

thm_2Ellist_2ELTL_LGENLIST
(TMHurMf384UAA1MX7Njn9Ed3NuDXo5QGHB8)

October 26, 2020

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ecombin_2ES$ to be $\lambda A.\lambda a : \iota.\lambda A.\lambda b : \iota.\lambda A.\lambda c : \iota.(\lambda V0f \in ((A_27c^{A_27b})^{A_27a}))$

Definition 3 We define $c_2Ecombin_2EC$ to be $\lambda A.\lambda a : \iota.\lambda A.\lambda b : \iota.\lambda A.\lambda c : \iota.(\lambda V0f \in ((A_27c^{A_27b})^{A_27a}))$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{1}$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{2}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{3}$$

Definition 4 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 5 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \tag{4}$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \tag{5}$$

Definition 6 We define c_2Ebool_2ET to be $(ap\ (ap\ (c_2Emin_2E_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

Definition 7 We define $c_2Ebool_2E_21$ to be $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A_27a})).(ap\ (ap\ (c_2Emin_2E_3D\ (2^{A_27a})))$

Definition 8 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num\ (c_2Enum_2ESUC_REP\ m))$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum)^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (6)$$

Definition 9 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_2Earithmetic_2E_2B))$

Definition 10 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let $ty_2Eoption_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Eoption_2Eoption A0) \quad (7)$$

Let $ty_2Ellist_2Ellist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Ellist_2Ellist A0) \quad (8)$$

Let $c_2Ellist_2Ellist_rep : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Ellist_2Ellist_rep A_27a \in ((ty_2Eoption_2Eoption A_27a)^{ty_2Enum_2Enum})^{(ty_2Ellist_2Ellist A_27a)} \quad (9)$$

Let $c_2Ellist_2Ellist_abs : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Ellist_2Ellist_abs A_27a \in ((ty_2Ellist_2Ellist A_27a)^{(ty_2Eoption_2Eoption A_27a)^{ty_2Enum_2Enum}}) \quad (10)$$

Let $ty_2Eone_2Eone : \iota$ be given. Assume the following.

$$nonempty ty_2Eone_2Eone \quad (11)$$

Definition 11 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 12 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.t1 t2))))$

Let $ty_2Esum_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Esum_2Esum A0 A1) \quad (12)$$

Let $c_2Esum_2EABS_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Esum_2EABS_sum A_27a A_27b \in ((ty_2Esum_2Esum A_27a A_27b)^{((2^{A_27b})^{A_27a})^2}) \quad (13)$$

Definition 13 We define c_2Esum_2EINL to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0e \in A_27a.(ap (c_2Esum_2EABS_sum))$

Let $c_2Eoption_2Eoption_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Eoption_2Eoption_ABS\ A_27a \in ((ty_2Eoption_2Eoption\ A_27a)^{(ty_2Esum_2Esum\ A_27a\ ty_2Eone_2Eone)}) \quad (14)$$

Definition 14 We define $c_2Eoption_2ESOME$ to be $\lambda A_27a : \iota.\lambda V0x \in A_27a.(ap\ (c_2Eoption_2Eoption_ABS\ A_27a)\ (V0x))$

Definition 15 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.\text{if } (\exists x \in A.p\ (ap\ P\ x)) \text{ then } (the\ (\lambda x.x \in A)\ P)$ of type $\iota \Rightarrow \iota$.

Definition 16 We define c_2Eone_2Eone to be $(ap\ (c_2Emin_2E_40\ ty_2Eone_2Eone)\ (\lambda V0x \in ty_2Eone_2Eone.\ V0x))$

Definition 17 We define c_2Ebool_2EF to be $(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 18 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap\ (ap\ c_2Emin_2E_3D_3D_3E\ V0t)\ c_2Ebool_2E_21\ 2))$

Definition 19 We define c_2Esum_2EINR to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0e \in A_27b.(ap\ (c_2Esum_2EABS\ A_27a\ A_27b)\ (V0e))$

Definition 20 We define $c_2Eoption_2ENONE$ to be $\lambda A_27a : \iota.(ap\ (c_2Eoption_2Eoption_ABS\ A_27a)\ (V0))$

Definition 21 We define $c_2Ellist_2ELHD$ to be $\lambda A_27a : \iota.\lambda V0ll \in (ty_2Ellist_2Ellist\ A_27a).(ap\ (ap\ (c_2Eoption_2ENONE\ A_27a)\ V0ll))$

Let $c_2Eoption_2Eoption_CASE : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Eoption_2Eoption_CASE\ A_27a\ A_27b \in (((A_27b^{(A_27b^{A_27a})})^{A_27b})^{(ty_2Eoption_2Eoption\ A_27a)}) \quad (15)$$

Definition 22 We define $c_2Ellist_2ELTL$ to be $\lambda A_27a : \iota.\lambda V0ll \in (ty_2Ellist_2Ellist\ A_27a).(ap\ (ap\ (ap\ (c_2Eoption_2ENONE\ A_27a)\ V0ll)\ V0ll))$

Definition 23 We define $c_2Ecombin_2EK$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0x \in A_27a.(\lambda V1y \in A_27b.V0x))$

Definition 24 We define $c_2Ecombin_2EI$ to be $\lambda A_27a : \iota.(ap\ (ap\ (c_2Ecombin_2ES\ A_27a\ (A_27a^{A_27a})\ A_27a))\ (V0))$

