

thm_2Ellist_2ELTL__HD__HD
(TMEjV1xx481vGr6iAt1pKUkatmdBtLC4Mgi)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{1}$$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \tag{2}$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \tag{3}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{4}$$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 4 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap c_2Enum_2EABS_num$

Definition 5 We define $c_2Ecombin_2Eo$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in (A_27b^{A_27c}).\lambda V1g$

Let $ty_2Ellist_2Ellist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Ellist_2Ellist\ A0) \tag{5}$$

Let $ty_2Eoption_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Eoption_2Eoption\ A0) \tag{6}$$

Let $c_2Ellist_2Ellist_abs : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ellist_2Ellist_abs\ A_27a \in ((ty_2Ellist_2Ellist\ A_27a)^{(ty_2Eoption_2Eoption\ A_27a)^{ty_2Enum_2Enum}}) \quad (7)$$

Let $c_2Ellist_2Ellist_rep : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ellist_2Ellist_rep\ A_27a \in (((ty_2Eoption_2Eoption\ A_27a)^{ty_2Enum_2Enum})^{(ty_2Ellist_2Ellist\ A_27a)}) \quad (8)$$

Definition 6 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o\ (p\ P \Rightarrow p\ Q)$ of type ι .

Definition 7 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2.$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \quad (9)$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \quad (10)$$

Definition 8 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap\ (c_2E$

Let $ty_2Eone_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eone_2Eone \quad (11)$$

Let $ty_2Esum_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Esum_2Esum\ A0\ A1) \quad (12)$$

Let $c_2Esum_2EABS_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Esum_2EABS_sum\ A_27a\ A_27b \in ((ty_2Esum_2Esum\ A_27a\ A_27b)^{((2^{A_27b})^{A_27a})^2}) \quad (13)$$

Definition 9 We define c_2Esum_2EINL to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0e \in A_27a.(ap\ (c_2Esum_2EABS_$

Let $c_2Eoption_2Eoption_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Eoption_2Eoption_ABS\ A_27a \in ((ty_2Eoption_2Eoption\ A_27a)^{(ty_2Esum_2Esum\ A_27a\ ty_2Eone_2Eone)}) \quad (14)$$

Definition 10 We define $c_2Eoption_2ESOME$ to be $\lambda A_27a : \iota. \lambda V0x \in A_27a. (ap (c_2Eoption_2Eoption_2ABS A_27a) V0x)$

Definition 11 We define $c_2Emin_2E_40$ to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. P (ap P x)) \text{ then } (the (\lambda x. x \in A) P)$ of type $\iota \Rightarrow \iota$.

Definition 12 We define c_2Eone_2Eone to be $(ap (c_2Emin_2E_40 ty_2Eone_2Eone) (\lambda V0x \in ty_2Eone_2Eone. V0x))$

Definition 13 We define $c_2Ebool_2E_21$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2. V0t))$.

Definition 14 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2. (ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_21) V0t))$

Definition 15 We define c_2Esum_2EINR to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0e \in A_27b. (ap (c_2Esum_2EABS A_27a A_27b) V0e)$

Definition 16 We define $c_2Eoption_2ENONE$ to be $\lambda A_27a : \iota. (ap (c_2Eoption_2Eoption_2ABS A_27a) (the (\lambda x. x \in A) P))$

Let $c_2Enum_2EZERO_2REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_2REP \in \omega \tag{15}$$

Definition 17 We define c_2Enum_2E0 to be $(ap c_2Enum_2EABS_2num c_2Enum_2EZERO_2REP)$.

Let $c_2Eoption_2Eoption_2CASE : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a. nonempty A_27a \Rightarrow \forall A_27b. nonempty A_27b \Rightarrow c_2Eoption_2Eoption_2CASE \\ A_27a A_27b \in (((A_27b^{(A_27b^{A_27a})})^{A_27b})^{(ty_2Eoption_2Eoption A_27a)}) \end{aligned} \tag{16}$$

Definition 18 We define $c_2Ellist_2ELTL_2HD$ to be $\lambda A_27a : \iota. \lambda V0ll \in (ty_2Ellist_2Ellist A_27a). (ap (ap (c_2Emin_2E_40) V0ll) V0ll)$

Definition 19 We define $c_2Ellist_2ELHD$ to be $\lambda A_27a : \iota. \lambda V0ll \in (ty_2Ellist_2Ellist A_27a). (ap (ap (c_2Emin_2E_40) V0ll) V0ll)$

