

thm\_2Ellist\_2ELTL\_\_THM  
(TMP9DSKdsnncSau41HS1QkpL8odpVa8EYM5)

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**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2E\_2T$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2))) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x)$

**Definition 3** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a})))$

**Definition 4** We define  $c\_2Ebool\_2E\_2F$  to be  $(ap (c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 5** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p \Rightarrow q)$  of type  $\iota$ .

**Definition 6** We define  $c\_2Ebool\_2E\_27E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2E\_2F))$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{1}$$

Let  $c\_2Earithmetic\_2E\_2D : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{2}$$

**Definition 7** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V2t)))$

**Definition 8** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A \wedge p))$  of type  $\iota \Rightarrow \iota$ .

**Definition 9** We define  $c\_2Ebool\_2ECOND$  to be  $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.(ap$

**Definition 10** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V2t)))$

**Definition 11** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap V0P (ap (c\_2Emin\_2E\_40$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \tag{3}$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \tag{4}$$

**Definition 12** We define  $c\_2Enum\_2E0$  to be  $(ap\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2EZERO\_REP)$ .

**Definition 13** We define  $c\_2Earithmetic\_2EZERO$  to be  $c\_2Enum\_2E0$ .

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \tag{5}$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \tag{6}$$

**Definition 14** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_num\ m)$

Let  $c\_2Earithmetic\_2E\_2B : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{7}$$

**Definition 15** We define  $c\_2Earithmetic\_2EBIT1$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic\_2E\_2B\ n))$

**Definition 16** We define  $c\_2Earithmetic\_2ENUMERAL$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.V0x$ .

Let  $ty\_2Eone\_2Eone : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Eone\_2Eone \tag{8}$$

Let  $ty\_2Esum\_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Esum\_2Esum\ A0\ A1) \tag{9}$$

Let  $c\_2Esum\_2EABS\_sum : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Esum\_2EABS\_sum\ A\_27a\ A\_27b \in ((ty\_2Esum\_2Esum\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})^2}) \tag{10}$$

**Definition 17** We define  $c\_2Esum\_2EINL$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0e \in A\_27a.(ap\ (c\_2Esum\_2EABS\_sum\ A\_27a\ A\_27b)\ e)$

Let  $ty\_2Eoption\_2Eoption : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Eoption\_2Eoption\ A0) \tag{11}$$

Let  $c\_2Eoption\_2Eoption\_ABS : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Eoption\_2Eoption\_ABS\ A\_27a \in ((ty\_2Eoption\_2Eoption\ A\_27a)^{ty\_2Esum\_2Esum\ A\_27a\ ty\_2Eone\_2Eone}) \tag{12}$$

**Definition 18** We define  $c\_2Eoption\_2ESOME$  to be  $\lambda A\_27a : \iota. \lambda V0x \in A\_27a. (ap (c\_2Eoption\_2Eoption\_2ABS A\_27a) (V0x))$

**Definition 19** We define  $c\_2Eone\_2Eone$  to be  $(ap (c\_2Emin\_2E40 ty\_2Eone\_2Eone) (\lambda V0x \in ty\_2Eone\_2Eone. V0x))$

**Definition 20** We define  $c\_2Esum\_2EINR$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0e \in A\_27b. (ap (c\_2Esum\_2EABS A\_27a A\_27b) (V0e))$

**Definition 21** We define  $c\_2Eoption\_2ENONE$  to be  $\lambda A\_27a : \iota. (ap (c\_2Eoption\_2Eoption\_2ABS A\_27a) (V0))$

**Definition 22** We define  $c\_2Ellist\_2Elrep\_ok$  to be  $\lambda A\_27a : \iota. (\lambda V0a0 \in ((ty\_2Eoption\_2Eoption A\_27a)^{ty\_2Eoption\_2Eoption A\_27a}))$

Let  $ty\_2Ellist\_2Ellist : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0. nonempty A0 \Rightarrow nonempty (ty\_2Ellist\_2Ellist A0) \quad (13)$$

Let  $c\_2Ellist\_2Ellist\_abs : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a. nonempty A\_27a \Rightarrow c\_2Ellist\_2Ellist\_abs A\_27a \in ((ty\_2Ellist\_2Ellist A\_27a)^{(ty\_2Eoption\_2Eoption A\_27a)^{ty\_2Enum\_2Enum}}) \quad (14)$$

**Definition 23** We define  $c\_2Ellist\_2ELNIL$  to be  $\lambda A\_27a : \iota. (ap (c\_2Ellist\_2Ellist\_abs A\_27a) (\lambda V0n \in ty\_2Ellist\_2Ellist A\_27a. V0n))$

Let  $c\_2Ellist\_2Ellist\_rep : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a. nonempty A\_27a \Rightarrow c\_2Ellist\_2Ellist\_rep A\_27a \in (((ty\_2Eoption\_2Eoption A\_27a)^{ty\_2Enum\_2Enum})^{(ty\_2Ellist\_2Ellist A\_27a)}) \quad (15)$$

**Definition 24** We define  $c\_2Ellist\_2ELHD$  to be  $\lambda A\_27a : \iota. \lambda V0ll \in (ty\_2Ellist\_2Ellist A\_27a). (ap (ap (c\_2Ellist\_2Ellist A\_27a) (V0ll)))$

Let  $c\_2Eoption\_2Eoption\_CASE : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a. nonempty A\_27a \Rightarrow \forall A\_27b. nonempty A\_27b \Rightarrow c\_2Eoption\_2Eoption\_CASE A\_27a A\_27b \in (((A\_27b^{(A\_27b^{A\_27a})})^{A\_27b})^{(ty\_2Eoption\_2Eoption A\_27a)}) \quad (16)$$