Definition 25 We define $c_2Ecombin_2Eo$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in (A_27b^{A_27c}).\lambda V1g \in (A_27c^{A_27a}).\lambda V2h \in (A_27a^{A_27b}).\lambda V3i \in (A_27b^{A_27c}).\lambda V4j \in (A_27c^{A_27a}).\lambda V5k \in (A_27a^{A_27b}).\lambda V6l \in (A_27b^{A_27c}).\lambda V7m \in (A_27c^{A_27a}).\lambda V8n \in (A_27a^{A_27b}).\lambda V9o \in (A_27b^{A_27c}).\lambda V10p \in (A_27c^{A_27a}).\lambda V11q \in (A_27a^{A_27b}).\lambda V12r \in (A_27b^{A_27c}).\lambda V13s \in (A_27c^{A_27a}).\lambda V14t \in (A_27a^{A_27b}).\lambda V15u \in (A_27b^{A_27c}).\lambda V16v \in (A_27c^{A_27a}).\lambda V17w \in (A_27a^{A_27b}).\lambda V18x \in (A_27b^{A_27c}).\lambda V19y \in (A_27c^{A_27a}).\lambda V20z \in (A_27a^{A_27b}).\lambda V21aa \in (A_27b^{A_27c}).\lambda V22ab \in (A_27c^{A_27a}).\lambda V23ba \in (A_27a^{A_27b}).\lambda V24bb \in (A_27b^{A_27c}).\lambda V25ca \in (A_27c^{A_27a}).\lambda V26cb \in (A_27a^{A_27b}).\lambda V27cb \in (A_27b^{A_27c}).\lambda V28cc \in (A_27c^{A_27a}).\lambda V29ca \in (A_27a^{A_27b}).\lambda V30cb \in (A_27b^{A_27c}).\lambda V31cc \in (A_27c^{A_27a}).\lambda V32ca \in (A_27a^{A_27b}).\lambda V33cb \in (A_27b^{A_27c}).\lambda V34cc \in (A_27c^{A_27a}).\lambda V35ca \in (A_27a^{A_27b}).\lambda V36cb \in (A_27b^{A_27c}).\lambda V37cc \in (A_27c^{A_27a}).\lambda V38ca \in (A_27a^{A_27b}).\lambda V39cb \in (A_27b^{A_27c}).\lambda V40cc \in (A_27c^{A_27a}).\lambda V41ca \in (A_27a^{A_27b}).\lambda V42cb \in (A_27b^{A_27c}).\lambda V43cc \in (A_27c^{A_27a}).\lambda V44ca \in (A_27a^{A_27b}).\lambda V45cb \in (A_27b^{A_27c}).\lambda V46cc \in (A_27c^{A_27a}).\lambda V47ca \in (A_27a^{A_27b}).\lambda V48cb \in (A_27b^{A_27c}).\lambda V49cc \in (A_27c^{A_27a}).\lambda V50ca \in (A_27a^{A_27b}).\lambda V51cb \in (A_27b^{A_27c}).\lambda V52cc \in (A_27c^{A_27a}).\lambda V53ca \in (A_27a^{A_27b}).\lambda V54cb \in (A_27b^{A_27c}).\lambda V55cc \in (A_27c^{A_27a}).\lambda V56ca \in (A_27a^{A_27b}).\lambda V57cb \in (A_27b^{A_27c}).\lambda V58cc \in (A_27c^{A_27a}).\lambda V59ca \in (A_27a^{A_27b}).\lambda V60cb \in (A_27b^{A_27c}).\lambda V61cc \in (A_27c^{A_27a}).\lambda V62ca \in (A_27a^{A_27b}).\lambda V63cb \in (A_27b^{A_27c}).\lambda V64cc \in (A_27c^{A_27a}).\lambda V65ca \in (A_27a^{A_27b}).\lambda V66cb \in (A_27b^{A_27c}).\lambda V67cc \in (A_27c^{A_27a}).\lambda V68ca \in (A_27a^{A_27b}).\lambda V69cb \in (A_27b^{A_27c}).\lambda V70cc \in (A_27c^{A_27a}).\lambda V71ca \in (A_27a^{A_27b}).\lambda V72cb \in (A_27b^{A_27c}).\lambda V73cc \in (A_27c^{A_27a}).\lambda V74ca \in (A_27a^{A_27b}).\lambda V75cb \in (A_27b^{A_27c}).\lambda V76cc \in (A_27c^{A_27a}).\lambda V77ca \in (A_27a^{A_27b}).\lambda V78cb \in (A_27b^{A_27c}).\lambda V79cc \in (A_27c^{A_27a}).\lambda V80ca \in (A_27a^{A_27b}).\lambda V81cb \in (A_27b^{A_27c}).\lambda V82cc \in (A_27c^{A_27a}).\lambda V83ca \in (A_27a^{A_27b}).\lambda V84cb \in (A_27b^{A_27c}).\lambda V85cc \in (A_27c^{A_27a}).\lambda V86ca \in (A_27a^{A_27b}).\lambda V87cb \in (A_27b^{A_27c}).\lambda V88cc \in (A_27c^{A_27a}).\lambda V89ca \in (A_27a^{A_27b}).\lambda V90cb \in (A_27b^{A_27c}).\lambda V91cc \in (A_27c^{A_27a}).\lambda V92ca \in (A_27a^{A_27b}).\lambda V93cb \in (A_27b^{A_27c}).\lambda V94cc \in (A_27c^{A_27a}).\lambda V95ca \in (A_27a^{A_27b}).\lambda V96cb \in (A_27b^{A_27c}).\lambda V97cc \in (A_27c^{A_27a}).\lambda V98ca \in (A_27a^{A_27b}).\lambda V99cb \in (A_27b^{A_27c}).\lambda V100cc \in (A_27c^{A_27a}).$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \quad (16)$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \quad (17)$$

Definition 26 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap\ (c_2Epair_2EABS_prod\ A_27a\ A_27b)\ (V0x\ V1y))$

Definition 27 We define $c_2Ellist_2ELTL_HD$ to be $\lambda A_27a : \iota.\lambda V0ll \in (ty_2Ellist_2Ellist\ A_27a).(ap\ (ap\ (c_2Eoption_2ENONE\ A_27a)\ V0ll))$

Let $c_2Ellist_2ELNTH : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ellist_2ELNTH\ A_27a \in (((ty_2Eoption_2Eoption\ A_27a)^{(ty_2Ellist_2Ellist\ A_27a)})^{ty_2Enum_2Enum}) \quad (18)$$

Definition 28 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A_27a. (\lambda V2t2 \in A_27a. ($

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2ESND\ A_27a\ A_27b \in (A_27b)^{(ty_2Epair_2Eprod\ A_27a\ A_27b)} \quad (19)$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EFST\ A_27a\ A_27b \in (A_27a)^{(ty_2Epair_2Eprod\ A_27a\ A_27b)} \quad (20)$$

Definition 29 We define $c_2Epair_2EUNCURRY$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda A_27c : \iota. \lambda V0f \in ((A_27c)^{A_27a}$

Let $c_2Eoption_2EOPTION_BIND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Eoption_2EOPTION_BIND\ A_27a\ A_27b \in (((ty_2Eoption_2Eoption\ A_27a)^{(ty_2Eoption_2Eoption\ A_27a)^{A_27b}})^{(ty_2Eoption_2Eoption\ A_27b)}) \quad (21)$$

Let $c_2Earithmetic_2EFUNPOW : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Earithmetic_2EFUNPOW\ A_27a \in (((A_27a)^{A_27a})^{ty_2Enum_2Enum})^{(A_27a)^{A_27a}} \quad (22)$$

Let $c_2Eoption_2EOPTION_MAP : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Eoption_2EOPTION_MAP\ A_27a\ A_27b \in (((ty_2Eoption_2Eoption\ A_27b)^{(ty_2Eoption_2Eoption\ A_27a)})^{(A_27b)^{A_27a}}) \quad (23)$$

Definition 30 We define $c_2Ellist_2ELUNFOLD$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0f \in ((ty_2Eoption_2Eoption$

Let $c_2Ellist_2ELGENLIST : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ellist_2ELGENLIST\ A_27a \in (((ty_2Ellist_2Ellist\ A_27a)^{(ty_2Eoption_2Eoption\ ty_2Enum_2Enum)})^{(A_27a)^{ty_2Enum_2Enum}}) \quad (24)$$

Let $c_2Earithmetic_2EEVEN : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EEVEN \in (2^{ty_2Enum_2Enum}) \quad (25)$$

Let $c_2Earithmetic_2EODD : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EODD \in (2^{ty_2Enum_2Enum}) \quad (26)$$

Definition 31 We define $c_Ebool_2E_3F$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap\ V0P\ (ap\ (c_Emin_2E_40$

Definition 32 We define $c_Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum$

Definition 33 We define $c_Earithmetic_2E_3E$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum$

Definition 34 We define $c_Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c_Ebool_2E_21\ 2)\ (\lambda V2t \in$

Definition 35 We define $c_Earithmetic_2E_3E_3D$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum$

Definition 36 We define $c_Eprim_rec_2EPRE$ to be $\lambda V0m \in ty_2Enum_2Enum. (ap\ (ap\ (ap\ (c_Ebool_2E$

Let $c_Earithmetic_2EEXP : \iota$ be given. Assume the following.

$$c_Earithmetic_2EEXP \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (27)$$

Let $c_Earithmetic_2E_2D : \iota$ be given. Assume the following.

$$c_Earithmetic_2E_2D \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (28)$$

Let $c_Earithmetic_2E_2A : \iota$ be given. Assume the following.

$$c_Earithmetic_2E_2A \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (29)$$

Definition 37 We define $c_Enumeral_2EiiSUC$ to be $\lambda V0n \in ty_2Enum_2Enum. (ap\ c_Enum_2ESUC\ (ap$

Definition 38 We define $c_Enumeral_2EiZ$ to be $\lambda V0x \in ty_2Enum_2Enum. V0x$.

