

thm\_2Ellist\_2ELTAKE\_\_LENGTH\_\_NONE  
(TMUYoD-  
KAwuBvVq1Rdz7WdEpSWu3DQxeKai6)

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**Definition 1** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.$ if  $(\exists x \in A.p (ap P x))$  then (the  $(\lambda x.x \in A \wedge p x)$  of type  $\iota \Rightarrow \iota$ ).

**Definition 2** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 3** We define  $c\_2Ebool\_2ET$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2))) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x)$

Let  $ty\_2Eone\_2Eone : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Eone\_2Eone \tag{1}$$

**Definition 4** We define  $c\_2Eone\_2Eone$  to be  $(ap (c\_2Emin\_2E\_40 ty\_2Eone\_2Eone) (\lambda V0x \in ty\_2Eone\_2Eone))$

**Definition 5** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A.\lambda 27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A-27a})) (2^{A-27a}))))$

**Definition 6** We define  $c\_2Ebool\_2EF$  to be  $(ap (c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 7** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 8** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2EF))$

**Definition 9** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V2t))))$

Let  $ty\_2Esum\_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Esum\_2Esum\ A0\ A1) \tag{2}$$

Let  $c\_2Esum\_2EABS\_sum : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Esum\_2EABS\_sum\ A\_27a\ A\_27b \in ((ty\_2Esum\_2Esum\ A\_27a\ A\_27b)^{((2^{A-27b})^{A-27a})^2}) \tag{3}$$

**Definition 10** We define  $c\_Esum\_2EINR$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0e \in A\_27b. (ap (c\_Esum\_2EABS$   
Let  $ty\_2Eoption\_2Eoption : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0. nonempty A0 \Rightarrow nonempty (ty\_2Eoption\_2Eoption A0) \quad (4)$$

Let  $c\_2Eoption\_2Eoption\_ABS : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a. nonempty A\_27a \Rightarrow c\_2Eoption\_2Eoption\_ABS A\_27a \in ((ty\_2Eoption\_2Eoption A\_27a)^{(ty\_2Esum\_2Esum A\_27a ty\_2Eone\_2Eone)}) \quad (5)$$

**Definition 11** We define  $c\_2Eoption\_2ENONE$  to be  $\lambda A\_27a : \iota. (ap (c\_2Eoption\_2Eoption\_ABS A\_27a) (c$   
Let  $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0. nonempty A0 \Rightarrow nonempty (ty\_2Elist\_2Elist A0) \quad (6)$$

Let  $ty\_2Ellist\_2Ellist : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0. nonempty A0 \Rightarrow nonempty (ty\_2Ellist\_2Ellist A0) \quad (7)$$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty ty\_2Enum\_2Enum \quad (8)$$

Let  $c\_2Ellist\_2ELTAKE : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a. nonempty A\_27a \Rightarrow c\_2Ellist\_2ELTAKE A\_27a \in (((ty\_2Eoption\_2Eoption (ty\_2Elist\_2Elist A\_27a))^{(ty\_2Ellist\_2Ellist A\_27a)})^{ty\_2Enum\_2Enum}) \quad (9)$$

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \quad (10)$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \quad (11)$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \quad (12)$$

**Definition 12** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. (ap c\_2Enum\_2EABS\_num$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \quad (13)$$

**Definition 13** We define  $c\_2Enum\_2E0$  to be  $(ap c\_2Enum\_2EABS\_num c\_2Enum\_2EZERO\_REP)$ .

**Definition 14** We define  $c\_2Earithmic\_2EZERO$  to be  $c\_2Enum\_2E0$ .

Let  $c\_2Earithmetic\_2E\_2B : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (14)$$

**Definition 15** We define  $c\_2Earithmetic\_2EBIT1$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap (ap c\_2Earithmetic\_2E\_2B))$

**Definition 16** We define  $c\_2Earithmetic\_2ENUMERAL$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.V0x$ .

Let  $c\_2Earithmetic\_2E\_2D : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (15)$$

Let  $c\_2Ellist\_2Ellist\_rep : \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty A\_27a \Rightarrow c\_2Ellist\_2Ellist\_rep A\_27a \in \\ & (((ty\_2Eoption\_2Eoption A\_27a)^{ty\_2Enum\_2Enum})^{(ty\_2Ellist\_2Ellist A\_27a)}) \end{aligned} \quad (16)$$

