

thm_2Elist_2ELPREFIX_LCONS
(TMcSMKJBA3wdRzS5yn4WaD5rFk5G1j3J6af)

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Definition 1 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Elist_2Elist A0) \quad (1)$$

Let $c_2Elist_2Elist_CASE : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Elist_2Elist_CASE \\ & A_27a A_27b \in (((A_27b^{(A_27b^{(ty_2Elist_2Elist A_27a)})^{A_27a}})_{A_27b})^{(ty_2Elist_2Elist A_27a)}) \end{aligned} \quad (2)$$

Definition 2 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a})))$

Definition 5 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty ty_2Enum_2Enum \quad (3)$$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (omega^{ty_2Enum_2Enum}) \quad (4)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (omega^{omega}) \quad (5)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{omega}) \quad (6)$$

Definition 6 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num$
Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{7}$$

Definition 7 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Let $c_2Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow c_2Elist_2ECONS\ A.27a \in (((ty_2Elist_2Elist\ A.27a)^{(ty_2Elist_2Elist\ A.27a)})^{A.27a}) \tag{8}$$

Definition 8 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \tag{9}$$

Definition 9 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ (ap\ c_2Earithmetic_2E_2B\ n))$

Definition 10 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let $c_2Earithmetic_2E_2D : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2D \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \tag{10}$$

Let $ty_2Eoption_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Eoption_2Eoption\ A0) \tag{11}$$

Let $ty_2Ellist_2Ellist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Ellist_2Ellist\ A0) \tag{12}$$

Let $c_2Ellist_2Ellist_rep : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow c_2Ellist_2Ellist_rep\ A.27a \in (((ty_2Eoption_2Eoption\ A.27a)^{(ty_2Enum_2Enum)})^{(ty_2Ellist_2Ellist\ A.27a)}) \tag{13}$$

Let $ty_2Eone_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eone_2Eone \tag{14}$$

Definition 11 We define $c_2Ebool_2E_2F.5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ t1\ t2))$

Let $ty_2Esum_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Esum_2Esum\ A0\ A1) \quad (15)$$

Let $c_2Esum_2EABS_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Esum_2EABS_sum\ A_27a\ A_27b \in ((ty_2Esum_2Esum\ A_27a\ A_27b)^{((2^{A_27b})^{A_27a})^2}) \quad (16)$$

Definition 12 We define c_2Esum_2EINL to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0e \in A_27a.(ap\ (c_2Esum_2EABS_sum\ A_27a\ A_27b)\ V0e)$

Let $c_2Eoption_2Eoption_abs : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Eoption_2Eoption_abs\ A_27a \in ((ty_2Eoption_2Eoption\ A_27a)^{(ty_2Esum_2Esum\ A_27a\ ty_2Eone_2Eone)}) \quad (17)$$

Definition 13 We define $c_2Eoption_2ESOME$ to be $\lambda A_27a : \iota.\lambda V0x \in A_27a.(ap\ (c_2Eoption_2Eoption_abs\ A_27a)\ V0x)$

Definition 14 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.\mathbf{if}\ (\exists x \in A.p\ (ap\ P\ x))\ \mathbf{then}\ (the\ (\lambda x.x \in A \wedge P\ x))\ \mathbf{else}\ (ty_2Eone_2Eone)$ of type $\iota \Rightarrow \iota$.

Definition 15 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.(ap\ (c_2Emin_2E_40\ A_27a)\ V2t2)\ V1t1))\ V0t)$

Let $c_2Ellist_2Ellist_abs : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ellist_2Ellist_abs\ A_27a \in ((ty_2Ellist_2Ellist\ A_27a)^{(ty_2Eoption_2Eoption\ A_27a)^{ty_2Enum_2Enum}}) \quad (18)$$

Definition 16 We define $c_2Ellist_2ELCONS$ to be $\lambda A_27a : \iota.\lambda V0h \in A_27a.\lambda V1t \in (ty_2Ellist_2Ellist\ A_27a)\ (ap\ (c_2Ellist_2Ellist_abs\ A_27a)\ V1t)\ V0h$

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ENIL\ A_27a \in (ty_2Elist_2Elist\ A_27a) \quad (19)$$