Definition 39 We define $c_Earithmetic_2EBIT2$ to be $\lambda V0n \in ty_2Enum_2Enum. (ap\ (ap\ c_Earithmetic$

Definition 40 We define $c_Earithmetic_2E_3C_3D$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum$

Definition 41 We define $c_Epair_2E_23_23$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda A_27c : \iota. \lambda A_27d : \iota. \lambda V0f \in (A_27$

Definition 42 We define $c_Epair_2Epair_CASE$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda A_27c : \iota. \lambda V0p \in (ty_2Epair$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum. ((ap\ (ap\ c_Earithmetic_2E_2B\ V0m)\ c_Enum_2E0) = V0m)) \quad (30)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\ & \quad ((ap\ (ap\ c_Earithmetic_2E_2B\ c_Enum_2E0)\ V0m) = V0m) \wedge (((ap\ (\\ & \quad ap\ c_Earithmetic_2E_2B\ V0m)\ c_Enum_2E0) = V0m) \wedge (((ap\ (ap\ c_Earithmetic_2E_2B \\ & \quad (ap\ c_Enum_2ESUC\ V0m)\ V1n) = (ap\ c_Enum_2ESUC\ (ap\ (ap\ c_Earithmetic_2E_2B \\ & \quad V0m)\ V1n)))) \wedge ((ap\ (ap\ c_Earithmetic_2E_2B\ V0m)\ (ap\ c_Enum_2ESUC \\ & \quad V1n)) = (ap\ c_Enum_2ESUC\ (ap\ (ap\ c_Earithmetic_2E_2B\ V0m)\ V1n))))))))) \end{aligned} \quad (31)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (ap (ap c_2Earithmetic_2E_2B V0m) V1n) = (ap (ap c_2Earithmetic_2E_2B V1n) V0m)))) \quad (32)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\forall V2p \in ty_2Enum_2Enum. ((ap (ap c_2Earithmetic_2E_2B V0m) V1n) V2p) = (ap (ap c_2Earithmetic_2E_2B (ap (ap c_2Earithmetic_2E_2B V0m) V1n)) V2p)))))) \quad (33)$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum. ((\neg (p (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) V0n))) \Leftrightarrow (V0n = c_2Enum_2E0))) \quad (34)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (p (ap (ap c_2Eprim_rec_2E_3C V0m) V1n)) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Enum_2ESUC V0m)) V1n)))))) \quad (35)$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum. (p (ap (ap c_2Earithmetic_2E_3C_3D c_2Enum_2E0) V0n))) \quad (36)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\neg (p (ap (ap c_2Eprim_rec_2E_3C V0m) V1n)))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D V1n) V0m)))))) \quad (37)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum. ((ap c_2Enum_2ESUC V0m) = (ap (ap c_2Earithmetic_2E_2B V0m) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))))) \quad (38)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum. ((ap c_2Eprim_rec_2EPRE V0m) = (ap (ap c_2Earithmetic_2E_2D V0m) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))))) \quad (39)$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& ((ap (ap c_2Earithmetic_2E_2A c_2Enum_2E0) V0m) = c_2Enum_2E0) \wedge \\
& (((ap (ap c_2Earithmetic_2E_2A V0m) c_2Enum_2E0) = c_2Enum_2E0) \wedge \\
& ((ap (ap c_2Earithmetic_2E_2A (ap c_2Earithmetic_2ENUMERAL \\
& (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) V0m) = V0m) \wedge \\
& (((ap (ap c_2Earithmetic_2E_2A V0m) (ap c_2Earithmetic_2ENUMERAL \\
& (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) = V0m) \wedge (\\
& ((ap (ap c_2Earithmetic_2E_2A (ap c_2Enum_2ESUC V0m)) V1n) = (ap \\
& (ap c_2Earithmetic_2E_2B (ap (ap c_2Earithmetic_2E_2A V0m) V1n)) \\
& V1n)) \wedge ((ap (ap c_2Earithmetic_2E_2A V0m) (ap c_2Enum_2ESUC V1n)) = \\
& (ap (ap c_2Earithmetic_2E_2B V0m) (ap (ap c_2Earithmetic_2E_2A \\
& V0m) V1n))))))))))
\end{aligned} \tag{40}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& \forall V2p \in ty_2Enum_2Enum. (((p (ap (ap c_2Earithmetic_2E_3C_3D \\
& V0m) V1n)) \wedge (p (ap (ap c_2Earithmetic_2E_3C_3D V1n) V2p))) \Rightarrow (p (\\
& ap (ap c_2Earithmetic_2E_3C_3D V0m) V2p))))))
\end{aligned} \tag{41}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& (V0m = V1n) \Leftrightarrow ((p (ap (ap c_2Earithmetic_2E_3C_3D V0m) V1n)) \wedge (p (\\
& ap (ap c_2Earithmetic_2E_3C_3D V1n) V0m))))))
\end{aligned} \tag{42}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& \forall V2p \in ty_2Enum_2Enum. ((p (ap (ap c_2Earithmetic_2E_3C_3D \\
& (ap (ap c_2Earithmetic_2E_2B V0m) V1n)) (ap (ap c_2Earithmetic_2E_2B \\
& V0m) V2p))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D V1n) V2p))))))
\end{aligned} \tag{43}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& (\neg(V0m = V1n)) \Leftrightarrow ((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Enum_2ESUC \\
& V0m)) V1n)) \vee (p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Enum_2ESUC \\
& V1n)) V0m))))))
\end{aligned} \tag{44}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. ((ap c_2Enum_2ESUC V0n) = (ap (ap \\
& c_2Earithmetic_2E_2B (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\
& c_2Earithmetic_2EZERO))) V0n)))
\end{aligned} \tag{45}$$

Assume the following.

$$\begin{aligned}
& (\forall V0P \in (2^{ty_2Enum_2Enum}).(\forall V1a \in ty_2Enum_2Enum. \\
& (\forall V2b \in ty_2Enum_2Enum.((p (ap V0P (ap (ap c_2Earithmic_2E_2D \\
& V1a) V2b))) \Leftrightarrow (\forall V3d \in ty_2Enum_2Enum.(((V2b = (ap (ap c_2Earithmic_2E_2B \\
& V1a) V3d)) \Rightarrow (p (ap V0P c_2Enum_2E0))) \wedge ((V1a = (ap (ap c_2Earithmic_2E_2B \\
& V2b) V3d)) \Rightarrow (p (ap V0P V3d))))))))))
\end{aligned} \tag{46}$$

