Eoption_2ENONE$ to be $\lambda A_27a : \iota. (ap (c_2Eoption_2Eoption_2ABS A_27a) V0)$

Definition 21 We define $c_2Ellist_2ELNIL$ to be $\lambda A_27a : \iota. (ap (c_2Ellist_2Ellist_abs A_27a) (\lambda V0n \in ty_2Ellist_2Ellist A_27a V0n))$

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow c_2Elist_2ENIL A_27a \in (ty_2Elist_2Elist A_27a) \quad (18)$$

Definition 22 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap V0P (ap (c_2Emin_2E_40 V0P) c_2Ebool_2E_3F)))$

Definition 23 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c_2Ebool_2E_21_2) (\lambda V2t \in 2. (c_2Ebool_2E_5C_2F V0t1 V1t2 V2t))))$

Definition 24 We define $c_2Ellist_2ELFINITE$ to be $\lambda A_27a : \iota. (\lambda V0a0 \in (ty_2Ellist_2Ellist A_27a). (ap (c_2Ellist_2Ellist A_27a) V0a0))$

Definition 25 We define $c_2Ellist_2Ellength_rel$ to be $\lambda A_27a : \iota. (\lambda V0a0 \in (ty_2Ellist_2Ellist A_27a). (\lambda V0a1 \in (ty_2Ellist_2Ellist A_27a). (c_2Ellist_2Ellist A_27a V0a0 V0a1)))$

Definition 26 We define $c_2Ellist_2ELLENGTH$ to be $\lambda A_27a : \iota. \lambda V0ll \in (ty_2Ellist_2Ellist A_27a). (ap (c_2Ellist_2Ellist A_27a) V0ll)$

Definition 29 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 30 We define $c_2Earithmetic_2E_3E$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 31 We define $c_2Earithmetic_2E_3E_3D$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 32 We define $c_2Eprim_rec_2EPRE$ to be $\lambda V0m \in ty_2Enum_2Enum.(ap (ap (ap (c_2Ebool_2E$

Let $c_2Earithmetic_2EEXP : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EEXP \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (30)$$

Let $c_2Earithmetic_2E_2A : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2A \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (31)$$

Definition 33 We define $c_2Enumeral_2EiZ$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Definition 34 We define $c_2Earithmetic_2EBIT2$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_2Earithmetic$

Let $c_2Eoption_2EOPTION_MAP : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Eoption_2EOPTION_MAP A_27a A_27b \in (((ty_2Eoption_2Eoption A_27b)^{(ty_2Eoption_2Eoption A_27a)})^{(A_27b^{A_27a})}) \quad (32)$$

Let $c_2Eoption_2EIS_SOME : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Eoption_2EIS_SOME A_27a \in (2^{(ty_2Eoption_2Eoption A_27a)}) \quad (33)$$

Let $c_2Eoption_2EIS_NONE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Eoption_2EIS_NONE A_27a \in (2^{(ty_2Eoption_2Eoption A_27a)}) \quad (34)$$

Let $c_2Elist_2ETAKE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2ETAKE A_27a \in (((ty_2Elist_2Elist A_27a)^{(ty_2Elist_2Elist A_27a)})^{ty_2Enum_2Enum}) \quad (35)$$

Definition 35 We define $c_2Earithmetic_2E_3C_3D$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Let $c_2Elist_2EAPPEND : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2EAPPEND A_27a \in (((ty_2Elist_2Elist A_27a)^{(ty_2Elist_2Elist A_27a)})^{(ty_2Elist_2Elist A_27a)}) \quad (36)$$

Let $c_2Elist_2EEL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2EEL\ A_27a \in ((A_27a^{(ty_2Elist_2Elist\ A_27a)})^{ty_2Enum_2Enum}) \quad (37)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\ & ((ap\ (ap\ c_2Earithmetic_2E_2B\ c_2Enum_2E0)\ V0m) = V0m) \wedge (((ap\ (\\ & ap\ c_2Earithmetic_2E_2B\ V0m)\ c_2Enum_2E0) = V0m) \wedge (((ap\ (ap\ c_2Earithmetic_2E_2B \\ & (ap\ c_2Enum_2ESUC\ V0m))\ V1n) = (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\ & V0m)\ V1n)))) \wedge ((ap\ (ap\ c_2Earithmetic_2E_2B\ V0m)\ (ap\ c_2Enum_2ESUC \\ & V1n)) = (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0m)\ V1n))))))))) \end{aligned} \quad (38)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\ & (ap\ (ap\ c_2Earithmetic_2E_2B\ V0m)\ V1n) = (ap\ (ap\ c_2Earithmetic_2E_2B \\ & V1n)\ V0m)))) \end{aligned} \quad (39)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\ & (ap\ (ap\ c_2Earithmetic_2E_2B\ V0m)\ V1n) = (ap\ (ap\ c_2Earithmetic_2E_2B \\ & V1n)\ V0m)))) \end{aligned} \quad (40)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\ & \forall V2p \in ty_2Enum_2Enum. ((ap\ (ap\ c_2Earithmetic_2E_2B\ V0m) \\ & (ap\ (ap\ c_2Earithmetic_2E_2B\ V1n)\ V2p)) = (ap\ (ap\ c_2Earithmetic_2E_2B \\ & (ap\ (ap\ c_2Earithmetic_2E_2B\ V0m)\ V1n))\ V2p)))))) \end{aligned} \quad (41)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\ & (p\ (ap\ (ap\ c_2Eprim_rec_2E_3C\ V0m)\ V1n)) \Leftrightarrow (p\ (ap\ (ap\ c_2Earithmetic_2E_3C_3D \\ & (ap\ c_2Enum_2ESUC\ V0m))\ V1n)))))) \end{aligned} \quad (42)$$