**Definition 17** We define  $c\_2Esum\_2EINL$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0e \in A\_27a.(ap (c\_2Esum\_2EABS))$

**Definition 18** We define  $c\_2Eoption\_2ESOME$  to be  $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.(ap (c\_2Eoption\_2Eoption\_2EABS))$

**Definition 19** We define  $c\_2Ebool\_2ECOND$  to be  $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.))$

Let  $c\_2Ellist\_2Ellist\_abs : \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty A\_27a \Rightarrow c\_2Ellist\_2Ellist\_abs A\_27a \in \\ & ((ty\_2Ellist\_2Ellist A\_27a)^{(ty\_2Eoption\_2Eoption A\_27a)^{ty\_2Enum\_2Enum}}) \end{aligned} \quad (17)$$

**Definition 20** We define  $c\_2Ellist\_2ELCONS$  to be  $\lambda A\_27a : \iota.\lambda V0h \in A\_27a.\lambda V1t \in (ty\_2Ellist\_2Ellist A\_27a)$

**Definition 21** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap V0P (ap (c\_2Emin\_2E\_40))$

**Definition 22** We define  $c\_2Ellist\_2ELNIL$  to be  $\lambda A\_27a : \iota.(ap (c\_2Ellist\_2Ellist\_abs A\_27a)) (\lambda V0n \in ty\_2Enum\_2Enum)$

**Definition 23** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21) 2)) (\lambda V2t \in 2.))$

**Definition 24** We define  $c\_2Ellist\_2ELlength\_rel$  to be  $\lambda A\_27a : \iota.(\lambda V0a0 \in (ty\_2Ellist\_2Ellist A\_27a).(\lambda V1a1 \in (ty\_2Ellist\_2Ellist A\_27a).))$

**Definition 25** We define  $c\_2Ellist\_2ELFINITE$  to be  $\lambda A\_27a : \iota.(\lambda V0a0 \in (ty\_2Ellist\_2Ellist A\_27a).(ap (c\_2Ellist\_2Ellist\_abs A\_27a))$

**Definition 26** We define  $c\_2Ellist\_2ELLENGTH$  to be  $\lambda A\_27a : \iota.\lambda V0ll \in (ty\_2Ellist\_2Ellist A\_27a).(ap (ap (c\_2Ellist\_2Ellist\_abs A\_27a))$

Let  $c\_2Eoption\_2EOPTION\_MAP : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Eoption\_2EOPTION\_MAP \\ & A\_27a A\_27b \in (((ty\_2Eoption\_2Eoption A\_27b)^{(ty\_2Eoption\_2Eoption A\_27a)})^{(A\_27b^{A\_27a})}) \end{aligned} \quad (18)$$

**Definition 27** We define  $c\_2Eprim\_rec\_2E\_3C$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.((V0m = c\_2Enum\_2E0) \vee (\exists V1n \in ty\_2Enum\_2Enum.(V0m = (ap\ c\_2Enum\_2ESUC\ V1n)))))) \quad (19)$$

Assume the following.

$$(p\ (ap\ (ap\ c\_2Eprim\_rec\_2E\_3C\ (ap\ c\_2Enum\_2ESUC\ V0m))\ (ap\ c\_2Enum\_2ESUC\ V1n))) \Leftrightarrow (p\ (ap\ (ap\ c\_2Eprim\_rec\_2E\_3C\ V0m)\ V1n))) \quad (20)$$

Assume the following.