Assume the following.

$$True \tag{47}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p \\
V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \tag{48}$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p V0t))) \tag{49}$$

Assume the following.

$$(\forall V0t \in 2.((p V0t) \vee \neg(p V0t))) \tag{50}$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in \\
A_27a.(p V0t)) \Leftrightarrow (p V0t))) \tag{51}$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0t \in 2.((\exists V1x \in \\
A_27a.(p V0t)) \Leftrightarrow (p V0t))) \tag{52}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \wedge \\
((p V1t2) \wedge (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \wedge (p V2t3)))))) \tag{53}$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow \\
(p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge \\
(((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \tag{54}$$

Assume the following.

$$(\forall V0t \in 2.(((True \vee (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \vee True) \Leftrightarrow True) \wedge \\
(((False \vee (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee False) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee \\
(p V0t)) \Leftrightarrow (p V0t)))))) \tag{55}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (56)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t)) \wedge ((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (57)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (58)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (59)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow \forall A_27b.nonempty \ A_27b \Rightarrow (\forall V0f \in (A_27b^{A_27a}).(\forall V1g \in (A_27b^{A_27a}).((V0f = V1g) \Leftrightarrow (\forall V2x \in A_27a.((ap \ V0f \ V2x) = (ap \ V1g \ V2x)))))) \quad (60)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (61)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0t1 \in A_27a.(\forall V1t2 \in A_27a.(((ap \ (ap \ (ap \ (c_2Ebool_2ECOND \ A_27a) \ c_2Ebool_2ET) \ V0t1) \ V1t2) = V0t1) \wedge ((ap \ (ap \ (ap \ (c_2Ebool_2ECOND \ A_27a) \ c_2Ebool_2EF) \ V0t1) \ V1t2) = V1t2)))))) \quad (62)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (2^{A_27a}).((\forall V2x \in A_27a.((p \ V0P) \Rightarrow (p \ (ap \ V1Q \ V2x)))) \Leftrightarrow ((p \ V0P) \Rightarrow (\forall V3x \in A_27a.(p \ (ap \ V1Q \ V3x))))))) \quad (63)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p \ V0A) \vee (p \ V1B) \vee (p \ V2C)) \Leftrightarrow (((p \ V0A) \vee (p \ V1B)) \vee (p \ V2C)))))) \quad (64)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \wedge (p V1B))) \Leftrightarrow ((\neg(p V0A)) \vee (\neg(p V1B)))) \wedge ((\neg((p V0A) \vee (p V1B))) \Leftrightarrow ((\neg(p V0A)) \wedge (\neg(p V1B)))))))) \quad (65)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((p V0A) \Rightarrow (p V1B)) \Leftrightarrow ((\neg(p V0A)) \vee (p V1B)))) \quad (66)$$

Assume the following.

$$(\forall V0t \in 2. (((p V0t) \Rightarrow \text{False}) \Leftrightarrow ((p V0t) \Leftrightarrow \text{False}))) \quad (67)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (68)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Leftrightarrow (p V1t2)) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \vee ((\neg(p V0t1)) \wedge (\neg(p V1t2)))))) \quad (69)$$

Assume the following.

$$(\forall V0x \in 2. (\forall V1x_{.27} \in 2. (\forall V2y \in 2. (\forall V3y_{.27} \in 2. (((((p V0x) \Leftrightarrow (p V1x_{.27})) \wedge ((p V1x_{.27}) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_{.27})))) \Rightarrow ((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_{.27}) \Rightarrow (p V3y_{.27})))))) \quad (70)$$

Assume the following.

$$\begin{aligned} & \forall A_{.27a}. \text{nonempty } A_{.27a} \Rightarrow (\forall V0P \in 2. (\forall V1Q \in 2. \\ & (\forall V2x \in A_{.27a}. (\forall V3x_{.27} \in A_{.27a}. (\forall V4y \in A_{.27a}. \\ & (\forall V5y_{.27} \in A_{.27a}. (((p V0P) \Leftrightarrow (p V1Q)) \wedge ((p V1Q) \Rightarrow (V2x = V3x_{.27})) \wedge \\ & ((\neg(p V1Q)) \Rightarrow (V4y = V5y_{.27})))) \Rightarrow ((ap (ap (ap (c_{.2Ebool_2ECOND } A_{.27a}) \\ & V0P) V2x) V4y) = (ap (ap (ap (c_{.2Ebool_2ECOND } A_{.27a}) V1Q) V3x_{.27} \\ & V5y_{.27})))))))))) \quad (71) \end{aligned}$$

Assume the following.

$$\forall A_{.27a}. \text{nonempty } A_{.27a} \Rightarrow (\forall V0P \in (2^{A_{.27a}}). (\forall V1a \in A_{.27a}. ((\exists V2x \in A_{.27a}. ((V2x = V1a) \wedge (p (ap V0P V2x)))) \Leftrightarrow (p (ap V0P V1a)))) \quad (72)$$

Assume the following.