Assume the following.

$$\begin{aligned} & (\forall V0n \in ty_2Enum_2Enum. (p\ (ap\ (ap\ c_2Earithmetic_2E_3C_3D \\ & c_2Enum_2E0)\ V0n))) \end{aligned} \quad (43)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\ & (\neg (p\ (ap\ (ap\ c_2Eprim_rec_2E_3C\ V0m)\ V1n))) \Leftrightarrow (p\ (ap\ (ap\ c_2Earithmetic_2E_3C_3D \\ & V1n)\ V0m)))))) \end{aligned} \quad (44)$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& ((ap (ap c_2Earithmetic_2E_2A c_2Enum_2E0) V0m) = c_2Enum_2E0) \wedge \\
& (((ap (ap c_2Earithmetic_2E_2A V0m) c_2Enum_2E0) = c_2Enum_2E0) \wedge \\
& (((ap (ap c_2Earithmetic_2E_2A (ap c_2Earithmetic_2ENUMERAL \\
& (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) V0m) = V0m) \wedge \\
& (((ap (ap c_2Earithmetic_2E_2A V0m) (ap c_2Earithmetic_2ENUMERAL \\
& (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) = V0m) \wedge (\\
& ((ap (ap c_2Earithmetic_2E_2A (ap c_2Enum_2ESUC V0m)) V1n) = (ap \\
& (ap c_2Earithmetic_2E_2B (ap (ap c_2Earithmetic_2E_2A V0m) V1n)) \\
& V1n)) \wedge ((ap (ap c_2Earithmetic_2E_2A V0m) (ap c_2Enum_2ESUC V1n)) = \\
& (ap (ap c_2Earithmetic_2E_2B V0m) (ap (ap c_2Earithmetic_2E_2A \\
& V0m) V1n))))))))))
\end{aligned} \tag{45}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& \forall V2p \in ty_2Enum_2Enum. (((p (ap (ap c_2Earithmetic_2E_3C_3D \\
& V0m) V1n)) \wedge (p (ap (ap c_2Earithmetic_2E_3C_3D V1n) V2p))) \Rightarrow (p (\\
& ap (ap c_2Earithmetic_2E_3C_3D V0m) V2p))))))
\end{aligned} \tag{46}$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum. (p (ap (ap c_2Earithmetic_2E_3C_3D V0m) V0m))) \tag{47}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& (V0m = V1n) \Leftrightarrow ((p (ap (ap c_2Earithmetic_2E_3C_3D V0m) V1n)) \wedge (p (\\
& ap (ap c_2Earithmetic_2E_3C_3D V1n) V0m))))))
\end{aligned} \tag{48}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& \forall V2p \in ty_2Enum_2Enum. (((p (ap (ap c_2Earithmetic_2E_3C_3D \\
& (ap (ap c_2Earithmetic_2E_2B V0m) V1n)) (ap (ap c_2Earithmetic_2E_2B \\
& V0m) V2p))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D V1n) V2p))))))
\end{aligned} \tag{49}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& (\neg (p (ap (ap c_2Earithmetic_2E_3C_3D V0m) V1n))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D \\
& (ap c_2Enum_2ESUC V1n)) V0m))))))
\end{aligned} \tag{50}$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum.((ap\ c_2Enum_2ESUC\ V0n) = (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT1\ c_2Earithmetic_2EZERO)))\ V0n))) \quad (51)$$

Assume the following.

$$True \quad (52)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (53)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \quad (54)$$

Assume the following.

$$(\forall V0t \in 2.((p\ V0t) \vee (\neg(p\ V0t)))) \quad (55)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \quad (56)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \wedge ((p\ V1t2) \wedge (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \wedge (p\ V2t3)))))) \quad (57)$$

Assume the following.

$$(\forall V0t \in 2.(((True \vee (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \vee True) \Leftrightarrow True) \wedge (((False \vee (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \vee False) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \vee (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \quad (58)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg(p\ V0t)))))) \quad (59)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (60)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.(V0x = V0x)) \quad (61)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (62)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (63)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg(p\ V0t)))))) \quad (64)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t1 \in A_27a. (\forall V1t2 \in A_27a. (((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ c_2Ebool_2ET)\ V0t1)\ V1t2) = V0t1) \wedge ((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ c_2Ebool_2EF)\ V0t1)\ V1t2) = V1t2)))))) \quad (65)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). (\forall V1Q \in 2. (((\forall V2x \in A_27a. (p\ (ap\ V0P\ V2x))) \wedge (p\ V1Q)) \Leftrightarrow (\forall V3x \in A_27a. ((p\ (ap\ V0P\ V3x)) \wedge (p\ V1Q)))))) \quad (66)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (2^{A_27a}). ((\forall V2x \in A_27a. ((p\ V0P) \vee (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow ((p\ V0P) \vee (\forall V3x \in A_27a. (p\ (ap\ V1Q\ V3x)))))) \quad (67)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p\ V0A) \vee ((p\ V1B) \vee (p\ V2C))) \Leftrightarrow (((p\ V0A) \vee (p\ V1B)) \vee (p\ V2C)))))) \quad (68)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p\ V0A) \wedge (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A) \vee \neg(p\ V1B)))) \wedge (((\neg(p\ V0A) \vee (p\ V1B)) \Leftrightarrow ((\neg(p\ V0A) \wedge \neg(p\ V1B)))))) \quad (69)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((p\ V0A) \Rightarrow (p\ V1B)) \Leftrightarrow ((\neg(p\ V0A)) \vee (p\ V1B)))) \quad (70)$$

Assume the following.

