

thm_2Ellist_2ELTL__LAPPEND
(TMUhou7CxTJKNnZEUH8cHPTrvg73Jn8CMM1)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 4 We define $c_2Ebool_2E_2F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{1}$$

Let $c_2Earithmetic_2E_2D : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2D \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \tag{2}$$

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p \Rightarrow q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))$

Definition 7 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A \wedge p))$ of type $\iota \Rightarrow \iota$.

Definition 8 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.(ap$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{3}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{4}$$

Definition 9 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 10 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (5)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (6)$$

Definition 11 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum)^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (7)$$

Definition 12 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ (ap\ c_2Earithmetic$

Definition 13 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let $ty_2Eone_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eone_2Eone \quad (8)$$

Let $ty_2Esum_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Esum_2Esum\ A0\ A1) \quad (9)$$

Let $c_2Esum_2EABS_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Esum_2EABS_sum\ A_27a\ A_27b \in ((ty_2Esum_2Esum\ A_27a\ A_27b)^{((2^{A_27b})^{A_27a})^2}) \quad (10)$$

Definition 14 We define c_2Esum_2EINL to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0e \in A_27a.(ap\ (c_2Esum_2EABS$

Let $ty_2Eoption_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Eoption_2Eoption\ A0) \quad (11)$$

Let $c_2Eoption_2Eoption_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Eoption_2Eoption_ABS\ A_27a \in ((ty_2Eoption_2Eoption\ A_27a)^{(ty_2Esum_2Esum\ A_27a\ ty_2Eone_2Eone)}) \quad (12)$$

Definition 15 We define $c_2Eoption_2ESOME$ to be $\lambda A_27a : \iota.\lambda V0x \in A_27a.(ap\ (c_2Eoption_2Eoption$

Definition 16 We define $c_Ebool_2E_3F$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap\ V0P\ (ap\ (c_2Emin_2E_40$

Definition 17 We define c_Eone_2Eone to be $(ap\ (c_2Emin_2E_40\ ty_2Eone_2Eone)\ (\lambda V0x \in ty_2Eone_2E$

Definition 18 We define $c_Ebool_2E_7E$ to be $(\lambda V0t \in 2. (ap\ (ap\ c_2Emin_2E_3D_3D_3E\ V0t)\ c_2Ebool_2E$

Definition 19 We define c_Esum_2EINR to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0e \in A_27b. (ap\ (c_2Esum_2EABS$

Definition 20 We define $c_Eoption_2ENONE$ to be $\lambda A_27a : \iota. (ap\ (c_2Eoption_2Eoption_ABS\ A_27a)\ ($

Definition 21 We define $c_Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in$

Definition 22 We define $c_Ellist_2Elrep_ok$ to be $\lambda A_27a : \iota. (\lambda V0a0 \in ((ty_2Eoption_2Eoption\ A_27a)^{ty$

Let $ty_2Ellist_2Ellist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. nonempty\ A0 \Rightarrow nonempty\ (ty_2Ellist_2Ellist\ A0) \quad (13)$$

Let $c_2Ellist_2Ellist_rep : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow c_2Ellist_2Ellist_rep\ A_27a \in \\ ((ty_2Eoption_2Eoption\ A_27a)^{ty_2Enum_2Enum})^{(ty_2Ellist_2Ellist\ A_27a)} \quad (14)$$

Let $c_2Ellist_2Ellist_abs : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow c_2Ellist_2Ellist_abs\ A_27a \in \\ ((ty_2Ellist_2Ellist\ A_27a)^{(ty_2Eoption_2Eoption\ A_27a)^{ty_2Enum_2Enum}}) \quad (15)$$

Definition 23 We define c_Ellist_2ELHD to be $\lambda A_27a : \iota. \lambda V0ll \in (ty_2Ellist_2Ellist\ A_27a). (ap\ (ap\ (c_2$

Let $c_2Eoption_2Eoption_CASE : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow \forall A_27b. nonempty\ A_27b \Rightarrow c_2Eoption_2Eoption_CASE \\ A_27a\ A_27b \in (((A_27b^{(A_27b^{A_27a})})^{A_27b})^{(ty_2Eoption_2Eoption\ A_27a)}) \quad (16)$$

Definition 24 We define c_Ellist_2ELTL to be $\lambda A_27a : \iota. \lambda V0ll \in (ty_2Ellist_2Ellist\ A_27a). (ap\ (ap\ (ap\$

Definition 25 We define $c_Ellist_2ELCONS$ to be $\lambda A_27a : \iota. \lambda V0h \in A_27a. \lambda V1t \in (ty_2Ellist_2Ellist\ A$

Definition 26 We define c_Ellist_2ELNIL to be $\lambda A_27a : \iota. (ap\ (c_2Ellist_2Ellist_abs\ A_27a)\ (\lambda V0n \in ty$

Let $c_2Ellist_2ELAPPEND : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow c_2Ellist_2ELAPPEND\ A_27a \in (((\\ ty_2Ellist_2Ellist\ A_27a)^{(ty_2Ellist_2Ellist\ A_27a)})^{(ty_2Ellist_2Ellist\ A_27a)}) \quad (17)$$

