

# thm\_2Ellist\_2ELTAKE\_\_NIL\_\_EQ\_\_NONE (TM-MMwSVuZ2WR87t8pQEGzWdCT725F8qzT1V)

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**Definition 1** We define `c_2Emin_2E_40` to be  $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p \text{ (ap } P \ x)) \text{ then (the } (\lambda x. x \in A \wedge p \text{ (ap } P \ x)) \text{ of type } \iota \Rightarrow \iota.$

**Definition 2** We define `c_2Emin_2E_3D` to be  $\lambda A. \lambda x \in A. \lambda y \in A. \text{inj\_o } (x = y)$  of type  $\iota \Rightarrow \iota.$

**Definition 3** We define `c_2Ebool_2E_3F` to be  $\lambda A. 27a : \iota. (\lambda V0P \in (2^{A-27a}). (\text{ap } V0P \text{ (ap (c_2Emin_2E_40 } A \text{ (ap } P \ x))$

**Definition 4** We define `c_2Emin_2E_3D_3D_3E` to be  $\lambda P \in 2. \lambda Q \in 2. \text{inj\_o } (p \Rightarrow q)$  of type  $\iota.$

**Definition 5** We define `c_2Ebool_2E_2T` to be  $(\text{ap (ap (c_2Emin_2E_3D } (2^2)) (\lambda V0x \in 2. V0x)) (\lambda V1x \in 2. V1x))$

**Definition 6** We define `c_2Ebool_2E_21` to be  $\lambda A. 27a : \iota. (\lambda V0P \in (2^{A-27a}). (\text{ap (ap (c_2Emin_2E_3D } (2^{A-27a})$

**Definition 7** We define `c_2Ebool_2E_5C_2F` to be  $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (\text{ap (c_2Ebool_2E_21 } 2) (\lambda V2t \in 2. V2t))$

**Definition 8** We define `c_2Ebool_2EF` to be  $(\text{ap (c_2Ebool_2E_21 } 2) (\lambda V0t \in 2. V0t)).$

Let `c_2Enum_2EZERO__REP` :  $\iota$  be given. Assume the following.

$$c_2Enum_2EZERO__REP \in \text{omega} \quad (1)$$

Let `ty_2Enum_2Enum` :  $\iota$  be given. Assume the following.

$$\text{nonempty ty_2Enum_2Enum} \quad (2)$$

Let `c_2Enum_2EABS__num` :  $\iota$  be given. Assume the following.

$$c_2Enum_2EABS__num \in (\text{ty_2Enum_2Enum}^{\text{omega}}) \quad (3)$$

**Definition 9** We define `c_2Enum_2E0` to be  $(\text{ap c_2Enum_2EABS__num c_2Enum_2EZERO__REP}).$

**Definition 10** We define `c_2Earithmetic_2EZERO` to be `c_2Enum_2E0`.

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \quad (4)$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \quad (5)$$

**Definition 11** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_num$

Let  $c\_2Earithmetic\_2E\_2B : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum)^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum} \quad (6)$$

**Definition 12** We define  $c\_2Earithmetic\_2EBIT1$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

**Definition 13** We define  $c\_2Earithmetic\_2ENUMERAL$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.V0x$ .

Let  $ty\_2Eoption\_2Eoption : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Eoption\_2Eoption\ A0) \quad (7)$$

Let  $ty\_2Ellist\_2Ellist : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Ellist\_2Ellist\ A0) \quad (8)$$

Let  $c\_2Ellist\_2Ellist\_rep : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ellist\_2Ellist\_rep\ A\_27a \in \\ (((ty\_2Eoption\_2Eoption\ A\_27a)^{ty\_2Enum\_2Enum})^{(ty\_2Ellist\_2Ellist\ A\_27a)}) \quad (9)$$

Let  $c\_2Ellist\_2Ellist\_abs : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ellist\_2Ellist\_abs\ A\_27a \in \\ ((ty\_2Ellist\_2Ellist\ A\_27a)^{((ty\_2Eoption\_2Eoption\ A\_27a)^{ty\_2Enum\_2Enum})}) \quad (10)$$

Let  $ty\_2Eone\_2Eone : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Eone\_2Eone \quad (11)$$

**Definition 14** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V2t$

Let  $ty\_2Esum\_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Esum\_2Esum\ A0\ A1) \quad (12)$$

Let  $c\_2Esum\_2EABS\_sum : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Esum\_2EABS\_sum\ A\_27a\ A\_27b \in \\ ((ty\_2Esum\_2Esum\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})^2}) \quad (13)$$

**Definition 15** We define  $c\_Esum\_2EINL$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0e \in A\_27a. (ap (c\_Esum\_2EABS$

Let  $c\_2Eoption\_2Eoption\_ABS : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a. nonempty A\_27a \Rightarrow c\_2Eoption\_2Eoption\_ABS A\_27a \in ((ty\_2Eoption\_2Eoption A\_27a)^{(ty\_2Esum\_2Esum A\_27a ty\_2Eone\_2Eone)}) \quad (14)$$