$$(\forall V0t \in 2.(((p V0t) \Rightarrow False) \Leftrightarrow ((p V0t) \Leftrightarrow False))) \quad (71)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow (p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (72)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Leftrightarrow (p V1t2)) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \vee ((\neg(p V0t1)) \wedge (\neg(p V1t2))))))) \quad (73)$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x_{.27} \in 2.(\forall V2y \in 2.(\forall V3y_{.27} \in 2.(((p V0x) \Leftrightarrow (p V1x_{.27})) \wedge ((p V1x_{.27}) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_{.27})))))) \Rightarrow ((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_{.27}) \Rightarrow (p V3y_{.27})))))) \quad (74)$$

Assume the following.

$$\begin{aligned} & \forall A_{.27a}.nonempty A_{.27a} \Rightarrow (\forall V0P \in 2.(\forall V1Q \in 2. \\ & (\forall V2x \in A_{.27a}.(\forall V3x_{.27} \in A_{.27a}.(\forall V4y \in A_{.27a}. \\ & (\forall V5y_{.27} \in A_{.27a}.(((p V0P) \Leftrightarrow (p V1Q)) \wedge ((p V1Q) \Rightarrow (V2x = V3x_{.27})) \wedge \\ & ((\neg(p V1Q)) \Rightarrow (V4y = V5y_{.27})))))) \Rightarrow ((ap (ap (ap (c_{.2Ebool_{.2ECOND}} A_{.27a}) \\ & V0P) V2x) V4y) = (ap (ap (ap (c_{.2Ebool_{.2ECOND}} A_{.27a}) V1Q) V3x_{.27} \\ & V5y_{.27})))))))))) \end{aligned} \quad (75)$$

Assume the following.

$$\begin{aligned} & \forall A_{.27a}.nonempty A_{.27a} \Rightarrow ((\forall V0t1 \in A_{.27a}.(\forall V1t2 \in \\ & A_{.27a}.((ap (ap (ap (c_{.2Ebool_{.2ECOND}} A_{.27a}) c_{.2Ebool_{.2ET}}) V0t1) \\ & V1t2) = V0t1))) \wedge (\forall V2t1 \in A_{.27a}.(\forall V3t2 \in A_{.27a}.((ap \\ & (ap (ap (c_{.2Ebool_{.2ECOND}} A_{.27a}) c_{.2Ebool_{.2EF}}) V2t1) V3t2) = V3t2)))))) \end{aligned} \quad (76)$$

Assume the following.

$$\begin{aligned} & \forall A_{.27a}.nonempty A_{.27a} \Rightarrow (((ap (c_{.2Elist_{.2ELENGTH}} A_{.27a}) \\ & (c_{.2Elist_{.2ENIL}} A_{.27a})) = c_{.2Enum_{.2E0}}) \wedge (\forall V0h \in A_{.27a}.(\\ & \forall V1t \in (ty_{.2Elist_{.2Elist}} A_{.27a}).((ap (c_{.2Elist_{.2ELENGTH}} \\ & A_{.27a}) (ap (ap (c_{.2Elist_{.2ECONS}} A_{.27a}) V0h) V1t)) = (ap c_{.2Enum_{.2ESUC}} \\ & (ap (c_{.2Elist_{.2ELENGTH}} A_{.27a}) V1t)))))) \end{aligned} \quad (77)$$

Assume the following.