Let $c_2Eoption_2ETHE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow c_2Eoption_2ETHE\ A_27a \in (A_27a^{(ty_2Eoption_2Eoption\ A_27a)}) \quad (18)$$

Assume the following.

$$True \quad (19)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p \ V0t1) \Rightarrow (p \ V1t2)) \Rightarrow (((p \ V1t2) \Rightarrow (p \ V0t1)) \Rightarrow ((p \ V0t1) \Leftrightarrow (p \ V1t2)))))) \quad (20)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p \ V0t))) \quad (21)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge ((p \ V0t) \Rightarrow False) \Leftrightarrow (\neg (p \ V0t)))))) \quad (22)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (23)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (24)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow True) \Leftrightarrow (p \ V0t)) \wedge (((False \Leftrightarrow (p \ V0t)) \Leftrightarrow (\neg (p \ V0t))) \wedge (((p \ V0t) \Leftrightarrow False) \Leftrightarrow (\neg (p \ V0t)))))) \quad (25)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p \ V0t1) \Rightarrow ((p \ V1t2) \Rightarrow (p \ V2t3))) \Leftrightarrow (((p \ V0t1) \wedge (p \ V1t2)) \Rightarrow (p \ V2t3)))))) \quad (26)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in 2. \\ & (\forall V2x \in A_27a.(\forall V3x_27 \in A_27a.(\forall V4y \in A_27a. \\ & (\forall V5y_27 \in A_27a.(((p \ V0P) \Leftrightarrow (p \ V1Q)) \wedge (((p \ V1Q) \Rightarrow (V2x = V3x_27)) \wedge \\ & ((\neg (p \ V1Q)) \Rightarrow (V4y = V5y_27)))))) \Rightarrow ((ap \ (ap \ (ap \ (c_2Ebool_2ECOND \ A_27a) \\ & V0P) \ V2x) \ V4y) = (ap \ (ap \ (ap \ (c_2Ebool_2ECOND \ A_27a) \ V1Q) \ V3x_27) \\ & \ V5y_27)))))) \quad (27) \end{aligned}$$

Assume the following.

$$2.(((p \ V0x) \Rightarrow (p \ V1y)) \wedge ((p \ V2z) \Rightarrow (p \ V3w))) \Rightarrow (((p \ V0x) \wedge (p \ V2z)) \Rightarrow ((p \ V1y) \wedge (p \ V3w)))) \quad (28)$$

Assume the following.

$$2.(((p \ V0x) \Rightarrow (p \ V1y)) \wedge ((p \ V2z) \Rightarrow (p \ V3w))) \Rightarrow (((p \ V0x) \vee (p \ V2z)) \Rightarrow ((p \ V1y) \vee (p \ V3w)))) \quad (29)$$

Assume the following.

$$\forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0P \in (2^{A.27a}). (\forall V1Q \in (2^{A.27a}). ((\forall V2x \in A.27a. ((p \ (ap \ V0P \ V2x)) \Rightarrow (p \ (ap \ V1Q \ V2x)))) \Rightarrow ((\exists V3x \in A.27a. (p \ (ap \ V0P \ V3x))) \Rightarrow (\exists V4x \in A.27a. (p \ (ap \ V1Q \ V4x))))))) \quad (30)$$

Assume the following.

$$\forall A.27a.nonempty \ A.27a \Rightarrow ((\forall V0t1 \in A.27a. (\forall V1t2 \in A.27a. ((ap \ (ap \ (ap \ (c.2Ebool.2ECOND \ A.27a) \ c.2Ebool.2ET) \ V0t1) \ V1t2) = V0t1))) \wedge (\forall V2t1 \in A.27a. (\forall V3t2 \in A.27a. ((ap \ (ap \ (c.2Ebool.2ECOND \ A.27a) \ c.2Ebool.2EF) \ V2t1) \ V3t2) = V3t2)))) \quad (31)$$

Assume the following.

$$\forall A.27a.nonempty \ A.27a \Rightarrow ((\forall V0a \in (ty.2Ellist.2Ellist \ A.27a). ((ap \ (c.2Ellist.2Ellist_abs \ A.27a) \ (ap \ (c.2Ellist.2Ellist_rep \ A.27a) \ V0a)) = V0a)) \wedge (\forall V1r \in ((ty.2Eoption.2Eoption \ A.27a)^{ty.2Enum.2Enum}). ((p \ (ap \ (c.2Ellist.2Elrep_ok \ A.27a) \ V1r)) \Leftrightarrow ((ap \ (c.2Ellist.2Ellist_rep \ A.27a) \ (ap \ (c.2Ellist.2Ellist_abs \ A.27a) \ V1r)) = V1r)))) \quad (32)$$

Assume the following.

