

thm_2Ellist_2ELTAKE__LNTH__EL (TMRRNTjf- BNEQGQQwiEwU6rXULAfDHnwEVWL)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 4 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Elist_2Elist A0) \quad (1)$$

Let $c_2Elist_2EHD : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2EHD A_27a \in (A_27a^{(ty_2Elist_2Elist A_27a)}) \quad (2)$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty ty_2Enum_2Enum \quad (3)$$

Let $c_2Elist_2EEL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2EEL A_27a \in ((A_27a^{(ty_2Elist_2Elist A_27a)})^{ty_2Enum_2Enum}) \quad (4)$$

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))$

Let $ty_2Eoption_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Eoption_2Eoption A0) \quad (5)$$

Let $ty_2Ellist_2Ellist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Ellist_2Ellist\ A0) \quad (6)$$

Let $c_2Ellist_2ELNTH : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ellist_2ELNTH\ A_27a \in (((ty_2Eoption_2Eoption\ A_27a)^{(ty_2Ellist_2Ellist\ A_27a)})^{ty_2Enum_2Enum}) \quad (7)$$

Let $c_2Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ECONS\ A_27a \in (((ty_2Elist_2Elist\ A_27a)^{(ty_2Elist_2Elist\ A_27a)})^{A_27a}) \quad (8)$$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (9)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (10)$$

Definition 7 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 8 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (11)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (12)$$

Definition 9 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num\ m)$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (13)$$

Definition 10 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ (ap\ c_2Earithmetic_2E_2B\ n))$

Definition 11 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let $c_2Earithmetic_2E_2D : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2D \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (14)$$

Let $c_2Ellist_2Ellist_rep : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ellist_2Ellist_rep\ A_27a \in (((ty_2Eoption_2Eoption\ A_27a)^{(ty_2Enum_2Enum)})^{(ty_2Ellist_2Ellist\ A_27a)}) \quad (15)$$

Let $ty_2Eone_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eone_2Eone \quad (16)$$

Definition 12 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21) 2) (\lambda V2t \in 2.))$
Let $ty_2Esum_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Esum_2Esum A0 A1) \quad (17)$$

Let $c_2Esum_2EABS_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Esum_2EABS_sum A_27a A_27b \in ((ty_2Esum_2Esum A_27a A_27b)^{((2^{A_27b})^{A_27a})^2}) \quad (18)$$

Definition 13 We define c_2Esum_2EINL to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0e \in A_27a.(ap (c_2Esum_2EABS_sum A_27a A_27b) V0e)$

Let $c_2Eoption_2Eoption_abs : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Eoption_2Eoption_abs A_27a \in (ty_2Eoption_2Eoption A_27a)^{(ty_2Esum_2Esum A_27a ty_2Eone_2Eone)} \quad (19)$$

Definition 14 We define $c_2Eoption_2ESOME$ to be $\lambda A_27a : \iota. \lambda V0x \in A_27a.(ap (c_2Eoption_2Eoption_abs A_27a) V0x)$

Definition 15 We define $c_2Emin_2E_40$ to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p (ap P x)) \text{ then } (the (\lambda x. x \in A \wedge P x))$
of type $\iota \Rightarrow \iota$.

Definition 16 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A_27a. (\lambda V2t2 \in A_27a. (ap (c_2Emin_2E_40) (V1t1 V2t2)) V0t)))$

Let $c_2Ellist_2Ellist_abs : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Ellist_2Ellist_abs A_27a \in (ty_2Ellist_2Ellist A_27a)^{(ty_2Eoption_2Eoption A_27a)^{ty_2Eenum_2Eenum}} \quad (20)$$

Definition 17 We define $c_2Ellist_2ELCONS$ to be $\lambda A_27a : \iota. \lambda V0h \in A_27a. \lambda V1t \in (ty_2Ellist_2Ellist A_27a) V0h$

Definition 18 We define c_2Eone_2Eone to be $(ap (c_2Emin_2E_40) ty_2Eone_2Eone) (\lambda V0x \in ty_2Eone_2Eone) V0x$

Definition 19 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2. (ap (ap (c_2Emin_2E_3D_3D_3E) V0t) c_2Ebool_2E_21) V0t)$