Definition 17 We define c_2Eone_2Eone to be $(ap\ (c_2Emin_2E_40\ ty_2Eone_2Eone)\ (\lambda V0x \in ty_2Eone_2Eone.\ V0x))$

Definition 18 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap\ (ap\ c_2Emin_2E_3D_3D_3E\ V0t)\ c_2Ebool_2ECOND\ A)\ V0t)$

Definition 19 We define c_2Esum_2EINR to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0e \in A_27b.(ap\ (c_2Esum_2EABS_sum\ A_27a\ A_27b)\ V0e)$

Definition 20 We define $c_2Eoption_2ENONE$ to be $\lambda A_27a : \iota.(ap\ (c_2Eoption_2Eoption_abs\ A_27a)\ (ty_2Eone_2Eone))$

Definition 21 We define $c_2Ellist_2ELNIL$ to be $\lambda A_27a : \iota.(ap\ (c_2Ellist_2Ellist_abs\ A_27a)\ (\lambda V0n \in ty_2Ellist_2Ellist\ A_27a.\ V0n))$

Let $c_2Elist_2EisPREFIX : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2EisPREFIX\ A_27a \in ((2^{(ty_2Elist_2Elist\ A_27a)})^{(ty_2Elist_2Elist\ A_27a)}) \quad (20)$$

Let $c_2Elist_2ELENGTH : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ELENGTH\ A_27a \in (ty_2Enum_2Enum^{(ty_2Elist_2Elist\ A_27a)}) \quad (21)$$

Let $c_2Ellist_2ELTAKE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ellist_2ELTAKE\ A_27a \in (((ty_2Eoption_2Eoption\ (ty_2Elist_2Elist\ A_27a))^{(ty_2Ellist_2Ellist\ A_27a)})^{ty_2Enum_2Enum}) \quad (22)$$

Definition 22 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap\ V0P\ (ap\ (c_2Emin_2E_40$

Definition 23 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in$

Definition 24 We define $c_2Ellist_2Elength_rel$ to be $\lambda A_27a : \iota. (\lambda V0a0 \in (ty_2Ellist_2Ellist\ A_27a). (\lambda V$

Definition 25 We define $c_2Ellist_2ELFINITE$ to be $\lambda A_27a : \iota. (\lambda V0a0 \in (ty_2Ellist_2Ellist\ A_27a). (ap\ (c$

Definition 26 We define $c_2Ellist_2ELLENGTH$ to be $\lambda A_27a : \iota. \lambda V0ll \in (ty_2Ellist_2Ellist\ A_27a). (ap\ (a$

Let $c_2Eoption_2ETHE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Eoption_2ETHE\ A_27a \in (A_27a^{(ty_2Eoption_2Eoption\ A_27a)}) \quad (23)$$

Definition 27 We define $c_2Ellist_2EtoList$ to be $\lambda A_27a : \iota. \lambda V0ll \in (ty_2Ellist_2Ellist\ A_27a). (ap\ (ap\ (ap$

Let $c_2Eoption_2Eoption_CASE : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Eoption_2Eoption_CASE\ A_27a\ A_27b \in (((A_27b^{(A_27b^{A_27a})})^{A_27b})^{(ty_2Eoption_2Eoption\ A_27a)}) \quad (24)$$

Definition 28 We define $c_2Ellist_2ELPREFIX$ to be $\lambda A_27a : \iota. \lambda V0l1 \in (ty_2Ellist_2Ellist\ A_27a). \lambda V1l2$

Let $c_2Eoption_2EOPTION_MAP : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Eoption_2EOPTION_MAP\ A_27a\ A_27b \in (((ty_2Eoption_2Eoption\ A_27b)^{(ty_2Eoption_2Eoption\ A_27a)})^{(A_27b^{A_27a})}) \quad (25)$$

Assume the following.

$$True \quad (26)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (27)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p V0t))) \quad (28)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0t \in 2.((\exists V1x \in A_27a.(p V0t)) \Leftrightarrow (p V0t))) \quad (29)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \end{aligned} \quad (30)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \vee (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \vee True) \Leftrightarrow True) \wedge \\ & (((False \vee (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee False) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee \\ & (p V0t)) \Leftrightarrow (p V0t)))))) \end{aligned} \quad (31)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((\\ & (p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))) \end{aligned} \quad (32)$$