**Definition 16** We define  $c\_2Eoption\_2ESOME$  to be  $\lambda A\_27a : \iota. \lambda V0x \in A\_27a. (ap (c\_2Eoption\_2Eoption\_ABS$

**Definition 17** We define  $c\_2Eone\_2Eone$  to be  $(ap (c\_2Emin\_2E\_40 ty\_2Eone\_2Eone) (\lambda V0x \in ty\_2Eone\_2Eone$

**Definition 18** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2. (ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2E\_7E$

**Definition 19** We define  $c\_2Esum\_2EINR$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0e \in A\_27b. (ap (c\_2Esum\_2EABS$

**Definition 20** We define  $c\_2Eoption\_2ENONE$  to be  $\lambda A\_27a : \iota. (ap (c\_2Eoption\_2Eoption\_ABS A\_27a) (c\_2Eoption\_2ENONE$

**Definition 21** We define  $c\_2Ellist\_2ELHD$  to be  $\lambda A\_27a : \iota. \lambda V0ll \in (ty\_2Ellist\_2Ellist A\_27a). (ap (ap (c\_2Ellist\_2ELHD$

Let  $c\_2Eoption\_2Eoption\_CASE : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a. nonempty A\_27a \Rightarrow \forall A\_27b. nonempty A\_27b \Rightarrow c\_2Eoption\_2Eoption\_CASE A\_27a A\_27b \in (((A\_27b)^{(A\_27b^{A\_27a})})^{A\_27b})^{(ty\_2Eoption\_2Eoption A\_27a)} \quad (15)$$

**Definition 22** We define  $c\_2Ellist\_2ELTL$  to be  $\lambda A\_27a : \iota. \lambda V0ll \in (ty\_2Ellist\_2Ellist A\_27a). (ap (ap (ap (c\_2Ellist\_2ELTL$

Let  $c\_2Eoption\_2ETHE : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a. nonempty A\_27a \Rightarrow c\_2Eoption\_2ETHE A\_27a \in (A\_27a)^{(ty\_2Eoption\_2Eoption A\_27a)} \quad (16)$$

Let  $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0. nonempty A0 \Rightarrow nonempty (ty\_2Elist\_2Elist A0) \quad (17)$$

Let  $c\_2Elist\_2ECONS : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a. nonempty A\_27a \Rightarrow c\_2Elist\_2ECONS A\_27a \in (((ty\_2Elist\_2Elist A\_27a)^{(ty\_2Elist\_2Elist A\_27a)})^{A\_27a}) \quad (18)$$

Let  $c\_2Eoption\_2EOPTION\_MAP : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a. nonempty A\_27a \Rightarrow \forall A\_27b. nonempty A\_27b \Rightarrow c\_2Eoption\_2EOPTION\_MAP A\_27a A\_27b \in (((ty\_2Eoption\_2Eoption A\_27b)^{(ty\_2Eoption\_2Eoption A\_27a)})^{(A\_27b^{A\_27a})}) \quad (19)$$

Let  $c\_2Earithmetic\_2E\_2D : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum)^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum} \quad (20)$$

**Definition 23** We define  $c\_Ebool\_2ECOND$  to be  $\lambda A\_27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A\_27a. (\lambda V2t2 \in A\_27a. ($

**Definition 24** We define  $c\_2Ellist\_2ELCONS$  to be  $\lambda A\_27a : \iota. \lambda V0h \in A\_27a. \lambda V1t \in (ty\_2Ellist\_2Ellist A$

**Definition 25** We define  $c\_2Ellist\_2ELNIL$  to be  $\lambda A\_27a : \iota. (ap (c\_2Ellist\_2Ellist\_abs A\_27a) (\lambda V0n \in ty$

Let  $c\_2Elist\_2ENIL : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a. nonempty A\_27a \Rightarrow c\_2Elist\_2ENIL A\_27a \in (ty\_2Elist\_2Elist A\_27a) \quad (21)$$

Let  $c\_2Ellist\_2ELTAKE : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a. nonempty A\_27a \Rightarrow c\_2Ellist\_2ELTAKE A\_27a \in (((ty\_2Eoption\_2Eoption (ty\_2Elist\_2Elist A\_27a))^{(ty\_2Ellist\_2Ellist A\_27a)} ty\_2Enum\_2Enum) \quad (22)$$

**Definition 26** We define  $c\_2Eprim\_rec\_2E\_3C$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. \lambda V1n \in ty\_2Enum\_2Enum$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum. ((V0m = c\_2Enum\_2E0) \vee (\exists V1n \in ty\_2Enum\_2Enum. (V0m = (ap c\_2Enum\_2ESUC V1n)))))) \quad (23)$$

Assume the following.

$$True \quad (24)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (25)$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p V0t))) \quad (26)$$

Assume the following.

$$((\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (27)$$

Assume the following.