$$True \quad (21)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (22)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \quad (23)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg(p\ V0t)))))) \quad (24)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (25)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(V0x = V0x)) \quad (26)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \quad (27)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (28)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg(p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p\ V0t)))))) \quad (29)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).(\forall V1Q \in \\ & 2.(((\forall V2x \in A.27a.(p\ (ap\ V0P\ V2x))) \wedge (p\ V1Q))) \Leftrightarrow (\forall V3x \in \\ & A.27a.((p\ (ap\ V0P\ V3x)) \wedge (p\ V1Q)))))) \end{aligned} \quad (30)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0Q \in 2.(\forall V1P \in ( \\ & 2^{A.27a}).((\forall V2x \in A.27a.((p\ (ap\ V1P\ V2x)) \vee (p\ V0Q))) \Leftrightarrow ((\forall V3x \in \\ & A.27a.(p\ (ap\ V1P\ V3x)) \vee (p\ V0Q)))))) \end{aligned} \quad (31)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in ( \\ & 2^{A.27a}).((\forall V2x \in A.27a.((p\ V0P) \vee (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow ((p \\ & V0P) \vee (\forall V3x \in A.27a.(p\ (ap\ V1Q\ V3x)))))) \end{aligned} \quad (32)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2.(\forall V1B \in 2.(((p\ V0A) \Rightarrow (p\ V1B)) \Leftrightarrow ((\neg(p\ V0A)) \vee \\ & (p\ V1B)))))) \end{aligned} \quad (33)$$

Assume the following.

