thm_2Ellist_2ELTAKE__SNOC__LNTH (TMQL-NGcd5dNRKeTLoDsYtiF4NGMSpNMRtYi)

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Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 4 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 5 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 6 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \mathsf{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \mathsf{c\_2Ebool\_2EF}$

Let $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Elist\_2Elist\ A0) \quad (1)$$

Let $c\_2Elist\_2EAPPEND : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2EAPPEND\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{(ty\_2Elist\_2Elist\ A\_27a)}) \quad (2)$$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \quad (3)$$

Let $c\_2Earithmetic\_2E\_2D : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (4)$$

Definition 7 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Definition 8 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.\mathbf{if}\ (\exists x \in A.p\ (ap\ P\ x))\ \mathbf{then}\ (the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

Definition 9 *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.(a$

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \tag{5}$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \tag{6}$$

Definition 10 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Definition 11 *We define* c_2Earithmetic_2EZERO *to be* c_2Enum_2E0*.*

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \tag{7}$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \tag{8}$$

Definition 12 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num$

Let $c\_2Earithmetic\_2E\_2B : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{9}$$

Definition 13 *We define* c_2Earithmetic_2EBIT1 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

Definition 14 *We define* c_2Earithmetic_2ENUMERAL *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x$*.*

Let $ty\_2Eone\_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Eone\_2Eone \tag{10}$$

Let $ty\_2Esum\_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Esum\_2Esum\ A0\ A1) \tag{11}$$

Let $c\_2Esum\_2EABS\_\_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Esum\_2EABS\_\_sum\ A\_27a\ A\_27b \in ((ty\_2Esum\_2Esum\ A\_27a\ A\_27b)^{(((2^{A\_27b})^{A\_27a})^{2})}) \tag{12}$$

Definition 15 *We define* c_2Esum_2EINL *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0e \in A\_27a.(ap\ (c\_2Esum\_2EABS$

Let $ty\_2Eoption\_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. nonempty\ A0 \Rightarrow nonempty\ (ty\_2Eoption\_2Eoption\ A0) \tag{13}$$

Let $c\_2Eoption\_2Eoption\_\_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a. nonempty\ A\_27a \Rightarrow c\_2Eoption\_2Eoption\_\_ABS\ A\_27a \in ((ty\_2Eoption\_2Eoption\ A\_27a)^{(ty\_2Esum\_2Esum\ A\_27a\ ty\_2Eone\_2Eone)}) \tag{14}$$

Definition 16 *We define* c_2Eoption_2ESOME *to be* $\lambda A\_27a : \iota. \lambda V0x \in A\_27a.(ap\ (c\_2Eoption\_2Eoption\_\_$

Definition 17 *We define* c_2Ebool_2E_3F *to be* $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ ($c_2Emin_2E_40

Definition 18 *We define* c_2Eone_2Eone *to be* $(ap\ ($c_2Emin_2E_40 $ty\_2Eone\_2Eone)\ (\lambda V0x \in ty\_2Eone\_2$

Definition 19 *We define* c_2Esum_2EINR *to be* $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0e \in A\_27b.(ap\ (c\_2Esum\_2EABS$

Definition 20 *We define* c_2Eoption_2ENONE *to be* $\lambda A\_27a : \iota.(ap\ (c\_2Eoption\_2Eoption\_\_ABS\ A\_27a)\ ($

Definition 21 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ ($c_2Ebool_2E_21 $2)\ (\lambda V2t \in$

Definition 22 *We define* c_2Ellist_2Elrep__ok *to be* $\lambda A\_27a : \iota.(\lambda V0a0 \in ((ty\_2Eoption\_2Eoption\ A\_27a)^{ty}$

Let $ty\_2Ellist\_2Ellist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. nonempty\ A0 \Rightarrow nonempty\ (ty\_2Ellist\_2Ellist\ A0) \tag{15}$$

Let $c\_2Ellist\_2Ellist\_\_rep : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a. nonempty\ A\_27a \Rightarrow c\_2Ellist\_2Ellist\_\_rep\ A\_27a \in (((ty\_2Eoption\_2Eoption\ A\_27a)^{ty\_2Enum\_2Enum})^{(ty\_2Ellist\_2Ellist\ A\_27a)}) \tag{16}$$