$$\forall A\_27a. nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (28)$$

Assume the following.

$$\forall A\_27a. nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a. (\forall V1y \in A\_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (29)$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\
& (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow \neg(p V0t)) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow \neg( \\
& p V0t))))))
\end{aligned} \tag{30}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty A.27a \Rightarrow ((\forall V0ll \in (ty\_2Ellist\_2Ellist \\
& A.27a).(ap (ap (c.2Ellist\_2ELTAK E A.27a) c.2Enum\_2E0) V0ll) = \\
& (ap (c.2Eoption\_2ESOME (ty\_2Elist\_2Elist A.27a)) (c.2Elist\_2ENIL \\
& A.27a)))) \wedge (\forall V1n \in ty\_2Enum\_2Enum.(\forall V2ll \in (ty\_2Ellist\_2Ellist \\
& A.27a).(ap (ap (c.2Ellist\_2ELTAK E A.27a) (ap c.2Enum\_2ESUC V1n)) \\
V2ll) = (ap (ap (ap (c.2Eoption\_2Eoption\_CASE A.27a (ty\_2Eoption\_2Eoption \\
& (ty\_2Elist\_2Elist A.27a))) (ap (c.2Ellist\_2ELHD A.27a) V2ll)) \\
& (c.2Eoption\_2ENONE (ty\_2Elist\_2Elist A.27a))) (\lambda V3hd \in A.27a. \\
& (ap (ap (ap (c.2Eoption\_2Eoption\_CASE (ty\_2Elist\_2Elist A.27a) \\
& (ty\_2Eoption\_2Eoption (ty\_2Elist\_2Elist A.27a))) (ap (ap (c.2Ellist\_2ELTAK E \\
& A.27a) V1n) (ap (c.2Eoption\_2ETHE (ty\_2Ellist\_2Ellist A.27a)) \\
& (ap (c.2Ellist\_2ELTL A.27a) V2ll)))))) (c.2Eoption\_2ENONE (ty\_2Elist\_2Elist \\
& A.27a))) (\lambda V4tl \in (ty\_2Elist\_2Elist A.27a).(ap (c.2Eoption\_2ESOME \\
& (ty\_2Elist\_2Elist A.27a)) (ap (ap (c.2Elist\_2ECONS A.27a) V3hd) \\
& V4tl)))))))))
\end{aligned} \tag{31}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty A.27a \Rightarrow \forall A.27b.nonempty A.27b \Rightarrow \forall A.27c. \\
& nonempty A.27c \Rightarrow ((\forall V0l \in (ty\_2Ellist\_2Ellist A.27a).(( \\
ap (ap (c.2Ellist\_2ELTAK E A.27a) c.2Enum\_2E0) V0l) = (ap (c.2Eoption\_2ESOME \\
& (ty\_2Elist\_2Elist A.27a)) (c.2Elist\_2ENIL A.27a)))) \wedge ((\forall V1n \in \\
& ty\_2Enum\_2Enum.((ap (ap (c.2Ellist\_2ELTAK E A.27b) (ap c.2Enum\_2ESUC \\
& V1n)) (c.2Ellist\_2ELNIL A.27b)) = (c.2Eoption\_2ENONE (ty\_2Elist\_2Elist \\
& A.27b)))) \wedge (\forall V2n \in ty\_2Enum\_2Enum.(\forall V3h \in A.27c. \\
& (\forall V4t \in (ty\_2Ellist\_2Ellist A.27c).((ap (ap (c.2Ellist\_2ELTAK E \\
& A.27c) (ap c.2Enum\_2ESUC V2n)) (ap (ap (c.2Ellist\_2ELCONS A.27c) \\
& V3h) V4t)) = (ap (ap (c.2Eoption\_2EOPTION\_MAP (ty\_2Elist\_2Elist \\
& A.27c) (ty\_2Elist\_2Elist A.27c)) (ap (c.2Elist\_2ECONS A.27c) \\
& V3h)) (ap (ap (c.2Ellist\_2ELTAK E A.27c) V2n) V4t)))))))))
\end{aligned} \tag{32}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty A.27a \Rightarrow (\forall V0x \in A.27a.(\neg((c.2Eoption\_2ENONE \\
& A.27a) = (ap (c.2Eoption\_2ESOME A.27a) V0x))))
\end{aligned} \tag{33}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum.(\neg(p (ap (ap c.2Eprim\_rec.2E.3C \\
& V0n) c.2Enum\_2E0))))
\end{aligned} \tag{34}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Enum\_2E0) (ap c\_2Enum\_2ESUC V0n)))) \quad (35)$$

**Theorem 1**

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0m \in ty\_2Enum\_2Enum. ((ap (ap (c\_2Ellist\_2ELTAKE A\_27a) V0m) (c\_2Ellist\_2ELNIL A\_27a)) = (c\_2Eoption\_2ENONE (ty\_2Elist\_2Elist A\_27a))) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Enum\_2E0) V0m))))$$