Definition 20 We define c_2Esum_2EINR to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0e \in A_27b. (ap (c_2Esum_2EABS_sum A_27a A_27b) V0e)$

Definition 21 We define $c_2Eoption_2ENONE$ to be $\lambda A_27a : \iota. (ap (c_2Eoption_2Eoption_abs A_27a) V0t)$

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2ENIL A_27a \in (ty_2Elist_2Elist A_27a) \quad (21)$$

Definition 22 We define $c_2Ellist_2ELNIL$ to be $\lambda A_27a : \iota. (ap (c_2Ellist_2Ellist_abs A_27a) (\lambda V0n \in ty_2Eone_2Eone) V0n)$

Let $c_2Ellist_2ELTAK E : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ellist_2ELTAK E\ A_27a \in (((ty_2Eoption_2Eoption\ (ty_2Elist_2Elist\ A_27a))^{(ty_2Ellist_2Ellist\ A_27a)} ty_2Enum_2Enum)) \quad (22)$$

Definition 23 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap\ V0P\ (ap\ (c_2Emin_2E_40$

Let $c_2Eoption_2EOPTION_MAP : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Eoption_2EOPTION_MAP\ A_27a\ A_27b \in (((ty_2Eoption_2Eoption\ A_27b)^{(ty_2Eoption_2Eoption\ A_27a)} (A_27b^{A_27a})) \quad (23)$$

Definition 24 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum. ((V0m = c_2Enum_2E0) \vee (\exists V1n \in ty_2Enum_2Enum. (V0m = (ap\ c_2Enum_2ESUC\ V1n)))))) \quad (24)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (p\ (ap\ (ap\ c_2Eprim_rec_2E_3C\ (ap\ c_2Enum_2ESUC\ V0m))\ (ap\ c_2Enum_2ESUC\ V1n))) \Leftrightarrow (p\ (ap\ (ap\ c_2Eprim_rec_2E_3C\ V0m)\ V1n)))))) \quad (25)$$

Assume the following.

$$True \quad (26)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (27)$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p\ V0t))) \quad (28)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in A_27a. (p\ V0t)) \Leftrightarrow (p\ V0t))) \quad (29)$$

Assume the following.