Let $c\_2Ellist\_2Ellist\_\_abs : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a. nonempty\ A\_27a \Rightarrow c\_2Ellist\_2Ellist\_\_abs\ A\_27a \in ((ty\_2Ellist\_2Ellist\ A\_27a)^{((ty\_2Eoption\_2Eoption\ A\_27a)^{ty\_2Enum\_2Enum})}) \tag{17}$$

Let $c\_2Eoption\_2EOPTION\_\_JOIN : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a. nonempty\ A\_27a \Rightarrow c\_2Eoption\_2EOPTION\_\_JOIN\ A\_27a \in ((ty\_2Eoption\_2Eoption\ A\_27a)^{(ty\_2Eoption\_2Eoption\ (ty\_2Eoption\_2Eoption\ A\_27a))}) \tag{18}$$

Let $c\_2Ellist\_2ELNTH : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a. nonempty\ A\_27a \Rightarrow c\_2Ellist\_2ELNTH\ A\_27a \in (((ty\_2Eoption\_2Eoption\ A\_27a)^{(ty\_2Ellist\_2Ellist\ A\_27a)})^{ty\_2Enum\_2Enum}) \tag{19}$$

Definition 23 *We define* c_2Ellist_2ELHD *to be* $\lambda A\_27a : \iota. \lambda V0ll \in (ty\_2Ellist\_2Ellist\ A\_27a).(ap\ (ap\ (c\_2$

Let $c\_2Eoption\_2Eoption\_\_CASE : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Eoption\_2Eoption\_\_CASE\ A\_27a\ A\_27b \in (((A\_27b^{(A\_27b^{A\_27a})})^{A\_27b})^{(ty\_2Eoption\_2Eoption\ A\_27a)}) \tag{20}$$

Definition 24 *We define* c_2Ellist_2ELTL *to be* $\lambda A\_27a : \iota.\lambda V0ll \in (ty\_2Ellist\_2Ellist\ A\_27a).(ap\ (ap\ (ap$

Let $c\_2Elist\_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ECONS\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{A\_27a}) \tag{21}$$

Definition 25 *We define* c_2Ellist_2ELCONS *to be* $\lambda A\_27a : \iota.\lambda V0h \in A\_27a.\lambda V1t \in (ty\_2Ellist\_2Ellist\ A$

Definition 26 *We define* c_2Ellist_2ELNIL *to be* $\lambda A\_27a : \iota.(ap\ (c\_2Ellist\_2Ellist\_\_abs\ A\_27a)\ (\lambda V0n \in ty$

Let $c\_2Elist\_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ENIL\ A\_27a \in (ty\_2Elist\_2Elist\ A\_27a) \tag{22}$$

Let $c\_2Ellist\_2ELTAKE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ellist\_2ELTAKE\ A\_27a \in (((ty\_2Eoption\_2Eoption\ (ty\_2Elist\_2Elist\ A\_27a))^{(ty\_2Ellist\_2Ellist\ A\_27a)})^{ty\_2Enum\_2Enum}) \tag{23}$$

Let $c\_2Eoption\_2EOPTION\_\_MAP : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Eoption\_2EOPTION\_\_MAP\ A\_27a\ A\_27b \in (((ty\_2Eoption\_2Eoption\ A\_27b)^{(ty\_2Eoption\_2Eoption\ A\_27a)})^{(A\_27b^{A\_27a})}) \tag{24}$$

Let $c\_2Eoption\_2ETHE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Eoption\_2ETHE\ A\_27a \in (A\_27a^{(ty\_2Eoption\_2Eoption\ A\_27a)}) \tag{25}$$

Assume the following.