$$\begin{aligned} & \forall A_{.27a}. \text{nonempty } A_{.27a} \Rightarrow ((\forall V0t1 \in A_{.27a}. (\forall V1t2 \in \\ & A_{.27a}. ((ap (ap (ap (c_{.2Ebool_2ECOND } A_{.27a}) c_{.2Ebool_2ET} V0t1) \\ & V1t2) = V0t1))) \wedge (\forall V2t1 \in A_{.27a}. (\forall V3t2 \in A_{.27a}. ((ap \\ & (ap (ap (c_{.2Ebool_2ECOND } A_{.27a}) c_{.2Ebool_2EF} V2t1) V3t2) = V3t2)))))) \quad (73) \end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow \forall A.27c. \\
& nonempty\ A.27c \Rightarrow (\forall V0f \in (A.27b^{A.27a}).(\forall V1g \in (A.27a^{A.27c}). \\
& (\forall V2x \in A.27c.((ap\ (ap\ (ap\ (c.2Ecombin_2Eo\ A.27c\ A.27b\ A.27a) \\
& V0f)\ V1g)\ V2x) = (ap\ V0f\ (ap\ V1g\ V2x))))))
\end{aligned} \tag{74}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0ll \in (ty_2Ellist_2Ellist \\
& A.27a).((ap\ (c.2Ellist_2ELTL\ A.27a)\ V0ll) = (ap\ (ap\ (c.2Eoption_2EOPTION_MAP \\
& (ty_2Epair_2Eprod\ (ty_2Ellist_2Ellist\ A.27a)\ A.27a)\ (ty_2Ellist_2Ellist \\
& A.27a))\ (c.2Epair_2EFST\ (ty_2Ellist_2Ellist\ A.27a)\ A.27a))\ (\\
& ap\ (c.2Ellist_2ELTL_HD\ A.27a)\ V0ll))))
\end{aligned} \tag{75}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0ll1 \in (ty_2Ellist_2Ellist \\
& A.27a).(\forall V1ll2 \in (ty_2Ellist_2Ellist\ A.27a).((V0ll1 = \\
& V1ll2) \Leftrightarrow (\forall V2n \in ty_2Enum_2Enum.((ap\ (ap\ (c.2Ellist_2ELNTH \\
& A.27a)\ V2n)\ V0ll1) = (ap\ (ap\ (c.2Ellist_2ELNTH\ A.27a)\ V2n)\ V1ll2))))))
\end{aligned} \tag{76}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\
& \forall V0f \in ((ty_2Eoption_2Eoption\ (ty_2Epair_2Eprod\ A.27b \\
& A.27a))^{A.27b}).(\forall V1x \in A.27b.((ap\ (c.2Ellist_2ELTL_HD \\
& A.27a)\ (ap\ (ap\ (c.2Ellist_2ELUNFOLD\ A.27a\ A.27b)\ V0f)\ V1x)) = (ap \\
& (ap\ (c.2Eoption_2EOPTION_MAP\ (ty_2Epair_2Eprod\ A.27b\ A.27a) \\
& (ty_2Epair_2Eprod\ (ty_2Ellist_2Ellist\ A.27a)\ A.27a))\ (ap\ (ap \\
& (c.2Epair_2E.23.23\ A.27b\ A.27a)\ (ty_2Ellist_2Ellist\ A.27a)\ A.27a) \\
& (ap\ (c.2Ellist_2ELUNFOLD\ A.27a\ A.27b)\ V0f))\ (c.2Ecombin_2EI\ A.27a))) \\
& (ap\ V0f\ V1x))))
\end{aligned} \tag{77}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0f \in ((ty_2Eoption_2Eoption\ (ty_2Epair_2Eprod\ A_27b \\
& \quad \quad A_27a))^{A_27b}).(\forall V1x \in A_27b.(\forall V2n \in ty_2Enum_2Enum. \\
& \quad ((ap\ (ap\ (c_2Ellist_2ELNTH\ A_27a)\ c_2Enum_2E0)\ (ap\ (ap\ (c_2Ellist_2ELUNFOLD \\
& \quad A_27a\ A_27b)\ V0f)\ V1x)) = (ap\ (ap\ (c_2Eoption_2EOPTION_MAP\ (ty_2Epair_2Eprod \\
& \quad \quad A_27b\ A_27a)\ A_27a)\ (c_2Epair_2ESND\ A_27b\ A_27a))\ (ap\ V0f\ V1x)))) \wedge \\
& \quad ((ap\ (ap\ (c_2Ellist_2ELNTH\ A_27a)\ (ap\ c_2Enum_2ESUC\ V2n))\ (ap\ (\\
& \quad ap\ (c_2Ellist_2ELUNFOLD\ A_27a\ A_27b)\ V0f)\ V1x)) = (ap\ (ap\ (ap\ (c_2Eoption_2Eoption_CASE \\
& \quad \quad (ty_2Epair_2Eprod\ A_27b\ A_27a)\ (ty_2Eoption_2Eoption\ A_27a)) \\
& \quad (ap\ V0f\ V1x))\ (c_2Eoption_2ENONE\ A_27a))\ (\lambda V3v \in (ty_2Epair_2Eprod \\
& \quad \quad A_27b\ A_27a).(ap\ (ap\ (c_2Epair_2Epair_CASE\ (ty_2Eoption_2Eoption \\
& \quad \quad \quad A_27a)\ A_27b\ A_27a)\ V3v)\ (\lambda V4tx \in A_27b.(\lambda V5hx \in A_27a.(ap \\
& \quad \quad \quad (ap\ (c_2Ellist_2ELNTH\ A_27a)\ V2n)\ (ap\ (ap\ (c_2Ellist_2ELUNFOLD \\
& \quad \quad \quad \quad A_27a\ A_27b)\ V0f)\ V4tx)))))))))) \\
& \hspace{15em} (78)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow ((\forall V0f \in (A_27a^{ty_2Enum_2Enum}). \\
& \quad ((ap\ (ap\ (c_2Ellist_2ELGENLIST\ A_27a)\ V0f)\ (c_2Eoption_2ENONE \\
& \quad ty_2Enum_2Enum)) = (ap\ (ap\ (c_2Ellist_2ELUNFOLD\ A_27a\ ty_2Enum_2Enum) \\
& \quad (\lambda V1n \in ty_2Enum_2Enum.(ap\ (c_2Eoption_2ESOME\ (ty_2Epair_2Eprod \\
& \quad \quad ty_2Enum_2Enum\ A_27a))\ (ap\ (ap\ (c_2Epair_2E_2C\ ty_2Enum_2Enum \\
& \quad \quad A_27a)\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V1n)\ (ap\ c_2Earithmetic_2ENUMERAL \\
& \quad \quad (ap\ c_2Earithmetic_2EBIT1\ c_2Earithmetic_2EZERO))))\ (ap\ V0f \\
& \quad \quad V1n))))\ c_2Enum_2E0))) \wedge (\forall V2f \in (A_27a^{ty_2Enum_2Enum}). \\
& \quad (\forall V3lim \in ty_2Enum_2Enum.((ap\ (ap\ (c_2Ellist_2ELGENLIST \\
& \quad \quad A_27a)\ V2f)\ (ap\ (c_2Eoption_2ESOME\ ty_2Enum_2Enum)\ V3lim)) = (\\
& \quad ap\ (ap\ (c_2Ellist_2ELUNFOLD\ A_27a\ ty_2Enum_2Enum)\ (\lambda V4n \in ty_2Enum_2Enum. \\
& \quad (ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ (ty_2Eoption_2Eoption\ (ty_2Epair_2Eprod \\
& \quad \quad ty_2Enum_2Enum\ A_27a))\ (ap\ (ap\ c_2Eprim_rec_2E_3C\ V4n)\ V3lim)) \\
& \quad (ap\ (c_2Eoption_2ESOME\ (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ A_27a)) \\
& \quad (ap\ (ap\ (c_2Epair_2E_2C\ ty_2Enum_2Enum\ A_27a)\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& \quad \quad V4n)\ (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT1 \\
& \quad \quad c_2Earithmetic_2EZERO))))\ (ap\ V2f\ V4n))))\ (c_2Eoption_2ENONE \\
& \quad (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ A_27a))))\ c_2Enum_2E0)))))) \\
& \hspace{15em} (79)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0P \in (2^{ty_2Enum_2Enum}).(((p\ (ap\ V0P\ c_2Enum_2E0)) \wedge \\
& (\forall V1n \in ty_2Enum_2Enum.((p\ (ap\ V0P\ V1n)) \Rightarrow (p\ (ap\ V0P\ (ap\ c_2Enum_2ESUC \\
& \quad V1n)))))) \Rightarrow (\forall V2n \in ty_2Enum_2Enum.(p\ (ap\ V0P\ V2n)))))) \\
& \hspace{15em} (80)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2B \\
& \quad c_2Enum_2E0) V0n) = V0n)) \wedge ((\forall V1n \in ty_2Enum_2Enum.((ap \\
& \quad (ap c_2Earithmetic_2E_2B V1n) c_2Enum_2E0) = V1n)) \wedge ((\forall V2n \in \\
& \quad ty_2Enum_2Enum.(\forall V3m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2B \\
& \quad (ap c_2Earithmetic_2ENUMERAL V2n)) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V3m)) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Enumeral_2EiZ (ap \\
& \quad (ap c_2Earithmetic_2E_2B V2n) V3m)))))) \wedge ((\forall V4n \in ty_2Enum_2Enum. \\
& \quad ((ap (ap c_2Earithmetic_2E_2A c_2Enum_2E0) V4n) = c_2Enum_2E0)) \wedge \\
& \quad ((\forall V5n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2A \\
& \quad V5n) c_2Enum_2E0) = c_2Enum_2E0)) \wedge ((\forall V6n \in ty_2Enum_2Enum. \\
& \quad (\forall V7m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2A (\\
& \quad ap c_2Earithmetic_2ENUMERAL V6n)) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V7m)) = (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2E_2A \\
& \quad V6n) V7m)))))) \wedge ((\forall V8n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2D \\
& \quad c_2Enum_2E0) V8n) = c_2Enum_2E0)) \wedge ((\forall V9n \in ty_2Enum_2Enum. \\
& \quad ((ap (ap c_2Earithmetic_2E_2D V9n) c_2Enum_2E0) = V9n)) \wedge ((\forall V10n \in \\
& \quad ty_2Enum_2Enum.(\forall V11m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2D \\
& \quad (ap c_2Earithmetic_2ENUMERAL V10n)) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V11m)) = (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2E_2D \\
& \quad V10n) V11m)))))) \wedge ((\forall V12n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2EEXP \\
& \quad c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\
& \quad V12n))) = c_2Enum_2E0)) \wedge ((\forall V13n \in ty_2Enum_2Enum.((ap \\
