

thm_2Ellist_2ELTAKE__LAPPEND2 (TMSg- pQo92v62ZgwiDFyDLjfU4XKxEdEUaMN)

October 26, 2020

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{1}$$

Let $c_2Earithmetic_2E_2D : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2D \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \tag{2}$$

Definition 1 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.$ if $(\exists x \in A.p (ap\ P\ x))$ **then** (the $(\lambda x.x \in A \wedge p\ x)$ of type $\iota \Rightarrow \iota$).

Definition 2 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o\ (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define c_2Ebool_2ET to be $(ap\ (ap\ (c_2Emin_2E_3D\ (2^2)))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x)$

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Elist_2Elist\ A0) \tag{3}$$

Let $c_2Elist_2EAPPEND : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2EAPPEND\ A_27a \in (((ty_2Elist_2Elist\ A_27a)^{(ty_2Elist_2Elist\ A_27a)})^{(ty_2Elist_2Elist\ A_27a)}) \tag{4}$$

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ (ap\ (c_2Emin_2E_3D\ (2^{A_27a}))\ P))\ V0t))$

Definition 5 We define c_2Ebool_2EF to be $(ap\ (c_2Ebool_2E_21\ 2))\ (\lambda V0t \in 2.V0t)$.

Definition 6 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o\ (p\ P \Rightarrow p\ Q)$ of type ι .

Definition 7 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap\ (ap\ c_2Emin_2E_3D_3D_3E\ V0t)\ c_2Ebool_2EF))$

Definition 8 We define $c_Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_Ebool_2E_21 2) (\lambda V2t \in 2$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (5)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (6)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (7)$$

Definition 9 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap c_2Enum_2EABS_num$

Definition 10 We define $c_Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap V0P (ap (c_2Emin_2E_40$

Definition 11 We define $c_Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Let $ty_2Ellist_2Ellist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Ellist_2Ellist A0) \quad (8)$$

Let $c_2Ellist_2ELAPPEND : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Ellist_2ELAPPEND A_27a \in (((ty_2Ellist_2Ellist A_27a)^{(ty_2Ellist_2Ellist A_27a)})^{(ty_2Ellist_2Ellist A_27a)}) \quad (9)$$

Let $ty_2Eoption_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Eoption_2Eoption A0) \quad (10)$$

Let $c_2Ellist_2ELTAKE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Ellist_2ELTAKE A_27a \in (((ty_2Eoption_2Eoption (ty_2Elist_2Elist A_27a))^{(ty_2Ellist_2Ellist A_27a)})^{ty_2Enum_2Enum}) \quad (11)$$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (12)$$

Definition 12 We define c_2Enum_2E0 to be $(ap c_2Enum_2EABS_num c_2Enum_2EZERO_REP)$.

Let $ty_2Eone_2Eone : \iota$ be given. Assume the following.

$$nonempty ty_2Eone_2Eone \quad (13)$$

Definition 13 We define c_2Eone_2Eone to be $(ap (c_2Emin_2E_40 ty_2Eone_2Eone) (\lambda V0x \in ty_2Eone_2Eone$

Let $ty_2Esum_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Esum_2Esum\ A0\ A1) \quad (14)$$

Let $c_2Esum_2EABS_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Esum_2EABS_sum\ A_27a\ A_27b \in ((ty_2Esum_2Esum\ A_27a\ A_27b)^{((2^{A_27b})^{A_27a})^2}) \quad (15)$$

Definition 14 We define c_2Esum_2EINR to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0e \in A_27b. (ap\ (c_2Esum_2EABS_sum\ A_27a\ A_27b)\ V0e)$

Let $c_2Eoption_2Eoption_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Eoption_2Eoption_ABS\ A_27a \in ((ty_2Eoption_2Eoption\ A_27a)^{ty_2Esum_2Esum\ A_27a\ ty_2Eone_2Eone}) \quad (16)$$

Definition 15 We define $c_2Eoption_2ENONE$ to be $\lambda A_27a : \iota. (ap\ (c_2Eoption_2Eoption_ABS\ A_27a)\ V0e)$

Definition 16 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (17)$$

Definition 17 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum. (ap\ (ap\ c_2Earithmetic_2E_2B)\ V0n)$

Definition 18 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum. V0x$.