$$True \tag{26}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \tag{27}$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge ((((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t))))))) \tag{28}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t)))))))) \tag{29}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{30}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg (p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg (p\ V0t))))))) \tag{31}$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1y \in 2.(\forall V2z \in 2.(\forall V3w \in 2.((((p\ V0x) \Rightarrow (p\ V1y)) \wedge ((p\ V2z) \Rightarrow (p\ V3w))) \Rightarrow (((p\ V0x) \wedge (p\ V2z)) \Rightarrow ((p\ V1y) \wedge (p\ V3w)))))))) \tag{32}$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1y \in 2.(\forall V2z \in 2.(\forall V3w \in 2.((((p\ V0x) \Rightarrow (p\ V1y)) \wedge ((p\ V2z) \Rightarrow (p\ V3w))) \Rightarrow (((p\ V0x) \vee (p\ V2z)) \Rightarrow ((p\ V1y) \vee (p\ V3w)))))))) \tag{33}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).(\forall V1Q \in (2^{A\_27a}).((\forall V2x \in A\_27a.((p\ (ap\ V0P\ V2x)) \Rightarrow (p\ (ap\ V1Q\ V2x)))) \Rightarrow ((\exists V3x \in A\_27a.(p\ (ap\ V0P\ V3x))) \Rightarrow (\exists V4x \in A\_27a.(p\ (ap\ V1Q\ V4x))))))) \tag{34}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow ((\forall V0l \in (ty\_2Elist\_2Elist\ A\_27a).((ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27a)\ (c\_2Elist\_2ENIL\ A\_27a))\ V0l) = V0l)) \wedge (\forall V1l1 \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V2l2 \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V3h \in A\_27a.((ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27a)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V3h)\ V1l1))\ V2l2) = (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V3h)\ (ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27a)\ V1l1)\ V2l2))))))) \tag{35}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0a0 \in A\_27a.(\forall V1a1 \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V2a0\_27 \in A\_27a.(\forall V3a1\_27 \in (ty\_2Elist\_2Elist\ A\_27a).(((ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V0a0)\ V1a1) = (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V2a0\_27)\ V3a1\_27)) \Leftrightarrow ((V0a0 = V2a0\_27) \wedge (V1a1 = V3a1\_27)))))))) \tag{36}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow ((\forall V0l1 \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V1l2 \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V2l3 \in (ty\_2Elist\_2Elist\ A\_27a).(((ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27a)\ V0l1)\ V1l2) = (ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27a)\ V0l1)\ V2l3)) \Leftrightarrow (V1l2 = V2l3))))) \wedge (\forall V3l1 \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V4l2 \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V5l3 \in (ty\_2Elist\_2Elist\ A\_27a).(((ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27a)\ V4l2)\ V3l1) = (ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27a)\ V5l3)\ V3l1)) \Leftrightarrow (V4l2 = V5l3)))))) \tag{37}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow ((\forall V0a \in (ty\_2Ellist\_2Ellist\ A\_27a).