& \quad (ap c_2Earithmetic_2EEXP c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap c_2Earithmetic_2EBIT2 V13n))) = c_2Enum_2E0)) \wedge ((\forall V14n \in \\
& \quad ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2EEXP V14n) c_2Enum_2E0) = \\
& \quad (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))))) \wedge \\
& \quad ((\forall V15n \in ty_2Enum_2Enum.(\forall V16m \in ty_2Enum_2Enum. \\
& \quad ((ap (ap c_2Earithmetic_2EEXP (ap c_2Earithmetic_2ENUMERAL V15n)) \\
& \quad (ap c_2Earithmetic_2ENUMERAL V16m)) = (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap (ap c_2Earithmetic_2EEXP V15n) V16m)))))) \wedge ((ap c_2Enum_2ESUC \\
& \quad c_2Enum_2E0) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\
& \quad c_2Earithmetic_2EZERO))) \wedge ((\forall V17n \in ty_2Enum_2Enum. (\\
& \quad (ap c_2Enum_2ESUC (ap c_2Earithmetic_2ENUMERAL V17n)) = (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap c_2Enum_2ESUC V17n)))))) \wedge ((ap c_2Eprim_rec_2EPRE c_2Enum_2E0) = \\
& \quad c_2Enum_2E0) \wedge ((\forall V18n \in ty_2Enum_2Enum.((ap c_2Eprim_rec_2EPRE \\
& \quad (ap c_2Earithmetic_2ENUMERAL V18n)) = (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap c_2Eprim_rec_2EPRE V18n)))))) \wedge ((\forall V19n \in ty_2Enum_2Enum. \\
& \quad (((ap c_2Earithmetic_2ENUMERAL V19n) = c_2Enum_2E0) \Leftrightarrow (V19n = c_2Earithmetic_2EZERO))) \wedge \\
& \quad ((\forall V20n \in ty_2Enum_2Enum.((c_2Enum_2E0 = (ap c_2Earithmetic_2ENUMERAL \\
& \quad V20n)) \Leftrightarrow (V20n = c_2Earithmetic_2EZERO))) \wedge ((\forall V21n \in ty_2Enum_2Enum. \\
& \quad (\forall V22m \in ty_2Enum_2Enum.(((ap c_2Earithmetic_2ENUMERAL \\
& \quad V21n) = (ap c_2Earithmetic_2ENUMERAL V22m)) \Leftrightarrow (V21n = V22m)))))) \wedge \\
& \quad ((\forall V23n \in ty_2Enum_2Enum.((p (ap (ap c_2Eprim_rec_2E_3C \\
& \quad V23n) c_2Enum_2E0)) \Leftrightarrow False)) \wedge ((\forall V24n \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V24n))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmetic_2EZERO) \\
& \quad V24n)))))) \wedge ((\forall V25n \in ty_2Enum_2Enum.(\forall V26m \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Eprim_rec_2E_3C (ap c_2Earithmetic_2ENUMERAL \\
& \quad V25n)) (ap c_2Earithmetic_2ENUMERAL V26m))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C \\
& \quad V25n) V26m)))))) \wedge ((\forall V27n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmetic_2E_3E \\
& \quad c_2Enum_2E0) V27n)) \Leftrightarrow False)) \wedge ((\forall V28n \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Earithmetic_2E_3E (ap c_2Earithmetic_2ENUMERAL \\
& \quad V28n)) c_2Enum_2E0)) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmetic_2EZERO) \\
& \quad V28n)))))) \wedge ((\forall V29n \in ty_2Enum_2Enum.(\forall V30m \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Earithmetic_2E_3E (ap c_2Earithmetic_2ENUMERAL \\
& \quad V29n)) (ap c_2Earithmetic_2ENUMERAL V30m))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C \\
& \quad V30m) V29n)))))) \wedge ((\forall V31n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmetic_2E_3C_3D \\
& \quad c_2Enum_2E0) V31n)) \Leftrightarrow True)) \wedge ((\forall V32n \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2ENUMERAL
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\\
& ((ap\ c_2Enumeral_2EiZ\ (ap\ (ap\ c_2Earithmetic_2E_2B\ c_2Earithmetic_2EZERO) \\
& V0n)) = V0n) \wedge (((ap\ c_2Enumeral_2EiZ\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& V0n)\ c_2Earithmetic_2EZERO)) = V0n) \wedge (((ap\ c_2Enumeral_2EiZ\ (\\
& ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT1\ V0n))\ (\\
& ap\ c_2Earithmetic_2EBIT1\ V1m))) = (ap\ c_2Earithmetic_2EBIT2\ (\\
& ap\ c_2Enumeral_2EiZ\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge \\
& (((ap\ c_2Enumeral_2EiZ\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT1 \\
& V0n))\ (ap\ c_2Earithmetic_2EBIT2\ V1m))) = (ap\ c_2Earithmetic_2EBIT1 \\
& (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge (\\
& ((ap\ c_2Enumeral_2EiZ\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT2 \\
& V0n))\ (ap\ c_2Earithmetic_2EBIT1\ V1m))) = (ap\ c_2Earithmetic_2EBIT1 \\
& (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge (\\
& ((ap\ c_2Enumeral_2EiZ\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT2 \\
& V0n))\ (ap\ c_2Earithmetic_2EBIT2\ V1m))) = (ap\ c_2Earithmetic_2EBIT2 \\
& (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge (\\
& ((ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ c_2Earithmetic_2EZERO) \\
& V0n)) = (ap\ c_2Enum_2ESUC\ V0n)) \wedge (((ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& V0n)\ c_2Earithmetic_2EZERO)) = (ap\ c_2Enum_2ESUC\ V0n)) \wedge (((ap \\
& c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT1 \\
& V0n))\ (ap\ c_2Earithmetic_2EBIT1\ V1m))) = (ap\ c_2Earithmetic_2EBIT1 \\
& (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge (\\
& ((ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT1 \\
& V0n))\ (ap\ c_2Earithmetic_2EBIT2\ V1m))) = (ap\ c_2Earithmetic_2EBIT2 \\
& (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge (\\
& ((ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT2 \\
& V0n))\ (ap\ c_2Earithmetic_2EBIT1\ V1m))) = (ap\ c_2Earithmetic_2EBIT2 \\
& (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge (\\
& ((ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT2 \\
& V0n))\ (ap\ c_2Earithmetic_2EBIT2\ V1m))) = (ap\ c_2Earithmetic_2EBIT1 \\
& (ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge \\
& (((ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ c_2Earithmetic_2EZERO) \\
& V0n)) = (ap\ c_2Enumeral_2EiiSUC\ V0n)) \wedge (((ap\ c_2Enumeral_2EiiSUC \\
& (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ c_2Earithmetic_2EZERO)) = (\\
& ap\ c_2Enumeral_2EiiSUC\ V0n)) \wedge (((ap\ c_2Enumeral_2EiiSUC\ (ap\ (\\
& ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT1\ V0n))\ (ap\ c_2Earithmetic_2EBIT1 \\
& V1m))) = (ap\ c_2Earithmetic_2EBIT2\ (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& V0n)\ V1m)))) \wedge (((ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& (ap\ c_2Earithmetic_2EBIT1\ V0n))\ (ap\ c_2Earithmetic_2EBIT2\ V1m))) = \\
& (ap\ c_2Earithmetic_2EBIT1\ (ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& V0n)\ V1m)))) \wedge (((ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& (ap\ c_2Earithmetic_2EBIT2\ V0n))\ (ap\ c_2Earithmetic_2EBIT1\ V1m))) = \\
& (ap\ c_2Earithmetic_2EBIT1\ (ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& V0n)\ V1m)))) \wedge (((ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& (ap\ c_2Earithmetic_2EBIT2\ V0n))\ (ap\ c_2Earithmetic_2EBIT2\ V1m))) = \\
& (ap\ c_2Earithmetic_2EBIT2\ (ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& V0n)\ V1m))))))))))))))))))))))))))))))
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\\
& ((p (ap (ap c_2Earithmic_2E_3C_3D c_2Earithmic_2EZERO) V0n)) \Leftrightarrow \\
& True) \wedge (((p (ap (ap c_2Earithmic_2E_3C_3D (ap c_2Earithmic_2EBIT1 \\
& V0n)) c_2Earithmic_2EZERO)) \Leftrightarrow False) \wedge (((p (ap (ap c_2Earithmic_2E_3C_3D \\