Let $c_2Ellist_2Ellist_rep : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ellist_2Ellist_rep\ A_27a \in (((ty_2Eoption_2Eoption\ A_27a)^{ty_2Enum_2Enum})^{(ty_2Ellist_2Ellist\ A_27a)}) \quad (18)$$

Definition 19 We define c_2Esum_2EINL to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0e \in A_27a. (ap\ (c_2Esum_2EABS_sum\ A_27a\ A_27b)\ V0e)$

Definition 20 We define $c_2Eoption_2ESOME$ to be $\lambda A_27a : \iota. \lambda V0x \in A_27a. (ap\ (c_2Eoption_2Eoption_ABS\ A_27a)\ V0x)$

Definition 21 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A_27a. (\lambda V2t2 \in A_27a. (ap\ (c_2Eoption_2Eoption_ABS\ A_27a)\ V2t2)\ V1t1)))$

Let $c_2Ellist_2Ellist_abs : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ellist_2Ellist_abs\ A_27a \in ((ty_2Ellist_2Ellist\ A_27a)^{(ty_2Eoption_2Eoption\ A_27a)^{ty_2Enum_2Enum}}) \quad (19)$$

Definition 22 We define $c_2Ellist_2ELCONS$ to be $\lambda A_27a : \iota. \lambda V0h \in A_27a. \lambda V1t \in (ty_2Ellist_2Ellist\ A_27a). (ap\ (c_2Ellist_2Ellist_abs\ A_27a)\ V1t)\ h$

Definition 23 We define $c_2Ellist_2ELNIL$ to be $\lambda A_27a : \iota. (ap\ (c_2Ellist_2Ellist_abs\ A_27a)\ V0e)$

Definition 24 We define $c_Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_Ebool_2E_21 2) (\lambda V2t \in$

Definition 25 We define $c_Ellist_2Ellength_rel$ to be $\lambda A_27a : \iota.(\lambda V0a0 \in (ty_2Ellist_2Ellist A_27a).(\lambda V$

Definition 26 We define $c_Ellist_2ELFINITE$ to be $\lambda A_27a : \iota.(\lambda V0a0 \in (ty_2Ellist_2Ellist A_27a).(ap (c$

Definition 27 We define $c_Ellist_2ELLENGTH$ to be $\lambda A_27a : \iota.\lambda V0ll \in (ty_2Ellist_2Ellist A_27a).(ap (a$

Let $c_2Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2ECONS A_27a \in (((ty_2Elist_2Elist A_27a)^{(ty_2Elist_2Elist A_27a)})^{A_27a}) \quad (20)$$

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2ENIL A_27a \in (ty_2Elist_2Elist A_27a) \quad (21)$$

Let $c_2Eoption_2ETHE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Eoption_2ETHE A_27a \in (A_27a)^{(ty_2Eoption_2Eoption A_27a)} \quad (22)$$

Definition 28 We define $c_Ellist_2EtoList$ to be $\lambda A_27a : \iota.\lambda V0ll \in (ty_2Ellist_2Ellist A_27a).(ap (ap (ap$

Definition 29 We define $c_Ecombin_2Eo$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in (A_27b^{A_27c}).\lambda V1g$

Let $c_2Eoption_2EOPTION_MAP : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Eoption_2EOPTION_MAP A_27a A_27b \in (((ty_2Eoption_2Eoption A_27b)^{(ty_2Eoption_2Eoption A_27a)})^{(A_27b^{A_27a})}) \quad (23)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.((V0m = c_2Enum_2E0) \vee (\exists V1n \in ty_2Enum_2Enum.(V0m = (ap c_2Enum_2ESUC V1n)))))) \quad (24)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.(((ap (ap c_2Earithmetic_2E_2D c_2Enum_2E0) V0m) = c_2Enum_2E0) \wedge ((ap (ap c_2Earithmetic_2E_2D V0m) c_2Enum_2E0) = V0m))) \quad (25)$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum.(\forall V1m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2D (ap c_2Enum_2ESUC V0n)) (ap c_2Enum_2ESUC V1m)) = (ap (ap c_2Earithmetic_2E_2D V0n) V1m)))) \quad (26)$$