$$\begin{aligned} & \forall A_{.27a}.nonempty A_{.27a} \Rightarrow (\forall V0P \in (2^{(ty_{.2Elist_{.2Elist}} A_{.27a})}). \\ & (((p (ap V0P (c_{.2Elist_{.2ENIL}} A_{.27a}))) \wedge (\forall V1t \in (ty_{.2Elist_{.2Elist}} \\ & A_{.27a}).((p (ap V0P V1t)) \Rightarrow (\forall V2h \in A_{.27a}.(p (ap V0P (ap (ap (\\ & c_{.2Elist_{.2ECONS}} A_{.27a}) V2h) V1t)))))) \Rightarrow (\forall V3l \in (ty_{.2Elist_{.2Elist}} \\ & A_{.27a}).(p (ap V0P V3l)))))) \end{aligned} \quad (78)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0l1 \in (ty_2Elist_2Elist \\ & \quad A.27a).(\forall V1l2 \in (ty_2Elist_2Elist\ A.27a).((ap\ (c_2Elist_2ELENGTH \\ A.27a)\ (ap\ (ap\ (c_2Elist_2EAPPEND\ A.27a)\ V0l1)\ V1l2)) = (ap\ (ap\ c_2Earithmetic_2E_2B \\ & \quad (ap\ (c_2Elist_2ELENGTH\ A.27a)\ V0l1))\ (ap\ (c_2Elist_2ELENGTH\ A.27a) \\ & \quad \quad V1l2)))))) \end{aligned} \tag{79}$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0l \in (ty_2Elist_2Elist \\ & \quad A.27a).((ap\ (ap\ (c_2Elist_2ETAKE\ A.27a)\ (ap\ (c_2Elist_2ELENGTH \\ & \quad \quad A.27a)\ V0l))\ V0l) = V0l)) \end{aligned} \tag{80}$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0l1 \in (ty_2Ellist_2Ellist \\ & \quad A.27a).(\forall V1l2 \in (ty_2Ellist_2Ellist\ A.27a).((V0l1 = \\ & \quad V1l2) \Leftrightarrow (\forall V2n \in ty_2Enum_2Enum.((ap\ (ap\ (c_2Ellist_2ELNTH \\ & \quad A.27a)\ V2n)\ V0l1) = (ap\ (ap\ (c_2Ellist_2ELNTH\ A.27a)\ V2n)\ V1l2)))))) \end{aligned} \tag{81}$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0l \in (ty_2Ellist_2Ellist \\ & \quad A.27a).((ap\ (ap\ (c_2Ellist_2ELAPPEND\ A.27a)\ V0l)\ (c_2Ellist_2ELNIL \\ & \quad \quad A.27a)) = V0l)) \end{aligned} \tag{82}$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0n \in ty_2Enum_2Enum.(\\ & \quad \forall V1l1 \in (ty_2Ellist_2Ellist\ A.27a).(\forall V2l2 \in (ty_2Ellist_2Ellist \\ & \quad \quad A.27a).((p\ (ap\ (c_2Eoption_2EIS_SOME\ (ty_2Elist_2Elist\ A.27a)) \\ & \quad (ap\ (ap\ (c_2Ellist_2ELTAKE\ A.27a)\ V0n)\ V1l1))) \Rightarrow ((ap\ (ap\ (c_2Ellist_2ELTAKE \\ & \quad \quad A.27a)\ V0n)\ (ap\ (ap\ (c_2Ellist_2ELAPPEND\ A.27a)\ V1l1)\ V2l2)) = (\\ & \quad \quad \quad ap\ (ap\ (c_2Ellist_2ELTAKE\ A.27a)\ V0n)\ V1l1)))))) \end{aligned} \tag{83}$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\ & \quad ((ap\ (c_2Ellist_2ELLENGTH\ A.27a)\ (c_2Ellist_2ELNIL\ A.27a)) = \\ & \quad (ap\ (c_2Eoption_2ESOME\ ty_2Enum_2Enum)\ c_2Enum_2E0)) \wedge (\forall V0h \in \\ & \quad A.27b.(\forall V1t \in (ty_2Ellist_2Ellist\ A.27b).((ap\ (c_2Ellist_2ELLENGTH \\ & \quad A.27b)\ (ap\ (ap\ (c_2Ellist_2ELCONS\ A.27b)\ V0h)\ V1t)) = (ap\ (ap\ (c_2Eoption_2EOPTION_MAP \\ & \quad \quad ty_2Enum_2Enum\ ty_2Enum_2Enum)\ c_2Enum_2ESUC)\ (ap\ (c_2Ellist_2ELLENGTH \\ & \quad \quad \quad A.27b)\ V1t)))))) \end{aligned} \tag{84}$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0ll \in (ty_2Ellist_2Ellist \\ A.27a).((p\ (ap\ (c_2Ellist_2ELFINITE\ A.27a)\ V0ll)) \Rightarrow (\exists V1n \in \\ ty_2Enum_2Enum.((ap\ (c_2Ellist_2ELLENGTH\ A.27a)\ V0ll) = (ap\ (\\ c_2Eoption_2ESOME\ ty_2Enum_2Enum)\ V1n)))))) \end{aligned} \quad (85)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0n \in ty_2Enum_2Enum.(\\ \forall V1ll \in (ty_2Ellist_2Ellist\ A.27a).(\forall V2m \in ty_2Enum_2Enum. \\ (\forall V3l \in (ty_2Elist_2Elist\ A.27a).((((ap\ (ap\ (c_2Ellist_2ELTAKE \\ A.27a)\ V0n)\ V1ll) = (ap\ (c_2Eoption_2ESOME\ (ty_2Elist_2Elist\ A.27a)) \\ V3l)) \wedge (p\ (ap\ (ap\ c_2Eprim_rec_2E_3C\ V2m)\ V0n))) \Rightarrow ((ap\ (ap\ (c_2Ellist_2ELNTH \\ A.27a)\ V2m)\ V1ll) = (ap\ (c_2Eoption_2ESOME\ A.27a)\ (ap\ (ap\ (c_2Elist_2EEL \\ A.27a)\ V2m)\ V3l)))))))))) \end{aligned} \quad (86)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0ll1 \in (ty_2Ellist_2Ellist \\ A.27a).(\forall V1ll2 \in (ty_2Ellist_2Ellist\ A.27a).((\neg(p\ (ap \\ (c_2Ellist_2ELFINITE\ A.27a)\ V0ll1))) \Rightarrow ((ap\ (ap\ (c_2Ellist_2ELAPPEND \\ A.27a)\ V0ll1)\ V1ll2) = V0ll1)))))) \end{aligned} \quad (87)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow (((ap\ (c_2Ellist_2EfromList\ A.27a) \\ (c_2Elist_2ENIL\ A.27a)) = (c_2Ellist_2ELNIL\ A.27a)) \wedge (\forall V0h \in \\ A.27a.(\forall V1t \in (ty_2Elist_2Elist\ A.27a).