Assume the following.

$$\begin{aligned} & ((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge \\ & ((\neg False) \Leftrightarrow True))) \end{aligned} \quad (33)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (34)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (35)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(\\ & p V0t)))))) \end{aligned} \quad (36)$$

Assume the following.

$$\begin{aligned} & (\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow \\ & ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \end{aligned} \quad (37)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Leftrightarrow (p V1t2)) \Leftrightarrow (((p V0t1) \Rightarrow (p V1t2)) \wedge ((p V1t2) \Rightarrow (p V0t1)))))) \quad (38)$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x_{.27} \in 2.(\forall V2y \in 2.(\forall V3y_{.27} \in 2.(((p V0x) \Leftrightarrow (p V1x_{.27})) \wedge ((p V1x_{.27}) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_{.27})))))) \Rightarrow ((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_{.27}) \Rightarrow (p V3y_{.27})))))) \quad (39)$$

Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow (\forall V0P \in (2^{A_{.27a}}).(\forall V1a \in A_{.27a}.((\exists V2x \in A_{.27a}.((V2x = V1a) \wedge (p (ap V0P V2x)))) \Leftrightarrow (p (ap V0P V1a)))))) \quad (40)$$

Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow (((ap (c_{.2Elist}_{.2ELENGTH} A_{.27a}) (c_{.2Elist}_{.2ENIL} A_{.27a})) = c_{.2Enum}_{.2E0}) \wedge (\forall V0h \in A_{.27a}.(\forall V1t \in (ty_{.2Elist}_{.2Elist} A_{.27a}).((ap (c_{.2Elist}_{.2ELENGTH} A_{.27a}) (ap (ap (c_{.2Elist}_{.2ECONS} A_{.27a}) V0h) V1t)) = (ap c_{.2Enum}_{.2ESUC} (ap (c_{.2Elist}_{.2ELENGTH} A_{.27a}) V1t))))))) \quad (41)$$

Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow (\forall V0a0 \in A_{.27a}.(\forall V1a1 \in (ty_{.2Elist}_{.2Elist} A_{.27a}).(\forall V2a0_{.27} \in A_{.27a}.(\forall V3a1_{.27} \in (ty_{.2Elist}_{.2Elist} A_{.27a}).(((ap (ap (c_{.2Elist}_{.2ECONS} A_{.27a}) V0a0) V1a1) = (ap (ap (c_{.2Elist}_{.2ECONS} A_{.27a}) V2a0_{.27}) V3a1_{.27})) \Leftrightarrow ((V0a0 = V2a0_{.27}) \wedge (V1a1 = V3a1_{.27}))))))) \quad (42)$$

Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow (\forall V0a1 \in (ty_{.2Elist}_{.2Elist} A_{.27a}).(\forall V1a0 \in A_{.27a}.(\neg((c_{.2Elist}_{.2ENIL} A_{.27a}) = (ap (ap (c_{.2Elist}_{.2ECONS} A_{.27a}) V1a0) V0a1)))))) \quad (43)$$

Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow (\forall V0l \in (ty_{.2Elist}_{.2Elist} A_{.27a}).(((ap (c_{.2Elist}_{.2ELENGTH} A_{.27a}) V0l) = c_{.2Enum}_{.2E0}) \Leftrightarrow (V0l = (c_{.2Elist}_{.2ENIL} A_{.27a})))) \quad (44)$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow ((\forall V0l \in (ty_2Elist_2Elist \\
& A_27a).(p\ (ap\ (ap\ (c_2Elist_2EisPREFIX\ A_27a)\ (c_2Elist_2ENIL \\
& A_27a))\ V0l)) \Leftrightarrow True)) \wedge (\forall V1h \in A_27a.(\forall V2t \in (ty_2Elist_2Elist \\
& A_27a).(\forall V3l \in (ty_2Elist_2Elist\ A_27a).((p\ (ap\ (ap\ (c_2Elist_2EisPREFIX \\
& A_27a)\ (ap\ (ap\ (c_2Elist_2ECONS\ A_27a)\ V1h)\ V2t))\ V3l)) \Leftrightarrow (p\ (ap\ (\\
& ap\ (ap\ (c_2Elist_2Elist_CASE\ A_27a\ 2)\ V3l)\ c_2Ebool_2EF)\ (\lambda V4h_27 \in \\
& A_27a.(\lambda V5t_27 \in (ty_2Elist_2Elist\ A_27a).(ap\ (ap\ c_2Ebool_2E_2F_5C \\
& (ap\ (ap\ (c_2Emin_2E_3D\ A_27a)\ V1h)\ V4h_27))\ (ap\ (ap\ (c_2Elist_2EisPREFIX \\
& A_27a)\ V2t)\ V5t_27))))))))))
\end{aligned} \tag{45}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0l \in (ty_2Elist_2Elist \\
& A_27a).(\forall V1h \in A_27a.(\forall V2t \in (ty_2Elist_2Elist\ A_27a). \\
& (\forall V3h1 \in A_27a.(\forall V4t1 \in (ty_2Elist_2Elist\ A_27a). \\
& (\forall V5h2 \in A_27a.(\forall V6t2 \in (ty_2Elist_2Elist\ A_27a). \\
& (((p\ (ap\ (ap\ (c_2Elist_2EisPREFIX\ A_27a)\ (c_2Elist_2ENIL\ A_27a)) \\
& V0l)) \Leftrightarrow True) \wedge (((p\ (ap\ (ap\ (c_2Elist_2EisPREFIX\ A_27a)\ (ap\ (ap\ (\\
& c_2Elist_2ECONS\ A_27a)\ V1h)\ V2t))\ (c_2Elist_2ENIL\ A_27a))) \Leftrightarrow False) \wedge \\
& ((p\ (ap\ (ap\ (c_2Elist_2EisPREFIX\ A_27a)\ (ap\ (ap\ (c_2Elist_2ECONS \\
& A_27a)\ V3h1)\ V4t1))\ (ap\ (ap\ (c_2Elist_2ECONS\ A_27a)\ V5h2)\ V6t2))) \Leftrightarrow \\
& ((V3h1 = V5h2) \wedge (p\ (ap\ (ap\ (c_2Elist_2EisPREFIX\ A_27a)\ V4t1)\ V6t2))))))))))
\end{aligned} \tag{46}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in (ty_2Elist_2Elist \\
& A_27a).(p\ (ap\ (ap\ (c_2Elist_2EisPREFIX\ A_27a)\ V0x)\ (c_2Elist_2ENIL \\
& A_27a))) \Leftrightarrow (V0x = (c_2Elist_2ENIL\ A_27a)))
\end{aligned} \tag{47}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0l \in (ty_2Ellist_2Ellist \\
& A_27a).((V0l = (c_2Ellist_2ELNIL\ A_27a)) \vee (\exists V1h \in A_27a. \\
& (\exists V2t \in (ty_2Ellist_2Ellist\ A_27a).(V0l = (ap\ (ap\ (c_2Ellist_2ELCONS \\
& A_27a)\ V1h)\ V2t))))))
\end{aligned} \tag{48}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0h \in A_27a.(\forall V1t \in \\
& (ty_2Ellist_2Ellist\ A_27a).((\neg((ap\ (ap\ (c_2Ellist_2ELCONS\ A_27a) \\
& V0h)\ V1t) = (c_2Ellist_2ELNIL\ A_27a))) \wedge (\neg((c_2Ellist_2ELNIL \\
& A_27a) = (ap\ (ap\ (c_2Ellist_2ELCONS\ A_27a)\ V0h)\ V1t))))))
\end{aligned} \tag{49}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0h1 \in A_27a. (\forall V1t1 \in \\
& \quad (ty_2Elist_2Elist A_27a). (\forall V2h2 \in A_27a. (\forall V3t2 \in \\
& \quad (ty_2Elist_2Elist A_27a). (((ap (ap (c_2Elist_2ELCONS A_27a) \\
& \quad V0h1) V1t1) = (ap (ap (c_2Elist_2ELCONS A_27a) V2h2) V3t2))) \Leftrightarrow ((\\
& \quad V0h1 = V2h2) \wedge (V1t1 = V3t2))))))
\end{aligned} \tag{50}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.\text{nonempty } A_27a \Rightarrow \forall A_27b.\text{nonempty } A_27b \Rightarrow \forall A_27c. \\
& \quad \text{nonempty } A_27c \Rightarrow ((\forall V0l \in (ty_2Elist_2Elist A_27a). ((\\
ap (ap (c_2Elist_2ELTAKE A_27a) c_2Enum_2E0) V0l) = (ap (c_2Eoption_2ESOME \\
& \quad (ty_2Elist_2Elist A_27a)) (c_2Elist_2ENIL A_27a)))) \wedge ((\forall V1n \in \\
ty_2Enum_2Enum. ((ap (ap (c_2Elist_2ELTAKE A_27b) (ap c_2Enum_2ESUC \\