Assume the following.

$$True \quad (27)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (28)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p V0t))) \quad (29)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p V0t)) \Leftrightarrow (p V0t))) \quad (30)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \quad (31)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow False) \Leftrightarrow (\neg (p V0t)))))) \quad (32)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True)))) \quad (33)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (34)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (35)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow (\forall V0f \in (A_27b^{A_27a}).(\forall V1g \in (A_27b^{A_27a}).((V0f = V1g) \Leftrightarrow (\forall V2x \in A_27a.((ap V0f V2x) = (ap V1g V2x)))))) \quad (36)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow \neg(p V0t)) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow \neg(p V0t)))))) \quad (37)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}).(\forall V1Q \in 2.(((\forall V2x \in A_27a.(p\ (ap\ V0P\ V2x))) \wedge (p\ V1Q)) \Leftrightarrow (\forall V3x \in A_27a.((p\ (ap\ V0P\ V3x)) \wedge (p\ V1Q)))))) \quad (38)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (2^{A_27a}).((\forall V2x \in A_27a.((p\ V0P) \vee (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow ((p\ V0P) \vee (\forall V3x \in A_27a.(p\ (ap\ V1Q\ V3x)))))) \quad (39)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \quad (40)$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x_27 \in 2.(\forall V2y \in 2.(\forall V3y_27 \in 2.(((p\ V0x) \Leftrightarrow (p\ V1x_27)) \wedge ((p\ V1x_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y_27)))))) \Rightarrow (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x_27) \Rightarrow (p\ V3y_27)))))) \quad (41)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0a \in A_27a.(\exists V1x \in A_27a.(V1x = V0a))) \quad (42)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c.nonempty\ A_27c \Rightarrow (\forall V0f \in (A_27b^{A_27a}).(\forall V1g \in (A_27a^{A_27c}).(\forall V2x \in A_27c.((ap\ (ap\ (ap\ (c_2Ecombin_2Eo\ A_27c\ A_27b\ A_27a)\ V0f)\ V1g)\ V2x) = (ap\ V0f\ (ap\ V1g\ V2x)))))) \quad (43)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow ((\forall V0l \in (ty_2Elist_2Elist\ A_27a).((ap\ (ap\ (c_2Elist_2EAPPEND\ A_27a)\ (c_2Elist_2ENIL\ A_27a))\ V0l) = V0l)) \wedge (\forall V1l1 \in (ty_2Elist_2Elist\ A_27a).(\forall V2l2 \in (ty_2Elist_2Elist\ A_27a).(\forall V3h \in A_27a.((ap\ (ap\ (c_2Elist_2EAPPEND\ A_27a)\ (ap\ (ap\ (c_2Elist_2ECONS\ A_27a)\ V3h)\ V1l1))\ V2l2) = (ap\ (ap\ (c_2Elist_2ECONS\ A_27a)\ V3h)\ (ap\ (ap\ (c_2Elist_2EAPPEND\ A_27a)\ V1l1)\ V2l2)))))) \quad (44)$$

Assume the following.

$$\begin{aligned} \forall A_27a. \text{nonempty } A_27a \Rightarrow (\forall V0a0 \in A_27a. (\forall V1a1 \in \\ (ty_2Elist_2Elist\ A_27a). (\forall V2a0_27 \in A_27a. (\forall V3a1_27 \in \\ (ty_2Elist_2Elist\ A_27a). (((ap\ (ap\ (c_2Elist_2ECONS\ A_27a)\ V0a0) \\ V1a1) = (ap\ (ap\ (c_2Elist_2ECONS\ A_27a)\ V2a0_27)\ V3a1_27))) \Leftrightarrow ((V0a0 = \\ V2a0_27) \wedge (V1a1 = V3a1_27)))))) \end{aligned} \quad (45)$$

Assume the following.