$$\begin{aligned} & (\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \Rightarrow \\ & ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \end{aligned} \quad (34)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in 2.(\forall V1x.27 \in 2.(\forall V2y \in 2.(\forall V3y.27 \in \\ & 2.((((p\ V0x) \Leftrightarrow (p\ V1x.27)) \wedge ((p\ V1x.27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y.27)))) \Rightarrow \\ & (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x.27) \Rightarrow (p\ V3y.27)))))) \end{aligned} \quad (35)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).(\forall V1a \in \\ & A.27a.((\exists V2x \in A.27a.((V2x = V1a) \wedge (p\ (ap\ V0P\ V2x)))) \Leftrightarrow (p\ ( \\ & ap\ V0P\ V1a)))))) \end{aligned} \quad (36)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0f \in (2^{A.27a}).(\forall V1v \in \\ & A.27a.((\forall V2x \in A.27a.((V2x = V1v) \Rightarrow (p\ (ap\ V0f\ V2x)))) \Leftrightarrow (p\ ( \\ & ap\ V0f\ V1v)))))) \end{aligned} \quad (37)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0m \in ty.2Enum.2Enum.( \\ & ((ap\ (ap\ (c.2Ellist.2ELTAKE\ A.27a)\ V0m)\ (c.2Ellist.2ELNIL\ A.27a)) = \\ & (c.2Eoption.2ENONE\ (ty.2Elist.2Elist\ A.27a))) \Leftrightarrow (p\ (ap\ (ap\ c.2Eprim\_rec.2E\_3C \\ & c.2Enum.2E0)\ V0m)))) \end{aligned} \quad (38)$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.\text{nonempty } A\_27a \Rightarrow (\forall V0m \in \text{ty\_2Enum\_2Enum}.) \\
& \quad \forall V1h \in A\_27a. (\forall V2t \in (\text{ty\_2Ellist\_2Ellist } A\_27a). \\
& \quad ((\text{ap } (\text{ap } (\text{c\_2Ellist\_2ELTAK}E \ A\_27a) \ V0m) \ (\text{ap } (\text{ap } (\text{c\_2Ellist\_2ELCONS} \\
& \quad A\_27a) \ V1h) \ V2t)) = (\text{c\_2Eoption\_2ENONE } (\text{ty\_2Ellist\_2Ellist } A\_27a))) \Leftrightarrow \\
& \quad (\exists V3n \in \text{ty\_2Enum\_2Enum}.) ((V0m = (\text{ap } \text{c\_2Enum\_2ESUC } \ V3n)) \wedge \\
& \quad ((\text{ap } (\text{ap } (\text{c\_2Ellist\_2ELTAK}E \ A\_27a) \ V3n) \ V2t) = (\text{c\_2Eoption\_2ENONE} \\
& \quad (\text{ty\_2Ellist\_2Ellist } A\_27a))))))
\end{aligned} \tag{39}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.\text{nonempty } A\_27a \Rightarrow \forall A\_27b.\text{nonempty } A\_27b \Rightarrow ( \\
& \quad ((\text{ap } (\text{c\_2Ellist\_2ELLENGTH } \ A\_27a) \ (\text{c\_2Ellist\_2ELNIL } \ A\_27a)) = \\
& \quad (\text{ap } (\text{c\_2Eoption\_2ESOME } \ \text{ty\_2Enum\_2Enum}) \ \text{c\_2Enum\_2E0})) \wedge (\forall V0h \in \\
& \quad A\_27b. (\forall V1t \in (\text{ty\_2Ellist\_2Ellist } \ A\_27b). ((\text{ap } (\text{c\_2Ellist\_2ELLENGTH} \\
& \quad A\_27b) \ (\text{ap } (\text{ap } (\text{c\_2Ellist\_2ELCONS } \ A\_27b) \ V0h) \ V1t)) = (\text{ap } (\text{ap } (\text{c\_2Eoption\_2EOPTION\_MAP} \\
& \quad \text{ty\_2Enum\_2Enum } \ \text{ty\_2Enum\_2Enum}) \ \text{c\_2Enum\_2ESUC}) \ (\text{ap } (\text{c\_2Ellist\_2ELLENGTH} \\
& \quad A\_27b) \ V1t))))))
\end{aligned} \tag{40}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.\text{nonempty } A\_27a \Rightarrow (\forall V0ll \in (\text{ty\_2Ellist\_2Ellist} \\
& \quad A\_27a). ((p \ (\text{ap } (\text{c\_2Ellist\_2ELFINITE } \ A\_27a) \ V0ll)) \Leftrightarrow (\exists V1n \in \\
& \quad \text{ty\_2Enum\_2Enum}.) ((\text{ap } (\text{c\_2Ellist\_2ELLENGTH } \ A\_27a) \ V0ll) = (\text{ap } ( \\
& \quad \text{c\_2Eoption\_2ESOME } \ \text{ty\_2Enum\_2Enum}) \ V1n))))))
\end{aligned} \tag{41}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.\text{nonempty } A\_27a \Rightarrow (\forall V0P \in (2^{(\text{ty\_2Ellist\_2Ellist } \ A\_27a)}). \\
& \quad (((p \ (\text{ap } \ V0P \ (\text{c\_2Ellist\_2ELNIL } \ A\_27a))) \wedge (\forall V1h \in A\_27a. ( \\
& \quad \forall V2t \in (\text{ty\_2Ellist\_2Ellist } \ A\_27a). ((p \ (\text{ap } \ V0P \ V2t)) \Rightarrow (p \ ( \\
& \quad \text{ap } \ V0P \ (\text{ap } (\text{ap } (\text{c\_2Ellist\_2ELCONS } \ A\_27a) \ V1h) \ V2t)))))) \Rightarrow (\forall V3a0 \in \\
& \quad (\text{ty\_2Ellist\_2Ellist } \ A\_27a). ((p \ (\text{ap } (\text{c\_2Ellist\_2ELFINITE } \ A\_27a) \\
& \quad V3a0)) \Rightarrow (p \ (\text{ap } \ V0P \ V3a0))))))
\end{aligned} \tag{42}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.\text{nonempty } A\_27a \Rightarrow (\forall V0x \in A\_27a. (\forall V1y \in \\
& \quad A\_27a. (((\text{ap } (\text{c\_2Eoption\_2ESOME } \ A\_27a) \ V0x) = (\text{ap } (\text{c\_2Eoption\_2ESOME} \\
& \quad A\_27a) \ V1y)) \Leftrightarrow (V0x = V1y))))
\end{aligned} \tag{43}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\
& \quad \forall V0f \in (A\_27b^{A\_27a}). (\forall V1x \in (ty\_2Eoption\_2Eoption \\
& \quad A\_27a). (\forall V2y \in A\_27b. (((ap\ (ap\ (c\_2Eoption\_2EOPTION\_MAP \\
& A\_27a\ A\_27b)\ V0f)\ V1x) = (ap\ (c\_2Eoption\_2ESOME\ A\_27b)\ V2y)) \Leftrightarrow (\exists V3z \in \\
& A\_27a. ((V1x = (ap\ (c\_2Eoption\_2ESOME\ A\_27a)\ V3z)) \wedge (V2y = (ap\ V0f \\
& \quad V3z))))))))) \\
\end{aligned} \tag{44}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
& ((ap\ c\_2Enum\_2ESUC\ V0m) = (ap\ c\_2Enum\_2ESUC\ V1n)) \Leftrightarrow (V0m = V1n))) \\
\end{aligned} \tag{45}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. (\neg (p\ (ap\ (ap\ c\_2Eprim\_rec\_2E\_3C \\
& \quad V0n)\ c\_2Enum\_2E0)))) \\
\end{aligned} \tag{46}$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \tag{47}$$