$$(\forall V0t \in 2. (((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \quad (30)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((p V0t) \Rightarrow False) \Leftrightarrow \neg(p V0t)))))) \quad (31)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (32)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (33)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow \neg(p V0t)) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow \neg(p V0t)))))) \quad (34)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p V0A) \vee (p V1B) \vee (p V2C)) \Leftrightarrow (((p V0A) \vee (p V1B)) \vee (p V2C)))))) \quad (35)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p V0A) \vee (p V1B)) \Leftrightarrow ((p V1B) \vee (p V0A)))) \quad (36)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p V0A) \Rightarrow (p V1B)) \Leftrightarrow (\neg(p V0A) \vee (p V1B)))) \quad (37)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (38)$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x_27 \in 2.(\forall V2y \in 2.(\forall V3y_27 \in 2.(((p V0x) \Leftrightarrow (p V1x_27)) \wedge ((p V1x_27) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_27)))))) \Rightarrow (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_27) \Rightarrow (p V3y_27)))))) \quad (39)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0f \in (2^{A_27a}).(\forall V1v \in A_27a.((\forall V2x \in A_27a.((V2x = V1v) \Rightarrow (p (ap V0f V2x)))) \Leftrightarrow (p (ap V0f V1v)))))) \quad (40)$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0P \in (2^{A.27a}). (\forall V1Q \in \\
& 2. (((\exists V2x \in A.27a. (p\ (ap\ V0P\ V2x))) \Rightarrow (p\ V1Q)) \Leftrightarrow (\forall V3x \in \\
& A.27a. ((p\ (ap\ V0P\ V3x)) \Rightarrow (p\ V1Q)))) \wedge (((\exists V4x \in A.27a. (p\ (\\
& ap\ V0P\ V4x))) \wedge (p\ V1Q)) \Leftrightarrow (\exists V5x \in A.27a. ((p\ (ap\ V0P\ V5x)) \wedge (p \\
& V1Q)))) \wedge (((p\ V1Q) \wedge (\exists V6x \in A.27a. (p\ (ap\ V0P\ V6x)))) \Leftrightarrow (\exists V7x \in \\
& A.27a. ((p\ V1Q) \wedge (p\ (ap\ V0P\ V7x)))))))))
\end{aligned} \tag{41}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0h \in A.27a. (\forall V1t \in \\
& (ty_2Elist_2Elist\ A.27a). ((ap\ (c_2Elist_2EHD\ A.27a)\ (ap\ (ap\ (\\
& c_2Elist_2ECONS\ A.27a)\ V0h)\ V1t)) = V0h)))
\end{aligned} \tag{42}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\
& \forall V0n \in ty_2Enum_2Enum. (\forall V1l \in A.27b. (\forall V2ls \in \\
& (ty_2Elist_2Elist\ A.27b). (((ap\ (c_2Elist_2EEL\ A.27a)\ c_2Enum_2E0) = \\
& (c_2Elist_2EHD\ A.27a)) \wedge ((ap\ (ap\ (c_2Elist_2EEL\ A.27b)\ (ap\ c_2Enum_2ESUC \\
& V0n))\ (ap\ (ap\ (c_2Elist_2ECONS\ A.27b)\ V1l)\ V2ls)) = (ap\ (ap\ (c_2Elist_2EEL \\
& A.27b)\ V0n)\ V2ls))))))
\end{aligned} \tag{43}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0l \in (ty_2Elist_2Elist \\
& A.27a). ((V0l = (c_2Elist_2ELNIL\ A.27a)) \vee (\exists V1h \in A.27a. \\
& (\exists V2t \in (ty_2Elist_2Elist\ A.27a). (V0l = (ap\ (ap\ (c_2Elist_2ELCONS \\
& A.27a)\ V1h)\ V2t))))))
\end{aligned} \tag{44}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow \forall A.27c. \\
& nonempty\ A.27c \Rightarrow ((\forall V0n \in ty_2Enum_2Enum. ((ap\ (ap\ (c_2Elist_2ELNTH \\
& A.27a)\ V0n)\ (c_2Elist_2ELNIL\ A.27a)) = (c_2Eoption_2ENONE\ A.27a))) \wedge \\
& ((\forall V1h \in A.27b. (\forall V2t \in (ty_2Elist_2Elist\ A.27b). \\
& ((ap\ (ap\ (c_2Elist_2ELNTH\ A.27b)\ c_2Enum_2E0)\ (ap\ (ap\ (c_2Elist_2ELCONS \\
& A.27b)\ V1h)\ V2t)) = (ap\ (c_2Eoption_2ESOME\ A.27b)\ V1h)))) \wedge (\forall V3n \in \\
& ty_2Enum_2Enum. (\forall V4h \in A.27c. (\forall V5t \in (ty_2Elist_2Elist \\