((ap\ (c_2Ellist_2EfromList \\ A.27a)\ (ap\ (ap\ (c_2Elist_2ECONS\ A.27a)\ V0h)\ V1t)) = (ap\ (ap\ (c_2Ellist_2ELCONS \\ A.27a)\ V0h)\ (ap\ (c_2Ellist_2EfromList\ A.27a)\ V1t)))))) \end{aligned} \quad (88)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0l \in (ty_2Elist_2Elist \\ A.27a).((ap\ (ap\ (c_2Ellist_2ELTAKE\ A.27a)\ (ap\ (c_2Elist_2ELENGTH \\ A.27a)\ V0l))\ (ap\ (c_2Ellist_2EfromList\ A.27a)\ V0l)) = (ap\ (c_2Eoption_2ESOME \\ (ty_2Elist_2Elist\ A.27a)\ V0l)))) \end{aligned} \quad (89)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0ll \in (ty_2Ellist_2Ellist \\ A.27a).((p\ (ap\ (c_2Ellist_2ELFINITE\ A.27a)\ V0ll)) \Rightarrow (\exists V1l \in \\ (ty_2Elist_2Elist\ A.27a).((ap\ (c_2Ellist_2EtoList\ A.27a)\ V0ll) = \\ (ap\ (c_2Eoption_2ESOME\ (ty_2Elist_2Elist\ A.27a)\ V1l)))))) \end{aligned} \quad (90)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0l \in (ty_2Ellist_2Ellist \\ A.27a).((p\ (ap\ (c_2Ellist_2ELFINITE\ A.27a)\ V0l)) \Rightarrow ((ap\ (c_2Ellist_2EfromList \\ A.27a)\ (ap\ (c_2Eoption_2ETHE\ (ty_2Elist_2Elist\ A.27a))\ (ap\ (c_2Ellist_2EtoList \\ A.27a)\ V0l)))) = V0l))) \end{aligned} \quad (91)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0n \in ty_2Enum_2Enum.(\\ \forall V1l \in (ty_2Ellist_2Ellist\ A.27a).(\forall V2l \in (ty_2Ellist_2Ellist \\ A.27a).((ap\ (ap\ (c_2Ellist_2ELNTH\ A.27a)\ V0n)\ (ap\ (ap\ (c_2Ellist_2ELAPPEND \\ A.27a)\ V1l)\ V2l)) = (ap\ (ap\ (ap\ (c_2Eoption_2Eoption_CASE\ ty_2Enum_2Enum \\ (ty_2Eoption_2Eoption\ A.27a))\ (ap\ (c_2Ellist_2ELLENGTH\ A.27a) \\ V1l))\ (ap\ (ap\ (c_2Ellist_2ELNTH\ A.27a)\ V0n)\ V1l))\ (\lambda V3m \in ty_2Enum_2Enum. \\ (ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ (ty_2Eoption_2Eoption\ A.27a))\ (ap \\ (ap\ c_2Eprim_rec_2E_3C\ V0n)\ V3m))\ (ap\ (ap\ (c_2Ellist_2ELNTH\ A.27a) \\ V0n)\ V1l))\ (ap\ (ap\ (c_2Ellist_2ELNTH\ A.27a)\ (ap\ (ap\ c_2Earithmetic_2E_2D \\ V0n)\ V3m))\ V2l)))))))))) \end{aligned} \quad (92)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0n \in ty_2Enum_2Enum.(\\ \forall V1l \in (ty_2Ellist_2Ellist\ A.27a).(\forall V2l \in (ty_2Elist_2Elist \\ A.27a).(((ap\ (ap\ (c_2Ellist_2ELTAKE\ A.27a)\ V0n)\ V1l) = (ap\ (c_2Eoption_2ESOME \\ (ty_2Elist_2Elist\ A.27a))\ V2l)) \Rightarrow (V0n = (ap\ (c_2Elist_2ELENGTH \\ A.27a)\ V2l)))))) \end{aligned} \quad (93)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0n \in ty_2Enum_2Enum.(\\ \forall V1l \in (ty_2Ellist_2Ellist\ A.27a).(\forall V2l \in (ty_2Elist_2Elist \\ A.27a).(\forall V3m \in ty_2Enum_2Enum.(((ap\ (ap\ (c_2Ellist_2ELTAKE \\ A.27a)\ V0n)\ V1l) = (ap\ (c_2Eoption_2ESOME\ (ty_2Elist_2Elist\ A.27a)) \\ V2l)) \wedge (p\ (ap\ (ap\ c_2Earithmetic_2E_3C_3D\ V3m)\ V0n))) \Rightarrow ((ap\ (ap \\ (c_2Ellist_2ELTAKE\ A.27a)\ V3m)\ V1l) = (ap\ (c_2Eoption_2ESOME \\ (ty_2Elist_2Elist\ A.27a))\ (ap\ (ap\ (c_2Elist_2ETAKE\ A.27a)\ V3m) \\ V2l)))))))))) \end{aligned} \quad (94)$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0l1 \in (ty_2Ellist_2Ellist \\
& \quad A_27a).(\forall V1l2 \in (ty_2Ellist_2Ellist A_27a).(\forall V2x \in \\
& \quad (ty_2Elist_2Elist A_27a).(((ap (c_2Ellist_2EtoList A_27a) (\\
ap (ap (c_2Ellist_2ELAPPEND A_27a) V0l1) V1l2)) = (ap (c_2Eoption_2ESOME \\
\quad (ty_2Elist_2Elist A_27a)) V2x)) \Rightarrow (V2x = (ap (ap (c_2Elist_2EAPPEND \\
A_27a) (ap (c_2Eoption_2ETHE (ty_2Elist_2Elist A_27a)) (ap (c_2Ellist_2EtoList \\
A_27a) V0l1))) (ap (c_2Eoption_2ETHE (ty_2Elist_2Elist A_27a)) \\
\quad (ap (c_2Ellist_2EtoList A_27a) V1l2))))))))))
\end{aligned} \tag{95}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0n \in ty_2Enum_2Enum.(\\
& \quad \forall V1l1 \in (ty_2Ellist_2Ellist A_27a).(\forall V2l1 \in (ty_2Elist_2Elist \\
A_27a).(((ap (ap (c_2Ellist_2ELTAKE A_27a) V0n) V1l1) = (ap (c_2Eoption_2ESOME \\
\quad (ty_2Elist_2Elist A_27a)) V2l1)) \Rightarrow (\exists V3l2 \in (ty_2Ellist_2Ellist \\
A_27a).(((ap (ap (c_2Ellist_2ELDROPP A_27a) V0n) V1l1) = (ap (c_2Eoption_2ESOME \\
\quad (ty_2Ellist_2Ellist A_27a)) V3l2)) \wedge ((ap (ap (c_2Ellist_2ELAPPEND \\
A_27a) (ap (c_2Ellist_2EfromList A_27a) V2l1)) V3l2) = V1l1))))))))))
\end{aligned} \tag{96}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2B \\
& \quad c_2Enum_2E0) V0n) = V0n)) \wedge ((\forall V1n \in ty_2Enum_2Enum.((ap \\
& \quad (ap c_2Earithmetic_2E_2B V1n) c_2Enum_2E0) = V1n)) \wedge ((\forall V2n \in \\
& \quad ty_2Enum_2Enum.(\forall V3m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2B \\