Assume the following.

$$(\forall V0A \in 2. ((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \tag{48}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\
& ((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))) \\
\end{aligned} \tag{49}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(p\ V0A)) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\
& ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))) \\
\end{aligned} \tag{50}$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p\ V0A)) \Rightarrow False) \Rightarrow (((p\ V0A) \Rightarrow False) \Rightarrow False))) \tag{51}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p\ V0p) \Leftrightarrow ( \\
& (p\ V1q) \Leftrightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((p\ V1q) \vee (p\ V2r))) \wedge (((p\ V0p) \vee ((\neg \\
& p\ V2r)) \vee (\neg(p\ V1q)))) \wedge (((p\ V1q) \vee ((\neg(p\ V2r)) \vee (\neg(p\ V0p)))) \wedge ((p\ V2r) \vee \\
& ((\neg(p\ V1q)) \vee (\neg(p\ V0p)))))))))) \\
\end{aligned} \tag{52}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow ( \\
& (p \ V1q) \wedge (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee (\neg(p \ V1q)) \vee (\neg(p \ V2r)))) \wedge (((p \ V1q) \vee \\
& (\neg(p \ V0p))) \wedge ((p \ V2r) \vee (\neg(p \ V0p))))))))))
\end{aligned} \tag{53}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow ( \\
& (p \ V1q) \vee (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee (\neg(p \ V1q))) \wedge ((p \ V0p) \vee (\neg(p \ V2r)))) \wedge \\
& ((p \ V1q) \vee ((p \ V2r) \vee (\neg(p \ V0p))))))))))
\end{aligned} \tag{54}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow ( \\
& (p \ V1q) \Rightarrow (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee (p \ V1q)) \wedge ((p \ V0p) \vee (\neg(p \ V2r))) \wedge (( \\
& \neg(p \ V1q)) \vee ((p \ V2r) \vee (\neg(p \ V0p))))))))))
\end{aligned} \tag{55}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (((p \ V0p) \Leftrightarrow (\neg(p \ V1q))) \Leftrightarrow (((p \ V0p) \vee \\
& (p \ V1q)) \wedge ((\neg(p \ V1q)) \vee (\neg(p \ V0p))))))
\end{aligned} \tag{56}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \ V0p) \Rightarrow (p \ V1q))) \Rightarrow (p \ V0p))) \tag{57}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \ V0p) \Rightarrow (p \ V1q))) \Rightarrow (\neg(p \ V1q)))) \tag{58}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \ V0p) \vee (p \ V1q))) \Rightarrow (\neg(p \ V0p)))) \tag{59}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \ V0p) \vee (p \ V1q))) \Rightarrow (\neg(p \ V1q)))) \tag{60}$$

Assume the following.

$$(\forall V0p \in 2. ((\neg(\neg(p \ V0p))) \Rightarrow (p \ V0p))) \tag{61}$$

**Theorem 1**

$$\begin{aligned}
& \forall A\_27a. nonempty \ A\_27a \Rightarrow (\forall V0ll \in (ty\_2Ellist\_2Ellist \\
& A\_27a). (\forall V1n \in ty\_2Enum\_2Enum. (\forall V2m \in ty\_2Enum\_2Enum. \\
& (((ap \ (c\_2Ellist\_2ELLENGTH \ A\_27a) \ V0ll) = (ap \ (c\_2Eoption\_2ESOME \\
& ty\_2Enum\_2Enum) \ V1n)) \wedge (p \ (ap \ (ap \ c\_2Eprim\_rec\_2E\_3C \ V1n) \ V2m))) \Rightarrow \\
& ((ap \ (ap \ (c\_2Ellist\_2ELTAKE \ A\_27a) \ V2m) \ V0ll) = (c\_2Eoption\_2ENONE \\
& (ty\_2Ellist\_2Ellist \ A\_27a))))))
\end{aligned}$$