Definition 20 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap V0P (ap (c_2Emin_2E_40) V0P)))$

Definition 21 We define $c_2Ebool_2E_5C_22F$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2. V2t)) V1t2) V0t1))$

Let $c_2Eoption_2EOPTION_2MAP : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a. nonempty A_27a \Rightarrow \forall A_27b. nonempty A_27b \Rightarrow c_2Eoption_2EOPTION_2MAP \\ A_27a A_27b \in (((ty_2Eoption_2Eoption A_27b)^{(ty_2Eoption_2Eoption A_27a)})^{(A_27b^{A_27a})}) \end{aligned} \tag{17}$$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a. nonempty A_27a \Rightarrow \forall A_27b. nonempty A_27b \Rightarrow c_2Epair_2ESND \\ A_27a A_27b \in (A_27b^{(ty_2Epair_2Eprod A_27a A_27b)}) \end{aligned} \tag{18}$$

Assume the following.

$$True \tag{19}$$

Assume the following.

$$\begin{aligned} \forall A_27a. nonempty A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow \\ True)) \end{aligned} \tag{20}$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0opt \in (ty_2Eoption_2Eoption \\ A.27a).((V0opt = (c_2Eoption_2ENONE\ A.27a)) \vee (\exists V1x \in A.27a. \\ (V0opt = (ap\ (c_2Eoption_2ESOME\ A.27a)\ V1x)))))) \end{aligned} \quad (21)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\ (\forall V0v \in A.27b.(\forall V1f \in (A.27b^{A.27a}).((ap\ (ap\ (ap\ (c_2Eoption_2Eoption_CASE \\ A.27a\ A.27b)\ (c_2Eoption_2ENONE\ A.27a))\ V0v)\ V1f) = V0v))) \wedge (\forall V2x \in \\ A.27a.(\forall V3v \in A.27b.(\forall V4f \in (A.27b^{A.27a}).((ap\ (ap \\ (ap\ (c_2Eoption_2Eoption_CASE\ A.27a\ A.27b)\ (ap\ (c_2Eoption_2ESOME \\ A.27a)\ V2x))\ V3v)\ V4f) = (ap\ V4f\ V2x))))))) \end{aligned} \quad (22)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0x \in A.27a.(\forall V1y \in \\ A.27a.(((ap\ (c_2Eoption_2ESOME\ A.27a)\ V0x) = (ap\ (c_2Eoption_2ESOME \\ A.27a)\ V1y)) \Leftrightarrow (V0x = V1y)))) \end{aligned} \quad (23)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\ (\forall V0f \in (A.27b^{A.27a}).(\forall V1x \in A.27a.((ap\ (ap\ (c_2Eoption_2EOPTION_MAP \\ A.27a\ A.27b)\ V0f)\ (ap\ (c_2Eoption_2ESOME\ A.27a)\ V1x)) = (ap\ (c_2Eoption_2ESOME \\ A.27b)\ (ap\ V0f\ V1x)))))) \wedge (\forall V2f \in (A.27b^{A.27a}).((ap\ (ap\ (c_2Eoption_2EOPTION_MAP \\ A.27a\ A.27b)\ V2f)\ (c_2Eoption_2ENONE\ A.27a)) = (c_2Eoption_2ENONE \\ A.27b)))) \end{aligned} \quad (24)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\ \forall V0x \in A.27a.(\forall V1y \in A.27b.((ap\ (c_2Epair_2ESND\ A.27a \\ A.27b)\ (ap\ (ap\ (c_2Epair_2E_2C\ A.27a\ A.27b)\ V0x)\ V1y)) = V1y))) \end{aligned} \quad (25)$$

Theorem 1

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0ll \in (ty_2Ellist_2Ellist \\ A.27a).((ap\ (c_2Ellist_2ELHD\ A.27a)\ V0ll) = (ap\ (ap\ (c_2Eoption_2EOPTION_MAP \\ (ty_2Epair_2Eprod\ (ty_2Ellist_2Ellist\ A.27a)\ A.27a)\ A.27a)\ (\\ c_2Epair_2ESND\ (ty_2Ellist_2Ellist\ A.27a)\ A.27a))\ (ap\ (c_2Ellist_2ELTL_HD \\ A.27a)\ V0ll)))) \end{aligned}$$