& \quad V1n)) (c_2Elist_2ELNIL A_27b)) = (c_2Eoption_2ENONE (ty_2Elist_2Elist \\
& \quad A_27b)))) \wedge (\forall V2n \in ty_2Enum_2Enum. (\forall V3h \in A_27c. \\
& \quad (\forall V4t \in (ty_2Elist_2Elist A_27c). ((ap (ap (c_2Elist_2ELTAKE \\
& \quad A_27c) (ap c_2Enum_2ESUC V2n)) (ap (ap (c_2Elist_2ELCONS A_27c) \\
& \quad V3h) V4t)) = (ap (ap (c_2Eoption_2EOPTION_MAP (ty_2Elist_2Elist \\
& \quad A_27c) (ty_2Elist_2Elist A_27c)) (ap (c_2Elist_2ECONS A_27c) \\
& \quad V3h)) (ap (ap (c_2Elist_2ELTAKE A_27c) V2n) V4t)))))))))
\end{aligned} \tag{51}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0l \in (ty_2Elist_2Elist \\
& \quad A_27a). (\forall V1m \in ty_2Enum_2Enum. (((ap (ap (c_2Elist_2ELTAKE \\
& \quad A_27a) V1m) (c_2Elist_2ELNIL A_27a)) = (ap (c_2Eoption_2ESOME \\
& \quad (ty_2Elist_2Elist A_27a)) V0l)) \Leftrightarrow ((V1m = c_2Enum_2E0) \wedge (V0l = (\\
& \quad c_2Elist_2ENIL A_27a))))))
\end{aligned} \tag{52}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.\text{nonempty } A_27a \Rightarrow \forall A_27b.\text{nonempty } A_27b \Rightarrow (\\
& \quad ((ap (c_2Elist_2EtoList A_27a) (c_2Elist_2ELNIL A_27a)) = (\\
ap (c_2Eoption_2ESOME (ty_2Elist_2Elist A_27a)) (c_2Elist_2ENIL \\
& \quad A_27a)))) \wedge (\forall V0h \in A_27b. (\forall V1t \in (ty_2Elist_2Elist \\
& \quad A_27b). ((ap (c_2Elist_2EtoList A_27b) (ap (ap (c_2Elist_2ELCONS \\
& \quad A_27b) V0h) V1t)) = (ap (ap (c_2Eoption_2EOPTION_MAP (ty_2Elist_2Elist \\
& \quad A_27b) (ty_2Elist_2Elist A_27b)) (ap (c_2Elist_2ECONS A_27b) \\
& \quad V0h)) (ap (c_2Elist_2EtoList A_27b) V1t))))))
\end{aligned} \tag{53}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0opt \in (ty_2Eoption_2Eoption \\
& \quad A_27a). ((V0opt = (c_2Eoption_2ENONE A_27a)) \vee (\exists V1x \in A_27a. \\
& \quad (V0opt = (ap (c_2Eoption_2ESOME A_27a) V1x))))))
\end{aligned} \tag{54}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& (\forall V0v \in A_27b. (\forall V1f \in (A_27b^{A_27a}). ((ap\ (ap\ (ap\ (c_2Eoption_2Eoption_CASE \\
& A_27a\ A_27b)\ (c_2Eoption_2ENONE\ A_27a))\ V0v)\ V1f) = V0v))) \wedge (\forall V2x \in \\
& A_27a. (\forall V3v \in A_27b. (\forall V4f \in (A_27b^{A_27a}). ((ap\ (ap \\
& (ap\ (c_2Eoption_2Eoption_CASE\ A_27a\ A_27b)\ (ap\ (c_2Eoption_2ESOME \\
& A_27a)\ V2x))\ V3v)\ V4f) = (ap\ V4f\ V2x))))))
\end{aligned} \tag{55}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in \\
& A_27a. (((ap\ (c_2Eoption_2ESOME\ A_27a)\ V0x) = (ap\ (c_2Eoption_2ESOME \\
& A_27a)\ V1y)) \Leftrightarrow (V0x = V1y))))
\end{aligned} \tag{56}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\neg((c_2Eoption_2ENONE \\
& A_27a) = (ap\ (c_2Eoption_2ESOME\ A_27a)\ V0x))))
\end{aligned} \tag{57}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \forall V0f \in (A_27b^{A_27a}). (\forall V1x \in (ty_2Eoption_2Eoption \\
& A_27a). (\forall V2y \in A_27b. (((ap\ (ap\ (c_2Eoption_2EOPTION_MAP \\
& A_27a\ A_27b)\ V0f)\ V1x) = (ap\ (c_2Eoption_2ESOME\ A_27b)\ V2y)) \Leftrightarrow (\exists V3z \in \\
& A_27a. ((V1x = (ap\ (c_2Eoption_2ESOME\ A_27a)\ V3z)) \wedge (V2y = (ap\ V0f \\
& V3z))))))
\end{aligned} \tag{58}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \forall V0f \in (A_27a^{A_27b}). (\forall V1x \in (ty_2Eoption_2Eoption \\
& A_27b). (((ap\ (ap\ (c_2Eoption_2EOPTION_MAP\ A_27b\ A_27a)\ V0f) \\
& V1x) = (c_2Eoption_2ENONE\ A_27a)) \Leftrightarrow (V1x = (c_2Eoption_2ENONE\ A_27b))) \wedge \\
& (((c_2Eoption_2ENONE\ A_27a) = (ap\ (ap\ (c_2Eoption_2EOPTION_MAP \\
& A_27b\ A_27a)\ V0f)\ V1x)) \Leftrightarrow (V1x = (c_2Eoption_2ENONE\ A_27b))))))
\end{aligned} \tag{59}$$