$$\begin{aligned} \forall A_27a. \text{nonempty } A_27a \Rightarrow (\forall V0l1 \in (ty_2Elist_2Elist \\ A_27a). (\forall V1l2 \in (ty_2Elist_2Elist\ A_27a). (\forall V2l3 \in \\ (ty_2Elist_2Elist\ A_27a). (((ap\ (ap\ (c_2Elist_2EAPPEND\ A_27a) \\ V0l1)\ (ap\ (ap\ (c_2Elist_2EAPPEND\ A_27a)\ V1l2)\ V2l3)) = (ap\ (ap\ (c_2Elist_2EAPPEND \\ A_27a)\ (ap\ (ap\ (c_2Elist_2EAPPEND\ A_27a)\ V0l1)\ V1l2))\ V2l3)))))) \end{aligned} \quad (46)$$

Assume the following.

$$\begin{aligned} \forall A_27a. \text{nonempty } A_27a \Rightarrow ((\forall V0l1 \in (ty_2Elist_2Elist \\ A_27a). (\forall V1l2 \in (ty_2Elist_2Elist\ A_27a). (\forall V2l3 \in \\ (ty_2Elist_2Elist\ A_27a). (((ap\ (ap\ (c_2Elist_2EAPPEND\ A_27a) \\ V0l1)\ V1l2) = (ap\ (ap\ (c_2Elist_2EAPPEND\ A_27a)\ V0l1)\ V2l3))) \Leftrightarrow (V1l2 = \\ V2l3)))))) \wedge (\forall V3l1 \in (ty_2Elist_2Elist\ A_27a). (\forall V4l2 \in \\ (ty_2Elist_2Elist\ A_27a). (\forall V5l3 \in (ty_2Elist_2Elist\ A_27a). \\ (((ap\ (ap\ (c_2Elist_2EAPPEND\ A_27a)\ V4l2)\ V3l1) = (ap\ (ap\ (c_2Elist_2EAPPEND \\ A_27a)\ V5l3)\ V3l1))) \Leftrightarrow (V4l2 = V5l3)))))) \end{aligned} \quad (47)$$

Assume the following.

$$\begin{aligned} \forall A_27a. \text{nonempty } A_27a \Rightarrow \forall A_27b. \text{nonempty } A_27b \Rightarrow \forall A_27c. \\ \text{nonempty } A_27c \Rightarrow ((\forall V0l \in (ty_2Elist_2Elist\ A_27a). ((\\ ap\ (ap\ (c_2Elist_2ELTAKE\ A_27a)\ c_2Enum_2E0)\ V0l) = (ap\ (c_2Eoption_2ESOME \\ (ty_2Elist_2Elist\ A_27a))\ (c_2Elist_2ENIL\ A_27a)))) \wedge ((\forall V1n \in \\ ty_2Enum_2Enum. ((ap\ (ap\ (c_2Elist_2ELTAKE\ A_27b)\ (ap\ c_2Enum_2ESUC \\ V1n))\ (c_2Elist_2ELNIL\ A_27b)) = (c_2Eoption_2ENONE\ (ty_2Elist_2Elist \\ A_27b)))) \wedge (\forall V2n \in ty_2Enum_2Enum. (\forall V3h \in A_27c. \\ (\forall V4t \in (ty_2Elist_2Elist\ A_27c). ((ap\ (ap\ (c_2Elist_2ELTAKE \\ A_27c)\ (ap\ c_2Enum_2ESUC\ V2n))\ (ap\ (ap\ (c_2Elist_2ELCONS\ A_27c) \\ V3h)\ V4t)) = (ap\ (ap\ (c_2Eoption_2EOPTION_MAP\ (ty_2Elist_2Elist \\ A_27c)\ (ty_2Elist_2Elist\ A_27c))\ (ap\ (c_2Elist_2ECONS\ A_27c) \\ V3h))\ (ap\ (ap\ (c_2Elist_2ELTAKE\ A_27c)\ V2n)\ V4t)))))) \end{aligned} \quad (48)$$

Assume the following.

