

thm_2Elist_2ELTAKE__CONS__EQ__SOME (TMGoQjRTRyUfJMTms4yr33Ltfdy2L7xHfpy)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Elist_2Elist A0) \quad (1)$$

Let $c_2Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2ECONS A_27a \in (((ty_2Elist_2Elist A_27a)^{(ty_2Elist_2Elist A_27a)})^{A_27a}) \quad (2)$$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (3)$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty ty_2Enum_2Enum \quad (4)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (5)$$

Definition 3 We define c_2Enum_2E0 to be $(ap c_2Enum_2EABS_num c_2Enum_2EZERO_REP)$.

Definition 4 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (6)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (7)$$

Definition 5 We define $c_Ebool_2E_21$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap (ap (c_Emin_2E_3D (2^{A_27a})))$

Definition 6 We define c_Eenum_2ESUC to be $\lambda V0m \in ty_2Eenum_2Eenum. (ap c_Eenum_2EABS_num$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Eenum_2Eenum^{ty_2Eenum_2Eenum})^{ty_2Eenum_2Eenum}) \quad (8)$$

Definition 7 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Eenum_2Eenum. (ap (ap c_2Earithmetic$

Definition 8 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Eenum_2Eenum.V0x$.

Let $c_2Earithmetic_2E_2D : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2D \in ((ty_2Eenum_2Eenum^{ty_2Eenum_2Eenum})^{ty_2Eenum_2Eenum}) \quad (9)$$

Let $ty_2Eoption_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. nonempty A0 \Rightarrow nonempty (ty_2Eoption_2Eoption A0) \quad (10)$$

Let $ty_2Ellist_2Ellist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. nonempty A0 \Rightarrow nonempty (ty_2Ellist_2Ellist A0) \quad (11)$$

Let $c_2Ellist_2Ellist_rep : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow c_2Ellist_2Ellist_rep A_27a \in ((ty_2Eoption_2Eoption A_27a)^{ty_2Eenum_2Eenum})^{(ty_2Ellist_2Ellist A_27a)} \quad (12)$$

Let $ty_2Eone_2Eone : \iota$ be given. Assume the following.

$$nonempty ty_2Eone_2Eone \quad (13)$$

Definition 9 We define $c_Emin_2E_3D_3D_3E$ to be $\lambda P \in 2. \lambda Q \in 2. inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 10 We define $c_Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c_Ebool_2E_21 2) (\lambda V2t \in$

Let $ty_2Esum_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. nonempty A0 \Rightarrow \forall A1. nonempty A1 \Rightarrow nonempty (ty_2Esum_2Esum A0 A1) \quad (14)$$

Let $c_2Esum_2EABS_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow \forall A_27b. nonempty A_27b \Rightarrow c_2Esum_2EABS_sum A_27a A_27b \in ((ty_2Esum_2Esum A_27a A_27b)^{((2^{A_27b})^{A_27a})^2}) \quad (15)$$

Definition 11 We define c_Esum_2EINL to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0e \in A_27a. (ap (c_Esum_2EABS$
Let $c_Eoption_2Eoption_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow c_Eoption_2Eoption_ABS A_27a \in ((ty_2Eoption_2Eoption A_27a)^{(ty_2Esum_2Esum A_27a ty_2Eone_2Eone)}) \quad (16)$$

Definition 12 We define $c_Eoption_2ESOME$ to be $\lambda A_27a : \iota. \lambda V0x \in A_27a. (ap (c_Eoption_2Eoption_ABS$

Definition 13 We define c_Ebool_2EF to be $(ap (c_Ebool_2E21 2) (\lambda V0t \in 2.V0t))$.

Definition 14 We define c_Emin_2E40 to be $\lambda A. \lambda P \in 2^A. \mathbf{if} (\exists x \in A. P (ap P x)) \mathbf{then} (the (\lambda x. x \in A \wedge P x))$
of type $\iota \Rightarrow \iota$.

Definition 15 We define c_Ebool_2ECOND to be $\lambda A_27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A_27a. (\lambda V2t2 \in A_27a. (ap (c_Ebool_2E21 2) (ap P x))$

Let $c_Ellist_2Ellist_abs : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow c_Ellist_2Ellist_abs A_27a \in ((ty_2Ellist_2Ellist A_27a)^{(ty_2Eoption_2Eoption A_27a)^{ty_2Enum_2Enum}}) \quad (17)$$

Definition 16 We define $c_Ellist_2ELCONS$ to be $\lambda A_27a : \iota. \lambda V0h \in A_27a. \lambda V1t \in (ty_2Ellist_2Ellist A_27a)$