& A.27c). ((ap\ (ap\ (c_2Elist_2ELNTH\ A.27c)\ (ap\ c_2Enum_2ESUC\ V3n)) \\
& (ap\ (ap\ (c_2Elist_2ELCONS\ A.27c)\ V4h)\ V5t)) = (ap\ (ap\ (c_2Elist_2ELNTH \\
& A.27c)\ V3n)\ V5t))))))
\end{aligned} \tag{45}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow \forall A.27c. \\
& \quad nonempty\ A.27c \Rightarrow ((\forall V0l \in (ty_2Ellist_2Ellist\ A.27a).((\\
ap\ (ap\ (c_2Ellist_2ELTAK E\ A.27a)\ c_2Enum_2E0)\ V0l) = (ap\ (c_2Eoption_2ESOME \\
& \quad (ty_2Elist_2Elist\ A.27a))\ (c_2Elist_2ENIL\ A.27a)))) \wedge ((\forall V1n \in \\
ty_2Enum_2Enum.((ap\ (ap\ (c_2Ellist_2ELTAK E\ A.27b)\ (ap\ c_2Enum_2ESUC \\
& \quad V1n))\ (c_2Ellist_2ELNIL\ A.27b)) = (c_2Eoption_2ENONE\ (ty_2Elist_2Elist \\
& \quad A.27b)))) \wedge (\forall V2n \in ty_2Enum_2Enum.(\forall V3h \in A.27c. \\
& \quad (\forall V4t \in (ty_2Ellist_2Ellist\ A.27c).((ap\ (ap\ (c_2Ellist_2ELTAK E \\
& \quad A.27c)\ (ap\ c_2Enum_2ESUC\ V2n))\ (ap\ (ap\ (c_2Ellist_2ELCONS\ A.27c) \\
& \quad V3h)\ V4t)) = (ap\ (ap\ (c_2Eoption_2EOPTION_MAP\ (ty_2Elist_2Elist \\
& \quad A.27c)\ (ty_2Elist_2Elist\ A.27c))\ (ap\ (c_2Elist_2ECONS\ A.27c) \\
& \quad V3h))\ (ap\ (ap\ (c_2Ellist_2ELTAK E\ A.27c)\ V2n)\ V4t))))))))) \\
& \hspace{15em} (46)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0l \in (ty_2Elist_2Elist \\
& \quad A.27a).(\forall V1m \in ty_2Enum_2Enum.(((ap\ (ap\ (c_2Ellist_2ELTAK E \\
& \quad A.27a)\ V1m)\ (c_2Ellist_2ELNIL\ A.27a)) = (ap\ (c_2Eoption_2ESOME \\
& \quad (ty_2Elist_2Elist\ A.27a))\ V0l)) \Leftrightarrow ((V1m = c_2Enum_2E0) \wedge (V0l = (\\
& \quad c_2Elist_2ENIL\ A.27a)))))) \\
& \hspace{15em} (47)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum.(\neg((ap\ c_2Enum_2ESUC\ V0n) = c_2Enum_2E0))) \\
& \hspace{15em} (48)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0P \in (2^{ty_2Enum_2Enum}).(((p\ (ap\ V0P\ c_2Enum_2E0)) \wedge \\
& \quad (\forall V1n \in ty_2Enum_2Enum.((p\ (ap\ V0P\ V1n)) \Rightarrow (p\ (ap\ V0P\ (ap\ c_2Enum_2ESUC \\
& \quad V1n)))))) \Rightarrow (\forall V2n \in ty_2Enum_2Enum.(p\ (ap\ V0P\ V2n)))))) \\
& \hspace{15em} (49)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0x \in A.27a.(\forall V1y \in \\
& \quad A.27a.(((ap\ (c_2Eoption_2ESOME\ A.27a)\ V0x) = (ap\ (c_2Eoption_2ESOME \\
& \quad A.27a)\ V1y)) \Leftrightarrow (V0x = V1y)))) \\
& \hspace{15em} (50)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\
& \quad \forall V0f \in (A.27b^{A.27a}).(\forall V1x \in (ty_2Eoption_2Eoption \\
& \quad A.27a).(\forall V2y \in A.27b.(((ap\ (ap\ (c_2Eoption_2EOPTION_MAP \\
& \quad A.27a\ A.27b)\ V0f)\ V1x) = (ap\ (c_2Eoption_2ESOME\ A.27b)\ V2y)) \Leftrightarrow (\exists V3z \in \\
& \quad A.27a.((V1x = (ap\ (c_2Eoption_2ESOME\ A.27a)\ V3z)) \wedge (V2y = (ap\ V0f \\
& \quad V3z))))))))) \\
& \hspace{15em} (51)
\end{aligned}$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum. (\neg(p (ap (ap c_2Eprim_rec_2E_3C V0n) c_2Enum_2E0)))) \quad (52)$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum. (p (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) (ap c_2Enum_2ESUC V0n)))) \quad (53)$$

Theorem 1

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow (\forall V0n \in ty_2Enum_2Enum. (\\ & \quad \forall V1ll \in (ty_2Elist_2Elist A_27a). (\forall V2m \in ty_2Enum_2Enum. \\ & \quad (\forall V3l \in (ty_2Elist_2Elist A_27a). (((ap (ap (c_2Elist_2ELTAKE \\ & \quad A_27a) V0n) V1ll) = (ap (c_2Eoption_2ESOME (ty_2Elist_2Elist A_27a) \\ & \quad V3l)) \wedge (p (ap (ap c_2Eprim_rec_2E_3C V2m) V0n))) \Rightarrow ((ap (ap (c_2Elist_2ELNTH \\ & \quad A_27a) V2m) V1ll) = (ap (c_2Eoption_2ESOME A_27a) (ap (ap (c_2Elist_2EEL \\ & \quad A_27a) V2m) V3l)))))))))) \end{aligned}$$