$$\forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0h \in A.27a. (\forall V1t \in (ty.2Ellist.2Ellist \ A.27a). (((ap \ (c.2Ellist.2ELHD \ A.27a) \ (ap \ (ap \ (c.2Ellist.2ELCONS \ A.27a) \ V0h) \ V1t)) = (ap \ (c.2Eoption.2ESOME \ A.27a) \ V0h)) \wedge ((ap \ (c.2Ellist.2ELTL \ A.27a) \ (ap \ (ap \ (c.2Ellist.2ELCONS \ A.27a) \ V0h) \ V1t)) = (ap \ (c.2Eoption.2ESOME \ (ty.2Ellist.2Ellist \ A.27a)) \ V1t)))))) \quad (33)$$

Assume the following.

$$\forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0l \in (ty.2Ellist.2Ellist \ A.27a). ((V0l = (c.2Ellist.2ELNIL \ A.27a)) \vee (\exists V1h \in A.27a. (\exists V2t \in (ty.2Ellist.2Ellist \ A.27a). (V0l = (ap \ (ap \ (c.2Ellist.2ELCONS \ A.27a) \ V1h) \ V2t)))))) \quad (34)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0h \in A_27a. (\forall V1t \in \\ (ty_2Ellist_2Ellist\ A_27a). ((\neg((ap\ (ap\ (c_2Ellist_2ELCONS\ A_27a) \\ V0h)\ V1t) = (c_2Ellist_2ELNIL\ A_27a)))) \wedge (\neg((c_2Ellist_2ELNIL \\ A_27a) = (ap\ (ap\ (c_2Ellist_2ELCONS\ A_27a)\ V0h)\ V1t)))))) \end{aligned} \quad (35)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in (ty_2Ellist_2Ellist \\ A_27a). ((ap\ (ap\ (c_2Ellist_2ELAPPEND\ A_27a)\ (c_2Ellist_2ELNIL \\ A_27a))\ V0x) = V0x)) \wedge (\forall V1h \in A_27a. (\forall V2t \in (ty_2Ellist_2Ellist \\ A_27a). (\forall V3x \in (ty_2Ellist_2Ellist\ A_27a). ((ap\ (ap\ (c_2Ellist_2ELAPPEND \\ A_27a)\ (ap\ (ap\ (c_2Ellist_2ELCONS\ A_27a)\ V1h)\ V2t))\ V3x) = (ap\ (ap \\ (c_2Ellist_2ELCONS\ A_27a)\ V1h)\ (ap\ (ap\ (c_2Ellist_2ELAPPEND\ A_27a) \\ V2t)\ V3x))))))))) \end{aligned} \quad (36)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ (\forall V0v \in A_27b. (\forall V1f \in (A_27b^{A_27a}). ((ap\ (ap\ (ap\ (c_2Eoption_2Eoption_CASE \\ A_27a\ A_27b)\ (c_2Eoption_2ENONE\ A_27a))\ V0v)\ V1f) = V0v))) \wedge (\forall V2x \in \\ A_27a. (\forall V3v \in A_27b. (\forall V4f \in (A_27b^{A_27a}). ((ap\ (ap \\ (ap\ (c_2Eoption_2Eoption_CASE\ A_27a\ A_27b)\ (ap\ (c_2Eoption_2ESOME \\ A_27a)\ V2x))\ V3v)\ V4f) = (ap\ V4f\ V2x))))))))) \end{aligned} \quad (37)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in \\ A_27a. (((ap\ (c_2Eoption_2ESOME\ A_27a)\ V0x) = (ap\ (c_2Eoption_2ESOME \\ A_27a)\ V1y)) \Leftrightarrow (V0x = V1y)))) \end{aligned} \quad (38)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. ((ap\ (c_2Eoption_2ETHE \\ A_27a)\ (ap\ (c_2Eoption_2ESOME\ A_27a)\ V0x)) = V0x)) \end{aligned} \quad (39)$$

Theorem 1

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0l1 \in (ty_2Ellist_2Ellist \\ A_27a). (\forall V1l2 \in (ty_2Ellist_2Ellist\ A_27a). ((ap\ (c_2Ellist_2ELTL \\ A_27a)\ (ap\ (ap\ (c_2Ellist_2ELAPPEND\ A_27a)\ V0l1)\ V1l2)) = (ap\ (ap \\ (ap\ (c_2Ebool_2ECOND\ (ty_2Eoption_2Eoption\ (ty_2Ellist_2Ellist \\ A_27a)))\ (ap\ (ap\ (c_2Emin_2E_3D\ (ty_2Ellist_2Ellist\ A_27a))\ V0l1) \\ (c_2Ellist_2ELNIL\ A_27a)))\ (ap\ (c_2Ellist_2ELTL\ A_27a)\ V1l2)) \\ (ap\ (c_2Eoption_2ESOME\ (ty_2Ellist_2Ellist\ A_27a))\ (ap\ (ap\ (c_2Ellist_2ELAPPEND \\ A_27a)\ (ap\ (c_2Eoption_2ETHE\ (ty_2Ellist_2Ellist\ A_27a))\ (ap \\ (c_2Ellist_2ELTL\ A_27a)\ V0l1)))\ V1l2))))))))) \end{aligned}$$