((ap\ (c\_2Ellist\_2Ellist\_\_abs\ A\_27a)\ (ap\ (c\_2Ellist\_2Ellist\_\_rep\ A\_27a)\ V0a)) = V0a)) \wedge (\forall V1r \in ((ty\_2Eoption\_2Eoption\ A\_27a)^{ty\_2Enum\_2Enum}).((p\ (ap\ (\mathsf{c\_2Ellist\_2Elrep\_\_ok}\ A\_27a)\ V1r)) \Leftrightarrow ((ap\ (c\_2Ellist\_2Ellist\_\_rep\ A\_27a)\ (ap\ (c\_2Ellist\_2Ellist\_\_abs\ A\_27a)\ V1r)) = V1r)))) \tag{38}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0h \in A\_27a.(\forall V1t \in (ty\_2Ellist\_2Ellist\ A\_27a).(((ap\ (\mathsf{c\_2Ellist\_2ELHD}\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Ellist\_2ELCONS}\ A\_27a)\ V0h)\ V1t)) = (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V0h)) \wedge ((ap\ (\mathsf{c\_2Ellist\_2ELTL}\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Ellist\_2ELCONS}\ A\_27a)\ V0h)\ V1t)) = (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ (ty\_2Ellist\_2Ellist\ A\_27a))\ V1t))))) \tag{39}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0l \in (ty\_2Ellist\_2Ellist\ A\_27a).((V0l = (\mathsf{c\_2Ellist\_2ELNIL}\ A\_27a)) \vee (\exists V1h \in A\_27a.(\exists V2t \in (ty\_2Ellist\_2Ellist\ A\_27a).(V0l = (ap\ (ap\ (\mathsf{c\_2Ellist\_2ELCONS}\ A\_27a)\ V1h)\ V2t)))))) \tag{40}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (((ap\ (\mathsf{c\_2Ellist\_2ELHD}\ A\_27a)\ (\mathsf{c\_2Ellist\_2ELNIL}\ A\_27a)) = (\mathsf{c\_2Eoption\_2ENONE}\ A\_27a)) \wedge (\forall V0h \in A\_27b.(\forall V1t \in (ty\_2Ellist\_2Ellist\ A\_27b).((ap\ (\mathsf{c\_2Ellist\_2ELHD}\ A\_27b)\ (ap\ (ap\ (\mathsf{c\_2Ellist\_2ELCONS}\ A\_27b)\ V0h)\ V1t)) = (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27b)\ V0h))))) \tag{41}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (((ap\ (\mathsf{c\_2Ellist\_2ELTL}\ A\_27a)\ (\mathsf{c\_2Ellist\_2ELNIL}\ A\_27a)) = (\mathsf{c\_2Eoption\_2ENONE}\ (ty\_2Ellist\_2Ellist\ A\_27a))) \wedge (\forall V0h \in A\_27b.(\forall V1t \in (ty\_2Ellist\_2Ellist\ A\_27b).((ap\ (\mathsf{c\_2Ellist\_2ELTL}\ A\_27b)\ (ap\ (ap\ (\mathsf{c\_2Ellist\_2ELCONS}\ A\_27b)\ V0h)\ V1t)) = (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ (ty\_2Ellist\_2Ellist\ A\_27b))\ V1t))))) \tag{42}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow ((\forall V0ll \in (ty\_2Ellist\_2Ellist\ A\_27a).((ap\ (ap\ (c\_2Ellist\_2ELNTH\ A\_27a)\ \mathsf{c\_2Enum\_2E0})\ V0ll) = (ap\ (\mathsf{c\_2Ellist\_2ELHD}\ A\_27a)\ V0ll))) \wedge (\forall V1n \in ty\_2Enum\_2Enum.(\forall V2ll \in (ty\_2Ellist\_2Ellist\ A\_27a).((ap\ (ap\ (c\_2Ellist\_2ELNTH\ A\_27a)\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V1n))\ V2ll) = (ap\ (c\_2Eoption\_2EOPTION\_\_JOIN\ A\_27a)\ (ap\ (ap\ (c\_2Eoption\_2EOPTION\_\_MAP\ (ty\_2Ellist\_2Ellist\ A\_27a)\ (ty\_2Eoption\_2Eoption\ A\_27a))\ (ap\ (c\_2Ellist\_2ELNTH\ A\_27a)\ V1n))\ (ap\ (\mathsf{c\_2Ellist\_2ELTL}\ A\_27a)\ V2ll))))))) \tag{43}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c.nonempty\ A\_27c \Rightarrow ((\forall V0n \in ty\_2Enum\_2Enum.