& (ap c_2Earithmic_2EBIT2 V0n)) c_2Earithmic_2EZERO)) \Leftrightarrow False) \wedge \\
& (((p (ap (ap c_2Earithmic_2E_3C_3D (ap c_2Earithmic_2EBIT1 \\
& V0n)) (ap c_2Earithmic_2EBIT1 V1m))) \Leftrightarrow (p (ap (ap c_2Earithmic_2E_3C_3D \\
& V0n) V1m))) \wedge (((p (ap (ap c_2Earithmic_2E_3C_3D (ap c_2Earithmic_2EBIT1 \\
& V0n)) (ap c_2Earithmic_2EBIT2 V1m))) \Leftrightarrow (p (ap (ap c_2Earithmic_2E_3C_3D \\
& V0n) V1m))) \wedge (((p (ap (ap c_2Earithmic_2E_3C_3D (ap c_2Earithmic_2EBIT2 \\
& V0n)) (ap c_2Earithmic_2EBIT1 V1m))) \Leftrightarrow (\neg (p (ap (ap c_2Earithmic_2E_3C_3D \\
& V1m) V0n)))) \wedge ((p (ap (ap c_2Earithmic_2E_3C_3D (ap c_2Earithmic_2EBIT2 \\
& V0n)) (ap c_2Earithmic_2EBIT2 V1m))) \Leftrightarrow (p (ap (ap c_2Earithmic_2E_3C_3D \\
& V0n) V1m))))))))))
\end{aligned} \tag{83}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty A_27a \Rightarrow (\forall V0opt \in (ty_2Eoption_2Eoption \\
& A_27a). ((V0opt = (c_2Eoption_2ENONE A_27a)) \vee (\exists V1x \in A_27a. \\
& (V0opt = (ap (c_2Eoption_2ESOME A_27a) V1x))))))
\end{aligned} \tag{84}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow (\\
& (\forall V0v \in A_27b. (\forall V1f \in (A_27b^{A_27a}). ((ap (ap (ap (c_2Eoption_2Eoption_2CASE \\
& A_27a A_27b) (c_2Eoption_2ENONE A_27a)) V0v) V1f) = V0v))) \wedge (\forall V2x \in \\
& A_27a. (\forall V3v \in A_27b. (\forall V4f \in (A_27b^{A_27a}). ((ap (ap \\
& (ap (c_2Eoption_2Eoption_2CASE A_27a A_27b) (ap (c_2Eoption_2ESOME \\
& A_27a) V2x)) V3v) V4f) = (ap V4f V2x))))))
\end{aligned} \tag{85}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in \\
& A_27a. (((ap (c_2Eoption_2ESOME A_27a) V0x) = (ap (c_2Eoption_2ESOME \\
& A_27a) V1y)) \Leftrightarrow (V0x = V1y))))
\end{aligned} \tag{86}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a. (\neg ((c_2Eoption_2ENONE \\
& A_27a) = (ap (c_2Eoption_2ESOME A_27a) V0x))))
\end{aligned} \tag{87}$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\ & (\forall V0f \in (A.27b^{A.27a}).(\forall V1x \in A.27a.((ap\ (ap\ (c.2Eoption.2EOPTION_MAP \\ & A.27a\ A.27b)\ V0f)\ (ap\ (c.2Eoption.2ESOME\ A.27a)\ V1x)) = (ap\ (c.2Eoption.2ESOME \\ & A.27b)\ (ap\ V0f\ V1x)))))) \wedge (\forall V2f \in (A.27b^{A.27a}).((ap\ (ap\ (c.2Eoption.2EOPTION_MAP \\ & A.27a\ A.27b)\ V2f)\ (c.2Eoption.2ENONE\ A.27a)) = (c.2Eoption.2ENONE \\ & A.27b)))))) \end{aligned} \quad (88)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\ & \forall V0f \in (A.27b^{A.27a}).(\forall V1x \in (ty.2Eoption.2Eoption \\ & A.27a).(\forall V2y \in A.27b.(((ap\ (ap\ (c.2Eoption.2EOPTION_MAP \\ & A.27a\ A.27b)\ V0f)\ V1x) = (ap\ (c.2Eoption.2ESOME\ A.27b)\ V2y))) \Leftrightarrow (\exists V3z \in \\ & A.27a.((V1x = (ap\ (c.2Eoption.2ESOME\ A.27a)\ V3z)) \wedge (V2y = (ap\ V0f \\ & V3z)))))))))) \end{aligned} \quad (89)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\ & \forall V0f \in (A.27a^{A.27b}).(\forall V1x \in (ty.2Eoption.2Eoption \\ & A.27b).(((ap\ (ap\ (c.2Eoption.2EOPTION_MAP\ A.27b\ A.27a)\ V0f) \\ & V1x) = (c.2Eoption.2ENONE\ A.27a)) \Leftrightarrow (V1x = (c.2Eoption.2ENONE\ A.27b)))) \wedge \\ & (((c.2Eoption.2ENONE\ A.27a) = (ap\ (ap\ (c.2Eoption.2EOPTION_MAP \\ & A.27b\ A.27a)\ V0f)\ V1x)) \Leftrightarrow (V1x = (c.2Eoption.2ENONE\ A.27b)))))) \end{aligned} \quad (90)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow \forall A.27c. \\ & nonempty\ A.27c \Rightarrow (\forall V0f \in (A.27b^{A.27c}).(\forall V1g \in (A.27c^{A.27a}). \\ & (\forall V2x \in (ty.2Eoption.2Eoption\ A.27a).((ap\ (ap\ (c.2Eoption.2EOPTION_MAP \\ & A.27c\ A.27b)\ V0f)\ (ap\ (ap\ (c.2Eoption.2EOPTION_MAP\ A.27a\ A.27c) \\ & V1g)\ V2x)) = (ap\ (ap\ (c.2Eoption.2EOPTION_MAP\ A.27a\ A.27b)\ (ap \\ & (ap\ (c.2Ecombin.2Eo\ A.27a\ A.27b\ A.27c)\ V0f)\ V1g))\ V2x)))))) \end{aligned} \quad (91)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\ & \forall V0x \in A.27a.(\forall V1y \in A.27b.((ap\ (c.2Epair.2EFST\ A.27a \\ & A.27b)\ (ap\ (ap\ (c.2Epair.2E_2C\ A.27a\ A.27b)\ V0x)\ V1y)) = V0x))) \end{aligned} \quad (92)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\ & \forall V0x \in A.27a.(\forall V1y \in A.27b.((ap\ (c.2Epair.2ESND\ A.27a \\ & A.27b)\ (ap\ (ap\ (c.2Epair.2E_2C\ A.27a\ A.27b)\ V0x)\ V1y)) = V1y))) \end{aligned} \quad (93)$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\
& nonempty\ A_27c \Rightarrow \forall A_27d.nonempty\ A_27d \Rightarrow (\forall V0p \in (ty_2Epair_2Eprod \\
& \quad A_27a\ A_27b).(\forall V1f \in (A_27c^{A_27a}).(\forall V2g \in (A_27d^{A_27b}). \\
& \quad ((ap\ (c_2Epair_2EFST\ A_27c\ A_27d)\ (ap\ (ap\ (ap\ (c_2Epair_2E_23_23 \\
& \quad A_27a\ A_27b\ A_27c\ A_27d)\ V1f)\ V2g)\ V0p))) = (ap\ V1f\ (ap\ (c_2Epair_2EFST \\
& \quad A_27a\ A_27b)\ V0p))))))
\end{aligned} \tag{94}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\
& nonempty\ A_27c \Rightarrow (\forall V0x \in A_27b.(\forall V1y \in A_27c.(\forall V2f \in \\
& \quad ((A_27a^{A_27c})^{A_27b}).((ap\ (ap\ (c_2Epair_2Epair_CASE\ A_27a\ A_27b \\
& \quad A_27c)\ (ap\ (ap\ (c_2Epair_2E_2C\ A_27b\ A_27c)\ V0x)\ V1y))\ V2f) = (ap \\
& \quad (ap\ V2f\ V0x)\ V1y))))))
\end{aligned} \tag{95}$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum.(\neg(p\ (ap\ (ap\ c_2Eprim_rec_2E_3C \\
\quad V0n)\ V0n)))) \tag{96}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \tag{97}$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \tag{98}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p\ V0A)) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\
\quad (((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))))) \tag{99}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p\ V0A)) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\
\quad ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))))) \tag{100}$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p\ V0A)) \Rightarrow False) \Rightarrow (((p\ V0A) \Rightarrow False) \Rightarrow False))) \tag{101}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow (\\
& \quad (p\ V1q) \Leftrightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((p\ V1q) \vee (p\ V2r))) \wedge (((p\ V0p) \vee ((\neg \\
& \quad p\ V2r)) \vee (\neg(p\ V1q)))) \wedge (((p\ V1q) \vee ((\neg(p\ V2r)) \vee (\neg(p\ V0p)))) \wedge ((p\ V2r) \vee \\
& \quad ((\neg(p\ V1q)) \vee (\neg(p\ V0p))))))))))
\end{aligned} \tag{102}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee \\
& (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{103}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge ((p V0p) \vee (\neg(p V2r)))) \wedge \\
& ((p V1q) \vee ((p V2r) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{104}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge (\\
& (\neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{105}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee \\
& (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p))))))
\end{aligned} \tag{106}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p))) \tag{107}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))) \tag{108}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V0p)))) \tag{109}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V1q)))) \tag{110}$$