Theorem 1

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \quad (\forall V0l \in (ty_2Ellist_2Ellist\ A_27a).(\forall V1h \in A_27a. \\ & \quad (\forall V2t \in (ty_2Ellist_2Ellist\ A_27a).((p\ (ap\ (ap\ (c_2Ellist_2ELPREFIX \\ & \quad A_27a)\ V0l)\ (ap\ (ap\ (c_2Ellist_2ELCONS\ A_27a)\ V1h)\ V2t))) \Leftrightarrow ((V0l = \\ & \quad (c_2Ellist_2ELNIL\ A_27a)) \vee (\exists V3l \in (ty_2Ellist_2Ellist \\ & \quad A_27a).((V0l = (ap\ (ap\ (c_2Ellist_2ELCONS\ A_27a)\ V1h)\ V3l)) \wedge (\\ & \quad p\ (ap\ (ap\ (c_2Ellist_2ELPREFIX\ A_27a)\ V3l)\ V2t)))))))))) \wedge (\forall V4h \in \\ & \quad A_27b.(\forall V5t \in (ty_2Ellist_2Ellist\ A_27b).(\forall V6l \in \\ & \quad (ty_2Ellist_2Ellist\ A_27b).((p\ (ap\ (ap\ (c_2Ellist_2ELPREFIX \\ & \quad A_27b)\ (ap\ (ap\ (c_2Ellist_2ELCONS\ A_27b)\ V4h)\ V5t))\ V6l)) \Leftrightarrow (\exists V7l \in \\ & \quad (ty_2Ellist_2Ellist\ A_27b).((V6l = (ap\ (ap\ (c_2Ellist_2ELCONS \\ & \quad A_27b)\ V4h)\ V7l)) \wedge (p\ (ap\ (ap\ (c_2Ellist_2ELPREFIX\ A_27b)\ V5t)\ V7l)))))))))) \end{aligned}$$