Definition 17 We define c_Eone_2Eone to be $(ap (c_Emin_2E40 ty_2Eone_2Eone) (\lambda V0x \in ty_2Eone_2Eone))$

Definition 18 We define c_Ebool_2E7E to be $(\lambda V0t \in 2. (ap (ap c_Emin_2E3D_3D_3E V0t) c_Ebool_2E21 2))$

Definition 19 We define c_Esum_2EINR to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0e \in A_27b. (ap (c_Esum_2EABS$

Definition 20 We define $c_Eoption_2ENONE$ to be $\lambda A_27a : \iota. (ap (c_Eoption_2Eoption_ABS A_27a) (ap P x))$

Definition 21 We define c_Ellist_2ELNIL to be $\lambda A_27a : \iota. (ap (c_Ellist_2Ellist_abs A_27a) (\lambda V0n \in ty_2Ellist_2Ellist A_27a))$

Let $c_Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow c_Elist_2ENIL A_27a \in (ty_2Elist_2Elist A_27a) \quad (18)$$

Let $c_Ellist_2ELTAKE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow c_Ellist_2ELTAKE A_27a \in (((ty_2Eoption_2Eoption A_27a)^{(ty_2Elist_2Elist A_27a)^{ty_2Enum_2Enum}})) \quad (19)$$

Definition 22 We define c_Ebool_2E3F to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap V0P (ap (c_Emin_2E40$

Let $c_Eoption_2EOPTION_MAP : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow \forall A_27b. nonempty A_27b \Rightarrow c_Eoption_2EOPTION_MAP A_27a A_27b \in (((ty_2Eoption_2Eoption A_27b)^{(ty_2Eoption_2Eoption A_27a)^{(A_27b^{A_27a}})}}) \quad (20)$$

Definition 23 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21) 2) (\lambda V2t \in$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.((V0m = c_2Enum_2E0) \vee (\exists V1n \in ty_2Enum_2Enum.(V0m = (ap c_2Enum_2ESUC V1n)))))) \quad (21)$$

Assume the following.

$$True \quad (22)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (23)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p V0t))) \quad (24)$$

Assume the following.

$$(\forall V0t \in 2.((p V0t) \vee (\neg(p V0t)))) \quad (25)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p V0t)) \Leftrightarrow (p V0t))) \quad (26)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0t \in 2.((\exists V1x \in A_27a.(p V0t)) \Leftrightarrow (p V0t))) \quad (27)$$

Assume the following.

$$(\forall V0t \in 2.(((p V0t) \Rightarrow False) \Rightarrow (\neg(p V0t)))) \quad (28)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(p V0t)) \Rightarrow ((p V0t) \Rightarrow False))) \quad (29)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \quad (30)$$

Assume the following.