Assume the following.

$$(\forall V0p \in 2. ((\neg(\neg(p V0p))) \Rightarrow (p V0p))) \tag{111}$$

Theorem 1
$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0f \in (A_27a^{ty_2Enum_2Enum}). \\ & \quad (\forall V1limopt \in (ty_2Eoption_2Eoption\ ty_2Enum_2Enum). (\\ & \quad (ap\ (c_2Ellist_2ELTL\ A_27a)\ (ap\ (ap\ (c_2Ellist_2ELGENLIST\ A_27a) \\ & \quad V0f)\ V1limopt)) = (ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ (ty_2Eoption_2Eoption \\ & \quad (ty_2Ellist_2Ellist\ A_27a)))\ (ap\ (ap\ (c_2Emin_2E_3D\ (ty_2Eoption_2Eoption \\ & \quad ty_2Enum_2Enum))\ V1limopt)\ (ap\ (c_2Eoption_2ESOME\ ty_2Enum_2Enum) \\ & \quad c_2Enum_2E0)))\ (c_2Eoption_2ENONE\ (ty_2Ellist_2Ellist\ A_27a))) \\ & \quad (ap\ (c_2Eoption_2ESOME\ (ty_2Ellist_2Ellist\ A_27a))\ (ap\ (ap\ (c_2Ellist_2ELGENLIST \\ & \quad A_27a)\ (ap\ (ap\ (c_2Ecombin_2Eo\ ty_2Enum_2Enum\ A_27a\ ty_2Enum_2Enum) \\ & \quad V0f)\ c_2Enum_2ESUC))\ (ap\ (ap\ (c_2Eoption_2EOPTION_MAP\ ty_2Enum_2Enum \\ & \quad ty_2Enum_2Enum)\ c_2Eprim_rec_2EPRE)\ V1limopt)))))) \end{aligned}$$