$$\begin{aligned} \forall A_27a. \text{nonempty } A_27a \Rightarrow (\forall V0m \in ty_2Enum_2Enum. (\\ ((ap\ (ap\ (c_2Elist_2ELTAKE\ A_27a)\ V0m)\ (c_2Elist_2ELNIL\ A_27a)) = \\ (c_2Eoption_2ENONE\ (ty_2Elist_2Elist\ A_27a))) \Leftrightarrow (p\ (ap\ (ap\ c_2Eprim_rec_2E_3C \\ c_2Enum_2E0)\ V0m)))) \end{aligned} \quad (49)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow & ((\forall V0x \in (ty_2Ellist_2Ellist \\ & A_27a).((ap\ (ap\ (c_2Ellist_2ELAPPEND\ A_27a)\ (c_2Ellist_2ELNIL \\ & A_27a))\ V0x) = V0x)) \wedge (\forall V1h \in A_27a.(\forall V2t \in (ty_2Ellist_2Ellist \\ & A_27a).(\forall V3x \in (ty_2Ellist_2Ellist\ A_27a).((ap\ (ap\ (c_2Ellist_2ELAPPEND \\ & A_27a)\ (ap\ (ap\ (c_2Ellist_2ELCONS\ A_27a)\ V1h)\ V2t))\ V3x) = (ap\ (ap \\ & (c_2Ellist_2ELCONS\ A_27a)\ V1h)\ (ap\ (ap\ (c_2Ellist_2ELAPPEND\ A_27a) \\ & V2t)\ V3x))))))))) \end{aligned} \quad (50)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow & (\forall V0ll \in (ty_2Ellist_2Ellist \\ & A_27a).((p\ (ap\ (c_2Ellist_2ELFINITE\ A_27a)\ V0ll)) \Leftrightarrow (\exists V1n \in \\ & ty_2Enum_2Enum.((ap\ (ap\ (c_2Ellist_2ELTAKE\ A_27a)\ V1n)\ V0ll) = \\ & (c_2Eoption_2ENONE\ (ty_2Ellist_2Ellist\ A_27a)))))) \end{aligned} \quad (51)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow & \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & ((ap\ (c_2Ellist_2ELLENGTH\ A_27a)\ (c_2Ellist_2ELNIL\ A_27a)) = \\ & (ap\ (c_2Eoption_2ESOME\ ty_2Enum_2Enum)\ c_2Enum_2E0)) \wedge (\forall V0h \in \\ & A_27b.(\forall V1t \in (ty_2Ellist_2Ellist\ A_27b).((ap\ (c_2Ellist_2ELLENGTH \\ & A_27b)\ (ap\ (ap\ (c_2Ellist_2ELCONS\ A_27b)\ V0h)\ V1t)) = (ap\ (ap\ (c_2Eoption_2EOPTION_MAP \\ & ty_2Enum_2Enum\ ty_2Enum_2Enum)\ c_2Enum_2ESUC)\ (ap\ (c_2Ellist_2ELLENGTH \\ & A_27b)\ V1t)))))) \end{aligned} \quad (52)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow & (\forall V0ll \in (ty_2Ellist_2Ellist \\ & A_27a).((p\ (ap\ (c_2Ellist_2ELFINITE\ A_27a)\ V0ll)) \Rightarrow (\exists V1n \in \\ & ty_2Enum_2Enum.((ap\ (c_2Ellist_2ELLENGTH\ A_27a)\ V0ll) = (ap\ (\\ & c_2Eoption_2ESOME\ ty_2Enum_2Enum)\ V1n)))))) \end{aligned} \quad (53)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow & (\forall V0P \in (2^{(ty_2Ellist_2Ellist\ A_27a)}). \\ & (((p\ (ap\ V0P\ (c_2Ellist_2ELNIL\ A_27a))) \wedge (\forall V1h \in A_27a.(\\ & \forall V2t \in (ty_2Ellist_2Ellist\ A_27a).((p\ (ap\ V0P\ V2t)) \Rightarrow (p\ (\\ & ap\ V0P\ (ap\ (ap\ (c_2Ellist_2ELCONS\ A_27a)\ V1h)\ V2t)))))) \Rightarrow (\forall V3a0 \in \\ & (ty_2Ellist_2Ellist\ A_27a).((p\ (ap\ (c_2Ellist_2ELFINITE\ A_27a) \\ & V3a0)) \Rightarrow (p\ (ap\ V0P\ V3a0)))))) \end{aligned} \quad (54)$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& ((ap\ (c_2Ellist_2EtoList\ A_27a)\ (c_2Ellist_2ELNIL\ A_27a)) = (\\
& ap\ (c_2Eoption_2ESOME\ (ty_2Elist_2Elist\ A_27a))\ (c_2Elist_2ENIL \\
& A_27a))) \wedge (\forall V0h \in A_27b. (\forall V1t \in (ty_2Ellist_2Ellist \\
& A_27b). ((ap\ (c_2Ellist_2EtoList\ A_27b)\ (ap\ (ap\ (c_2Ellist_2ELCONS \\
& A_27b)\ V0h)\ V1t)) = (ap\ (ap\ (c_2Eoption_2EOPTION_MAP\ (ty_2Elist_2Elist \\
& A_27b)\ (ty_2Elist_2Elist\ A_27b))\ (ap\ (c_2Elist_2ECONS\ A_27b) \\