& \quad (ap c_2Earithmetic_2ENUMERAL V2n)) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V3m)) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Enumeral_2EiZ (ap \\
& \quad (ap c_2Earithmetic_2E_2B V2n) V3m)))))) \wedge ((\forall V4n \in ty_2Enum_2Enum. \\
& \quad ((ap (ap c_2Earithmetic_2E_2A c_2Enum_2E0) V4n) = c_2Enum_2E0)) \wedge \\
& \quad ((\forall V5n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2A \\
& \quad V5n) c_2Enum_2E0) = c_2Enum_2E0)) \wedge ((\forall V6n \in ty_2Enum_2Enum. \\
& \quad (\forall V7m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2A (\\
& \quad ap c_2Earithmetic_2ENUMERAL V6n)) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V7m)) = (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2E_2A \\
& \quad V6n) V7m)))))) \wedge ((\forall V8n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2D \\
& \quad c_2Enum_2E0) V8n) = c_2Enum_2E0)) \wedge ((\forall V9n \in ty_2Enum_2Enum. \\
& \quad ((ap (ap c_2Earithmetic_2E_2D V9n) c_2Enum_2E0) = V9n)) \wedge ((\forall V10n \in \\
& \quad ty_2Enum_2Enum.(\forall V11m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2D \\
& \quad (ap c_2Earithmetic_2ENUMERAL V10n)) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V11m)) = (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2E_2D \\
& \quad V10n) V11m)))))) \wedge ((\forall V12n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2EEXP \\
& \quad c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\
& \quad V12n))) = c_2Enum_2E0)) \wedge ((\forall V13n \in ty_2Enum_2Enum.((ap \\
& \quad (ap c_2Earithmetic_2EEXP c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap c_2Earithmetic_2EBIT2 V13n))) = c_2Enum_2E0)) \wedge ((\forall V14n \in \\
& \quad ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2EEXP V14n) c_2Enum_2E0) = \\
& \quad (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))))) \wedge \\
& \quad ((\forall V15n \in ty_2Enum_2Enum.(\forall V16m \in ty_2Enum_2Enum. \\
& \quad ((ap (ap c_2Earithmetic_2EEXP (ap c_2Earithmetic_2ENUMERAL V15n)) \\
& \quad (ap c_2Earithmetic_2ENUMERAL V16m)) = (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap (ap c_2Earithmetic_2EEXP V15n) V16m)))))) \wedge ((ap c_2Enum_2ESUC \\
& \quad c_2Enum_2E0) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\
& \quad c_2Earithmetic_2EZERO))) \wedge ((\forall V17n \in ty_2Enum_2Enum. (\\
& \quad (ap c_2Enum_2ESUC (ap c_2Earithmetic_2ENUMERAL V17n)) = (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap c_2Enum_2ESUC V17n)))))) \wedge ((ap c_2Eprim_rec_2EPRE c_2Enum_2E0) = \\
& \quad c_2Enum_2E0) \wedge ((\forall V18n \in ty_2Enum_2Enum.((ap c_2Eprim_rec_2EPRE \\
& \quad (ap c_2Earithmetic_2ENUMERAL V18n)) = (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap c_2Eprim_rec_2EPRE V18n)))))) \wedge ((\forall V19n \in ty_2Enum_2Enum. \\
& \quad (((ap c_2Earithmetic_2ENUMERAL V19n) = c_2Enum_2E0) \Leftrightarrow (V19n = c_2Earithmetic_2EZERO))) \wedge \\
& \quad ((\forall V20n \in ty_2Enum_2Enum.((c_2Enum_2E0 = (ap c_2Earithmetic_2ENUMERAL \\
& \quad V20n)) \Leftrightarrow (V20n = c_2Earithmetic_2EZERO))) \wedge ((\forall V21n \in ty_2Enum_2Enum. \\
& \quad (\forall V22m \in ty_2Enum_2Enum.(((ap c_2Earithmetic_2ENUMERAL \\
& \quad V21n) = (ap c_2Earithmetic_2ENUMERAL V22m)) \Leftrightarrow (V21n = V22m)))))) \wedge \\
& \quad ((\forall V23n \in ty_2Enum_2Enum.((p (ap (ap c_2Eprim_rec_2E_3C \\
& \quad V23n) c_2Enum_2E0)) \Leftrightarrow False)) \wedge ((\forall V24n \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V24n))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmetic_2EZERO) \\
& \quad V24n)))))) \wedge ((\forall V25n \in ty_2Enum_2Enum.(\forall V26m \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Eprim_rec_2E_3C (ap c_2Earithmetic_2ENUMERAL \\
& \quad V25n)) (ap c_2Earithmetic_2ENUMERAL V26m))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C \\
& \quad V25n) V26m)))))) \wedge ((\forall V27n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmetic_2E_3E \\
& \quad c_2Enum_2E0) V27n)) \Leftrightarrow False)) \wedge ((\forall V28n \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Earithmetic_2E_3E (ap c_2Earithmetic_2ENUMERAL \\
& \quad V28n)) c_2Enum_2E0)) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmetic_2EZERO) \\