((ap\ (ap\ (c\_2Ellist\_2ELNTH\ A\_27a)\ V0n)\ (\mathsf{c\_2Ellist\_2ELNIL}\ A\_27a)) = (\mathsf{c\_2Eoption\_2ENONE}\ A\_27a))) \wedge ((\forall V1h \in A\_27b.(\forall V2t \in (ty\_2Ellist\_2Ellist\ A\_27b).((ap\ (ap\ (c\_2Ellist\_2ELNTH\ A\_27b)\ \mathsf{c\_2Enum\_2E0})\ (ap\ (ap\ (\mathsf{c\_2Ellist\_2ELCONS}\ A\_27b)\ V1h)\ V2t)) = (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27b)\ V1h)))) \wedge (\forall V3n \in ty\_2Enum\_2Enum.(\forall V4h \in A\_27c.(\forall V5t \in (ty\_2Ellist\_2Ellist\ A\_27c).((ap\ (ap\ (c\_2Ellist\_2ELNTH\ A\_27c)\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V3n))\ (ap\ (ap\ (\mathsf{c\_2Ellist\_2ELCONS}\ A\_27c)\ V4h)\ V5t)) = (ap\ (ap\ (c\_2Ellist\_2ELNTH\ A\_27c)\ V3n)\ V5t))))))) \tag{44}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow ((\forall V0ll \in (ty\_2Ellist\_2Ellist\ A\_27a).((ap\ (ap\ (c\_2Ellist\_2ELTAKE\ A\_27a)\ \mathsf{c\_2Enum\_2E0})\ V0ll) = (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ (ty\_2Elist\_2Elist\ A\_27a))\ (c\_2Elist\_2ENIL\ A\_27a)))) \wedge (\forall V1n \in ty\_2Enum\_2Enum.(\forall V2ll \in (ty\_2Ellist\_2Ellist\ A\_27a).((ap\ (ap\ (c\_2Ellist\_2ELTAKE\ A\_27a)\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V1n))\ V2ll) = (ap\ (ap\ (ap\ (c\_2Eoption\_2Eoption\_\_CASE\ A\_27a\ (ty\_2Eoption\_2Eoption\ (ty\_2Elist\_2Elist\ A\_27a)))\ (ap\ (\mathsf{c\_2Ellist\_2ELHD}\ A\_27a)\ V2ll))\ (\mathsf{c\_2Eoption\_2ENONE}\ (ty\_2Elist\_2Elist\ A\_27a)))\ (\lambda V3hd \in A\_27a.(ap\ (ap\ (ap\ (c\_2Eoption\_2Eoption\_\_CASE\ (ty\_2Elist\_2Elist\ A\_27a)\ (ty\_2Eoption\_2Eoption\ (ty\_2Elist\_2Elist\ A\_27a)))\ (ap\ (ap\ (c\_2Ellist\_2ELTAKE\ A\_27a)\ V1n)\ (ap\ (c\_2Eoption\_2ETHE\ (ty\_2Ellist\_2Ellist\ A\_27a))\ (ap\ (\mathsf{c\_2Ellist\_2ELTL}\ A\_27a)\ V2ll))))\ (\mathsf{c\_2Eoption\_2ENONE}\ (ty\_2Elist\_2Elist\ A\_27a)))\ (\lambda V4tl \in (ty\_2Elist\_2Elist\ A\_27a).(ap\ (\mathsf{c\_2Eoption\_2ESOME}\ (ty\_2Elist\_2Elist\ A\_27a))\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V3hd)\ V4tl)))))))))) \tag{45}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c.nonempty\ A\_27c \Rightarrow ((\forall V0l \in (ty\_2Ellist\_2Ellist\ A\_27a).((ap\ (ap\ (c\_2Ellist\_2ELTAKE\ A\_27a)\ \mathsf{c\_2Enum\_2E0})\ V0l) = (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ (ty\_2Elist\_2Elist\ A\_27a))\ (c\_2Elist\_2ENIL\ A\_27a)))) \wedge ((\forall V1n \in ty\_2Enum\_2Enum.((ap\ (ap\ (c\_2Ellist\_2ELTAKE\ A\_27b)\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V1n))\ (\mathsf{c\_2Ellist\_2ELNIL}\ A\_27b)) = (\mathsf{c\_2Eoption\_2ENONE}\ (ty\_2Elist\_2Elist\ A\_27b)))) \wedge (\forall V2n \in ty\_2Enum\_2Enum.(\forall V3h \in A\_27c.(\forall V4t \in (ty\_2Ellist\_2Ellist\ A\_27c).((ap\ (ap\ (c\_2Ellist\_2ELTAKE\ A\_27c)\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V2n))\ (ap\ (ap\ (\mathsf{c\_2Ellist\_2ELCONS}\ A\_27c)\ V3h)\ V4t)) = (ap\ (ap\ (c\_2Eoption\_2EOPTION\_\_MAP\ (ty\_2Elist\_2Elist\ A\_27c)\ (ty\_2Elist\_2Elist\ A\_27c))\ (ap\ (c\_2Elist\_2ECONS\ A\_27c)\ V3h))\ (ap\ (ap\ (c\_2Ellist\_2ELTAKE\ A\_27c)\ V2n)\ V4t)))))))) \tag{46}$$