& V0h))\ (ap\ (c_2Ellist_2EtoList\ A_27b)\ V1t))))))
\end{aligned} \tag{55}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0ll \in (ty_2Ellist_2Ellist \\
& A_27a). ((p\ (ap\ (c_2Ellist_2ELFINITE\ A_27a)\ V0ll)) \Rightarrow (\exists V1l \in \\
& (ty_2Elist_2Elist\ A_27a). ((ap\ (c_2Ellist_2EtoList\ A_27a)\ V0ll) = \\
& (ap\ (c_2Eoption_2ESOME\ (ty_2Elist_2Elist\ A_27a)\ V1l))))))
\end{aligned} \tag{56}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0opt \in (ty_2Eoption_2Eoption \\
& A_27a). ((V0opt = (c_2Eoption_2ENONE\ A_27a)) \vee (\exists V1x \in A_27a. \\
& (V0opt = (ap\ (c_2Eoption_2ESOME\ A_27a)\ V1x))))))
\end{aligned} \tag{57}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in \\
& A_27a. (((ap\ (c_2Eoption_2ESOME\ A_27a)\ V0x) = (ap\ (c_2Eoption_2ESOME \\
& A_27a)\ V1y)) \Leftrightarrow (V0x = V1y))))))
\end{aligned} \tag{58}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\neg((c_2Eoption_2ENONE \\
& A_27a) = (ap\ (c_2Eoption_2ESOME\ A_27a)\ V0x))))))
\end{aligned} \tag{59}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& (\forall V0f \in (A_27b^{A_27a}). (\forall V1x \in A_27a. ((ap\ (ap\ (c_2Eoption_2EOPTION_MAP \\
& A_27a\ A_27b)\ V0f)\ (ap\ (c_2Eoption_2ESOME\ A_27a)\ V1x)) = (ap\ (c_2Eoption_2ESOME \\
& A_27b)\ (ap\ V0f\ V1x)))))) \wedge (\forall V2f \in (A_27b^{A_27a}). ((ap\ (ap\ (c_2Eoption_2EOPTION_MAP \\
& A_27a\ A_27b)\ V2f)\ (c_2Eoption_2ENONE\ A_27a)) = (c_2Eoption_2ENONE \\
& A_27b))))))
\end{aligned} \tag{60}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. ((ap\ (c_2Eoption_2ETHE \\
& A_27a)\ (ap\ (c_2Eoption_2ESOME\ A_27a)\ V0x)) = V0x))
\end{aligned} \tag{61}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0f \in (A_27b^{A_27a}). (\forall V1x \in (ty_2Eoption_2Eoption \\
& \quad A_27a). (\forall V2y \in A_27b. (((ap\ (ap\ (c_2Eoption_2EOPTION_MAP \\
& A_27a\ A_27b)\ V0f)\ V1x) = (ap\ (c_2Eoption_2ESOME\ A_27b)\ V2y)) \Leftrightarrow (\exists V3z \in \\
& A_27a. ((V1x = (ap\ (c_2Eoption_2ESOME\ A_27a)\ V3z)) \wedge (V2y = (ap\ V0f \\
& \quad V3z))))))))) \\
& \hspace{15em} (62)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0f \in (A_27a^{A_27b}). (\forall V1x \in (ty_2Eoption_2Eoption \\
& \quad A_27b). (((ap\ (ap\ (c_2Eoption_2EOPTION_MAP\ A_27b\ A_27a)\ V0f) \\
& V1x) = (c_2Eoption_2ENONE\ A_27a)) \Leftrightarrow (V1x = (c_2Eoption_2ENONE\ A_27b))) \wedge \\
& \quad (((c_2Eoption_2ENONE\ A_27a) = (ap\ (ap\ (c_2Eoption_2EOPTION_MAP \\
& \quad A_27b\ A_27a)\ V0f)\ V1x)) \Leftrightarrow (V1x = (c_2Eoption_2ENONE\ A_27b)))))) \\
& \hspace{15em} (63)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\
& \quad nonempty\ A_27c \Rightarrow (\forall V0f \in (A_27b^{A_27c}). (\forall V1g \in (A_27c^{A_27a}). \\
& (\forall V2x \in (ty_2Eoption_2Eoption\ A_27a). ((ap\ (ap\ (c_2Eoption_2EOPTION_MAP \\
& A_27c\ A_27b)\ V0f)\ (ap\ (ap\ (c_2Eoption_2EOPTION_MAP\ A_27a\ A_27c) \\
& V1g)\ V2x)) = (ap\ (ap\ (c_2Eoption_2EOPTION_MAP\ A_27a\ A_27b)\ (ap \\
& \quad (ap\ (c_2Ecombin_2Eo\ A_27a\ A_27b\ A_27c)\ V0f)\ V1g))\ V2x)))))) \\
& \hspace{15em} (64)
\end{aligned}$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \hspace{10em} (65)$$