& \quad V28n)))))) \wedge ((\forall V29n \in ty_2Enum_2Enum.(\forall V30m \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Earithmetic_2E_3E (ap c_2Earithmetic_2ENUMERAL \\
& \quad V29n)) (ap c_2Earithmetic_2ENUMERAL V30m))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C \\
& \quad V30m) V29n)))))) \wedge ((\forall V31n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmetic_2E_3C_3D \\
& \quad c_2Enum_2E0) V31n)) \Leftrightarrow True)) \wedge ((\forall V32n \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2ENUMERAL
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\\
& ((p (ap (ap c_2Earithmic_2E_3C_3D c_2Earithmic_2EZERO) V0n)) \Leftrightarrow \\
& True) \wedge (((p (ap (ap c_2Earithmic_2E_3C_3D (ap c_2Earithmic_2EBIT1 \\
& V0n)) c_2Earithmic_2EZERO)) \Leftrightarrow False) \wedge (((p (ap (ap c_2Earithmic_2E_3C_3D \\
& (ap c_2Earithmic_2EBIT2 V0n)) c_2Earithmic_2EZERO)) \Leftrightarrow False) \wedge \\
& (((p (ap (ap c_2Earithmic_2E_3C_3D (ap c_2Earithmic_2EBIT1 \\
& V0n)) (ap c_2Earithmic_2EBIT1 V1m))) \Leftrightarrow (p (ap (ap c_2Earithmic_2E_3C_3D \\
& V0n) V1m))) \wedge (((p (ap (ap c_2Earithmic_2E_3C_3D (ap c_2Earithmic_2EBIT1 \\
& V0n)) (ap c_2Earithmic_2EBIT2 V1m))) \Leftrightarrow (p (ap (ap c_2Earithmic_2E_3C_3D \\
& V0n) V1m))) \wedge (((p (ap (ap c_2Earithmic_2E_3C_3D (ap c_2Earithmic_2EBIT2 \\
& V0n)) (ap c_2Earithmic_2EBIT1 V1m))) \Leftrightarrow (\neg (p (ap (ap c_2Earithmic_2E_3C_3D \\
& V1m) V0n)))) \wedge ((p (ap (ap c_2Earithmic_2E_3C_3D (ap c_2Earithmic_2EBIT2 \\
& V0n)) (ap c_2Earithmic_2EBIT2 V1m))) \Leftrightarrow (p (ap (ap c_2Earithmic_2E_3C_3D \\
& V0n) V1m))))))))))
\end{aligned} \tag{98}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty A_27a \Rightarrow (\forall V0opt \in (ty_2Eoption_2Eoption \\
& A_27a). ((V0opt = (c_2Eoption_2ENONE A_27a)) \vee (\exists V1x \in A_27a. \\
& (V0opt = (ap (c_2Eoption_2ESOME A_27a) V1x))))))
\end{aligned} \tag{99}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow (\\
& (\forall V0v \in A_27b. (\forall V1f \in (A_27b^{A_27a}). ((ap (ap (ap (c_2Eoption_2Eoption_CASE \\
& A_27a A_27b) (c_2Eoption_2ENONE A_27a)) V0v) V1f) = V0v))) \wedge (\forall V2x \in \\
& A_27a. (\forall V3v \in A_27b. (\forall V4f \in (A_27b^{A_27a}). ((ap (ap \\
& (ap (c_2Eoption_2Eoption_CASE A_27a A_27b) (ap (c_2Eoption_2ESOME \\
& A_27a) V2x)) V3v) V4f) = (ap V4f V2x))))))
\end{aligned} \tag{100}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in \\
& A_27a. (((ap (c_2Eoption_2ESOME A_27a) V0x) = (ap (c_2Eoption_2ESOME \\
& A_27a) V1y)) \Leftrightarrow (V0x = V1y))))
\end{aligned} \tag{101}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a. (\neg ((ap (c_2Eoption_2ESOME \\
& A_27a) V0x) = (c_2Eoption_2ENONE A_27a))))
\end{aligned} \tag{102}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & (\forall V0f \in (A_27b^{A_27a}).(\forall V1x \in A_27a.((ap\ (ap\ (c_2Eoption_2EOPTION_MAP \\ & A_27a\ A_27b)\ V0f)\ (ap\ (c_2Eoption_2ESOME\ A_27a)\ V1x)) = (ap\ (c_2Eoption_2ESOME \\ & A_27b)\ (ap\ V0f\ V1x)))))) \wedge (\forall V2f \in (A_27b^{A_27a}).((ap\ (ap\ (c_2Eoption_2EOPTION_MAP \\ & A_27a\ A_27b)\ V2f)\ (c_2Eoption_2ENONE\ A_27a)) = (c_2Eoption_2ENONE \\ & A_27b)))))) \end{aligned} \quad (103)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.((ap\ (c_2Eoption_2ETHE \\ & A_27a)\ (ap\ (c_2Eoption_2ESOME\ A_27a)\ V0x)) = V0x)) \end{aligned} \quad (104)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0opt \in (ty_2Eoption_2Eoption \\ & A_27a).((p\ (ap\ (c_2Eoption_2EIS_SOME\ A_27a)\ V0opt)) \Leftrightarrow (\exists V1x \in \\ & A_27a.(V0opt = (ap\ (c_2Eoption_2ESOME\ A_27a)\ V1x)))))) \end{aligned} \quad (105)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in (ty_2Eoption_2Eoption \\ & A_27a).((p\ (ap\ (c_2Eoption_2EIS_NONE\ A_27a)\ V0x)) \Leftrightarrow (V0x = (c_2Eoption_2ENONE \\ & A_27a)))))) \end{aligned} \quad (106)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in 2.(\forall V1X \in (\\ & ty_2Eoption_2Eoption\ A_27a).(\forall V2x \in A_27a.