Assume the following.

$$(\forall V0P \in (2^{ty\_2Enum\_2Enum}).(((p\ (ap\ V0P\ \mathsf{c\_2Enum\_2E0})) \wedge (\forall V1n \in ty\_2Enum\_2Enum.((p\ (ap\ V0P\ V1n)) \Rightarrow (p\ (ap\ V0P\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V1n)))))) \Rightarrow (\forall V2n \in ty\_2Enum\_2Enum.(p\ (ap\ V0P\ V2n))))) \tag{47}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0opt \in (ty\_2Eoption\_2Eoption\ A\_27a).((V0opt = (\mathsf{c\_2Eoption\_2ENONE}\ A\_27a)) \vee (\exists V1x \in A\_27a.(V0opt = (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V1x))))) \tag{48}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ((\forall V0v \in A\_27b.(\forall V1f \in (A\_27b^{A\_27a}).((ap\ (ap\ (ap\ (c\_2Eoption\_2Eoption\_\_CASE\ A\_27a\ A\_27b)\ (\mathsf{c\_2Eoption\_2ENONE}\ A\_27a))\ V0v)\ V1f) = V0v))) \wedge (\forall V2x \in A\_27a.(\forall V3v \in A\_27b.(\forall V4f \in (A\_27b^{A\_27a}).((ap\ (ap\ (ap\ (c\_2Eoption\_2Eoption\_\_CASE\ A\_27a\ A\_27b)\ (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V2x))\ V3v)\ V4f) = (ap\ V4f\ V2x)))))) \tag{49}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.(((ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V0x) = (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V1y)) \Leftrightarrow (V0x = V1y)))) \tag{50}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ((\forall V0f \in (A\_27b^{A\_27a}).(\forall V1x \in A\_27a.((ap\ (ap\ (c\_2Eoption\_2EOPTION\_\_MAP\ A\_27a\ A\_27b)\ V0f)\ (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V1x)) = (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27b)\ (ap\ V0f\ V1x))))) \wedge (\forall V2f \in (A\_27b^{A\_27a}).((ap\ (ap\ (c\_2Eoption\_2EOPTION\_\_MAP\ A\_27a\ A\_27b)\ V2f)\ (\mathsf{c\_2Eoption\_2ENONE}\ A\_27a)) = (\mathsf{c\_2Eoption\_2ENONE}\ A\_27b)))) \tag{51}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((ap\ (c\_2Eoption\_2ETHE\ A\_27a)\ (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V0x)) = V0x)) \tag{52}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0n \in ty\_2Enum\_2Enum.(\forall V1ll \in (ty\_2Ellist\_2Ellist\ A\_27a).((ap\ (ap\ (c\_2Ellist\_2ELTAKE\ A\_27a)\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V0n))\ V1ll) = (ap\ (ap\ (ap\ (c\_2Eoption\_2Eoption\_\_CASE\ (ty\_2Elist\_2Elist\ A\_27a)\ (ty\_2Eoption\_2Eoption\ (ty\_2Elist\_2Elist\ A\_27a)))\ (ap\ (ap\ (c\_2Ellist\_2ELTAKE\ A\_27a)\ V0n)\ V1ll))\ (\mathsf{c\_2Eoption\_2ENONE}\ (ty\_2Elist\_2Elist\ A\_27a)))\ (\lambda V2l \in (ty\_2Elist\_2Elist\ A\_27a).\ (ap\ (ap\ (ap\ (c\_2Eoption\_2Eoption\_\_CASE\ A\_27a\ (ty\_2Eoption\_2Eoption\ (ty\_2Elist\_2Elist\ A\_27a)))\ (ap\ (ap\ (c\_2Ellist\_2ELNTH\ A\_27a)\ V0n)\ V1ll))\ (\mathsf{c\_2Eoption\_2ENONE}\ (ty\_2Elist\_2Elist\ A\_27a)))\ (\lambda V3e \in A\_27a.(ap\ (\mathsf{c\_2Eoption\_2ESOME}\ (ty\_2Elist\_2Elist\ A\_27a))\ (ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27a)\ V2l)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V3e)\ (c\_2Elist\_2ENIL\ A\_27a)))))))))))$$