Assume the following.

$$(\forall V0A \in 2. ((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \hspace{10em} (66)$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\
& \quad (((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))))) \\
& \hspace{15em} (67)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(p\ V0A)) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\
& \quad ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))))) \\
& \hspace{15em} (68)
\end{aligned}$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p\ V0A)) \Rightarrow False) \Rightarrow (((p\ V0A) \Rightarrow False) \Rightarrow False))) \hspace{10em} (69)$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg(\\
& p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee \\
& ((\neg(p V1q)) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{70}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee \\
& (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))))
\end{aligned} \tag{71}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge \\
& ((p V1q) \vee ((p V2r) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{72}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee \\
& (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p))))))
\end{aligned} \tag{73}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p))) \tag{74}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))) \tag{75}$$

Theorem 1

$$\begin{aligned}
& \forall A_27a.nonempty A_27a \Rightarrow (\forall V0n \in ty_2Enum_2Enum. (\\
& \forall V1l1 \in (ty_2Ellist_2Ellist A_27a). (\forall V2l2 \in (ty_2Ellist_2Ellist \\
& A_27a). (((ap (ap (c_2Ellist_2ELTAK E A_27a) V0n) V1l1) = (c_2Eoption_2ENONE \\
& (ty_2Ellist_2Ellist A_27a))) \Rightarrow ((ap (ap (c_2Ellist_2ELTAK E A_27a) \\
& V0n) (ap (ap (c_2Ellist_2ELAPPEND A_27a) V1l1) V2l2)) = (ap (ap (\\
& c_2Eoption_2EOPTION_MAP (ty_2Ellist_2Ellist A_27a) (ty_2Ellist_2Ellist \\
& A_27a)) (ap (c_2Ellist_2EAPPEND A_27a) (ap (c_2Eoption_2ETHE (\\
& ty_2Ellist_2Ellist A_27a)) (ap (c_2Ellist_2EtoList A_27a) V1l1)))) \\
& (ap (ap (c_2Ellist_2ELTAK E A_27a) (ap (ap c_2Earithmetic_2E_2D \\
& V0n) (ap (c_2Eoption_2ETHE ty_2Enum_2Enum) (ap (c_2Ellist_2ELLENGTH \\
& A_27a) V1l1)))))) V2l2))))))
\end{aligned}$$