(((ap\ (ap\ (\\ & ap\ (c_2Ebool_2ECOND\ (ty_2Eoption_2Eoption\ A_27a))\ V0P)\ V1X)\ (\\ & c_2Eoption_2ENONE\ A_27a)) = (c_2Eoption_2ENONE\ A_27a)) \Leftrightarrow ((p\ V0P) \Rightarrow \\ & (p\ (ap\ (c_2Eoption_2EIS_NONE\ A_27a)\ V1X)))))) \wedge (((ap\ (ap\ (ap\ (c_2Ebool_2ECOND \\ & (ty_2Eoption_2Eoption\ A_27a))\ V0P)\ (c_2Eoption_2ENONE\ A_27a)) \\ & V1X) = (c_2Eoption_2ENONE\ A_27a)) \Leftrightarrow ((p\ (ap\ (c_2Eoption_2EIS_SOME \\ & A_27a)\ V1X)) \Rightarrow (p\ V0P)))) \wedge (((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ (ty_2Eoption_2Eoption \\ & A_27a))\ V0P)\ V1X)\ (c_2Eoption_2ENONE\ A_27a)) = (ap\ (c_2Eoption_2ESOME \\ & A_27a)\ V2x)) \Leftrightarrow ((p\ V0P) \wedge (V1X = (ap\ (c_2Eoption_2ESOME\ A_27a)\ V2x)))) \wedge \\ & (((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ (ty_2Eoption_2Eoption\ A_27a)) \\ & V0P)\ (c_2Eoption_2ENONE\ A_27a))\ V1X) = (ap\ (c_2Eoption_2ESOME \\ & A_27a)\ V2x)) \Leftrightarrow ((\neg(p\ V0P)) \wedge (V1X = (ap\ (c_2Eoption_2ESOME\ A_27a)\ \\ & V2x))))))))) \end{aligned} \quad (107)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(\\ & ((ap\ c_2Enum_2ESUC\ V0m) = (ap\ c_2Enum_2ESUC\ V1n)) \Leftrightarrow (V0m = V1n)))) \end{aligned} \quad (108)$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum.(\neg(p (ap (ap c_2Eprim_rec_2E_3C V0n) V0n)))) \quad (109)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0l1 \in (ty_2Elist_2Elist \\ A_27a).(\forall V1l2 \in (ty_2Elist_2Elist\ A_27a).((p (ap (ap (c_2Elist_2EisPREFIX \\ A_27a) V1l2) V0l1)) \Leftrightarrow (\exists V2l \in (ty_2Elist_2Elist\ A_27a). \\ V0l1 = (ap (ap (c_2Elist_2EAPPEND\ A_27a) V1l2) V2l)))))) \end{aligned} \quad (110)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0n \in ty_2Enum_2Enum.(\\ \forall V1l1 \in (ty_2Elist_2Elist\ A_27a).((p (ap (ap c_2Earithmic_2E_3C_3D \\ V0n) (ap (c_2Elist_2ELENGTH\ A_27a) V1l1))) \Rightarrow (\forall V2l2 \in (ty_2Elist_2Elist \\ A_27a).((ap (ap (c_2Elist_2ETAKE\ A_27a) V0n) (ap (ap (c_2Elist_2EAPPEND \\ A_27a) V1l1) V2l2)) = (ap (ap (c_2Elist_2ETAKE\ A_27a) V0n) V1l1)))))) \end{aligned} \quad (111)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0n \in ty_2Enum_2Enum.(\\ \forall V1l1 \in (ty_2Elist_2Elist\ A_27a).(\forall V2l2 \in (ty_2Elist_2Elist \\ A_27a).((p (ap (ap c_2Eprim_rec_2E_3C\ V0n) (ap (c_2Elist_2ELENGTH \\ A_27a) V1l1))) \Rightarrow ((ap (ap (c_2Elist_2EEL\ A_27a) V0n) (ap (ap (c_2Elist_2EAPPEND \\ A_27a) V1l1) V2l2)) = (ap (ap (c_2Elist_2EEL\ A_27a) V0n) V1l1)))))) \end{aligned} \quad (112)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \quad (113)$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \quad (114)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\ ((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))) \quad (115)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p\ V0A)) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\ ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))) \quad (116)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p\ V0A)) \Rightarrow False) \Rightarrow (((p\ V0A) \Rightarrow False) \Rightarrow False))) \quad (117)$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg(\\
& p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee \\
& ((\neg(p V1q)) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{118}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee \\
& (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p))))))
\end{aligned} \tag{119}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge \\
& ((p V1q) \vee ((p V2r) \vee (\neg(p V0p))))))
\end{aligned} \tag{120}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((\\
& \neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p))))))
\end{aligned} \tag{121}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee \\
& (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p))))))
\end{aligned} \tag{122}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p))) \tag{123}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))) \tag{124}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V0p)))) \tag{125}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V1q)))) \tag{126}$$

Assume the following.

$$(\forall V0p \in 2. ((\neg(\neg(p V0p))) \Rightarrow (p V0p))) \tag{127}$$

Theorem 1

$$\begin{aligned}
& \forall A_27a. nonempty A_27a \Rightarrow (\forall V0l1 \in (ty_2Ellist_2Ellist \\
& A_27a). (\forall V1l2 \in (ty_2Ellist_2Ellist A_27a). ((p (ap (ap \\
& (c_2Ellist_2ELPREFIX A_27a) V0l1) V1l2)) \Leftrightarrow (\exists V2ll \in (ty_2Ellist_2Ellist \\
& A_27a). (V1l2 = (ap (ap (c_2Ellist_2ELAPPEND A_27a) V0l1) V2ll))))))
\end{aligned}$$