$$(\forall V0t \in 2.(((True \vee (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \vee True) \Leftrightarrow True) \wedge (((False \vee (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee False) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee (p V0t)) \Leftrightarrow (p V0t)))))) \quad (31)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((p V0t) \Rightarrow False) \Leftrightarrow \neg(p V0t)))))) \quad (32)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \wedge ((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True)) \quad (33)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0x \in A.27a.((V0x = V0x) \Leftrightarrow True)) \quad (34)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0x \in A.27a.(\forall V1y \in A.27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (35)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow \neg(p V0t)) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow \neg(p V0t)))))) \quad (36)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).(\neg(\forall V1x \in A.27a.(p(ap V0P V1x)))) \Leftrightarrow (\exists V2x \in A.27a.(\neg(p(ap V0P V2x))))) \quad (37)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p V0A) \vee (p V1B) \vee (p V2C)) \Leftrightarrow (((p V0A) \vee (p V1B)) \vee (p V2C))))) \quad (38)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p V0A) \vee (p V1B)) \Leftrightarrow ((p V1B) \vee (p V0A)))) \quad (39)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \wedge (p V1B))) \Leftrightarrow ((\neg(p V0A) \vee \neg(p V1B)))) \wedge ((\neg((p V0A) \vee (p V1B))) \Leftrightarrow ((\neg(p V0A) \wedge \neg(p V1B))))) \quad (40)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).(\forall V1a \in A.27a.((\exists V2x \in A.27a.((V2x = V1a) \wedge (p(ap V0P V2x)))) \Leftrightarrow (p(ap V0P V1a)))) \quad (41)$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow \forall A.27c. \\
& \quad nonempty\ A.27c \Rightarrow ((\forall V0l \in (ty_2Ellist_2Ellist\ A.27a).((\\
ap\ (ap\ (c_2Ellist_2ELTAK E\ A.27a)\ c_2Enum_2E0)\ V0l) = (ap\ (c_2Eoption_2ESOME \\
& \quad (ty_2Elist_2Elist\ A.27a))\ (c_2Elist_2ENIL\ A.27a)))) \wedge ((\forall V1n \in \\
ty_2Enum_2Enum.((ap\ (ap\ (c_2Ellist_2ELTAK E\ A.27b)\ (ap\ c_2Enum_2ESUC \\
& \quad V1n))\ (c_2Ellist_2ELNIL\ A.27b)) = (c_2Eoption_2ENONE\ (ty_2Elist_2Elist \\
& \quad A.27b)))) \wedge (\forall V2n \in ty_2Enum_2Enum.(\forall V3h \in A.27c. \\
& \quad (\forall V4t \in (ty_2Ellist_2Ellist\ A.27c).((ap\ (ap\ (c_2Ellist_2ELTAK E \\
& \quad A.27c)\ (ap\ c_2Enum_2ESUC\ V2n))\ (ap\ (ap\ (c_2Ellist_2ELCONS\ A.27c) \\
& \quad V3h)\ V4t)) = (ap\ (ap\ (c_2Eoption_2EOPTION_MAP\ (ty_2Elist_2Elist \\
& \quad A.27c)\ (ty_2Elist_2Elist\ A.27c))\ (ap\ (c_2Elist_2ECONS\ A.27c) \\
& \quad V3h))\ (ap\ (ap\ (c_2Ellist_2ELTAK E\ A.27c)\ V2n)\ V4t))))))))) \\
& \hspace{15em} (42)
\end{aligned}$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum.(\neg((ap\ c_2Enum_2ESUC\ V0n) = c_2Enum_2E0))) \quad (43)$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0x \in A.27a.(\forall V1y \in \\
& \quad A.27a.(((ap\ (c_2Eoption_2ESOME\ A.27a)\ V0x) = (ap\ (c_2Eoption_2ESOME \\
& \quad A.27a)\ V1y)) \Leftrightarrow (V0x = V1y)))) \\
& \hspace{15em} (44)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\
& \quad \forall V0f \in (A.27b^{A.27a}).(\forall V1x \in (ty_2Eoption_2Eoption \\
& \quad A.27a).(\forall V2y \in A.27b.(((ap\ (ap\ (c_2Eoption_2EOPTION_MAP \\
& \quad A.27a\ A.27b)\ V0f)\ V1x) = (ap\ (c_2Eoption_2ESOME\ A.27b)\ V2y)) \Leftrightarrow (\exists V3z \in \\
& \quad A.27a.((V1x = (ap\ (c_2Eoption_2ESOME\ A.27a)\ V3z)) \wedge (V2y = (ap\ V0f \\
& \quad V3z))))))))) \\
& \hspace{15em} (45)
\end{aligned}$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.((ap\ c_2Enum_2ESUC\ V0m) = (ap\ c_2Enum_2ESUC\ V1n)) \Leftrightarrow (V0m = V1n))) \quad (46)$$

Assume the following.

$$(\forall V0t \in 2.(\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t)) \quad (47)$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A) \Rightarrow (\neg(p\ V0A)) \Rightarrow False)) \quad (48)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow ((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))) \quad (49)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg(\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow (p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (50)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \quad (51)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \Leftrightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee (\neg(p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \quad (52)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \wedge (p V2r)) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))) \quad (53)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \vee (p V2r)) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))) \quad (54)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \Rightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((\neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))) \quad (55)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \quad (56)$$

Theorem 1

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0m \in ty.2Enum.2Enum.(\forall V1h \in A.27a.(\forall V2t \in (ty.2Ellist.2Ellist A.27a).(\forall V3l \in (ty.2Ellist.2Ellist A.27a).(((ap (ap (c.2Ellist.2ELTAK E A.27a) V0m) (ap (ap (c.2Ellist.2ELCONS A.27a) V1h) V2t)) = (ap (c.2Eoption.2ESOME (ty.2Ellist.2Ellist A.27a) V3l)) \Leftrightarrow (((V0m = c.2Enum.2E0) \wedge (V3l = (c.2Ellist.2ENIL A.27a))) \vee (\exists V4n \in ty.2Enum.2Enum.(\exists V5l.27 \in (ty.2Ellist.2Ellist A.27a).((V0m = (ap c.2Enum.2ESUC V4n)) \wedge ((ap (ap (c.2Ellist.2ELTAK E A.27a) V4n) V2t) = (ap (c.2Eoption.2ESOME (ty.2Ellist.2Ellist A.27a) V5l.27)) \wedge (V3l = (ap (ap (c.2Ellist.2ECONS A.27a) V1h) V5l.27))))))))))$$