**Definition 25** We define  $c\_2Ellist\_2ELTL$  to be  $\lambda A\_27a : \iota. \lambda V0ll \in (ty\_2Ellist\_2Ellist A\_27a). (ap (ap (ap (c\_2Ellist\_2Ellist A\_27a) (V0ll))))$

**Definition 26** We define  $c\_2Ellist\_2ELCONS$  to be  $\lambda A\_27a : \iota. \lambda V0h \in A\_27a. \lambda V1t \in (ty\_2Ellist\_2Ellist A\_27a). (ap (ap (c\_2Ellist\_2Ellist A\_27a) (V1t)))$

Assume the following.

$$True \quad (17)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (18)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((p V0t) \Rightarrow False) \Leftrightarrow (\neg (p V0t)))))) \quad (19)$$

Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0x \in A.27a.((V0x = V0x) \Leftrightarrow True)) \quad (20)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg( \\ & p\ V0t)))))) \end{aligned} \quad (21)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in 2.(\forall V1y \in 2.(\forall V2z \in 2.(\forall V3w \in \\ & 2.(((p\ V0x) \Rightarrow (p\ V1y)) \wedge ((p\ V2z) \Rightarrow (p\ V3w))) \Rightarrow (((p\ V0x) \wedge (p\ V2z)) \Rightarrow \\ & ((p\ V1y) \wedge (p\ V3w)))))) \end{aligned} \quad (22)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in 2.(\forall V1y \in 2.(\forall V2z \in 2.(\forall V3w \in \\ & 2.(((p\ V0x) \Rightarrow (p\ V1y)) \wedge ((p\ V2z) \Rightarrow (p\ V3w))) \Rightarrow (((p\ V0x) \vee (p\ V2z)) \Rightarrow \\ & ((p\ V1y) \vee (p\ V3w)))))) \end{aligned} \quad (23)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).(\forall V1Q \in \\ & (2^{A.27a}).(\forall V2x \in A.27a.((p\ (ap\ V0P\ V2x)) \Rightarrow (p\ (ap\ V1Q\ V2x)))) \Rightarrow \\ & ((\exists V3x \in A.27a.(p\ (ap\ V0P\ V3x))) \Rightarrow (\exists V4x \in A.27a.(p\ ( \\ & ap\ V1Q\ V4x)))))) \end{aligned} \quad (24)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow ((\forall V0a \in (ty\_2Ellist\_2Ellist \\ & A.27a).((ap\ (c\_2Ellist\_2Ellist\_abs\ A.27a)\ (ap\ (c\_2Ellist\_2Ellist\_rep \\ & A.27a)\ V0a)) = V0a) \wedge (\forall V1r \in ((ty\_2Eoption\_2Eoption\ A.27a)^{ty\_2Enum\_2Enum}). \\ & ((p\ (ap\ (c\_2Ellist\_2Ellist\_ok\ A.27a)\ V1r)) \Leftrightarrow ((ap\ (c\_2Ellist\_2Ellist\_rep \\ & A.27a)\ (ap\ (c\_2Ellist\_2Ellist\_abs\ A.27a)\ V1r)) = V1r)))) \end{aligned} \quad (25)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0h \in A.27a.(\forall V1t \in \\ & (ty\_2Ellist\_2Ellist\ A.27a).(((ap\ (c\_2Ellist\_2ELHD\ A.27a)\ (ap \\ & (ap\ (c\_2Ellist\_2ELCONS\ A.27a)\ V0h)\ V1t)) = (ap\ (c\_2Eoption\_2ESOME \\ & A.27a)\ V0h)) \wedge ((ap\ (c\_2Ellist\_2ELTL\ A.27a)\ (ap\ (ap\ (c\_2Ellist\_2ELCONS \\ & A.27a)\ V0h)\ V1t)) = (ap\ (c\_2Eoption\_2ESOME\ (ty\_2Ellist\_2Ellist \\ & A.27a)\ V1t)))))) \end{aligned} \quad (26)$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\
& (\forall V0v \in A\_27b. (\forall V1f \in (A\_27b^{A\_27a}). ((ap\ (ap\ (ap\ (c\_2Eoption\_2Eoption\_CASE \\
& A\_27a\ A\_27b)\ (c\_2Eoption\_2ENONE\ A\_27a))\ V0v)\ V1f) = V0v))) \wedge (\forall V2x \in \\
& A\_27a. (\forall V3v \in A\_27b. (\forall V4f \in (A\_27b^{A\_27a}). ((ap\ (ap \\
& (ap\ (c\_2Eoption\_2Eoption\_CASE\ A\_27a\ A\_27b)\ (ap\ (c\_2Eoption\_2ESOME \\
& A\_27a)\ V2x))\ V3v)\ V4f) = (ap\ V4f\ V2x))))))
\end{aligned} \tag{27}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a. (\forall V1y \in \\
& A\_27a. (((ap\ (c\_2Eoption\_2ESOME\ A\_27a)\ V0x) = (ap\ (c\_2Eoption\_2ESOME \\
& A\_27a)\ V1y)) \Leftrightarrow (V0x = V1y))))
\end{aligned} \tag{28}$$

**Theorem 1**

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\
& ((ap\ (c\_2Ellist\_2ELTL\ A\_27a)\ (c\_2Ellist\_2ELNIL\ A\_27a)) = (c\_2Eoption\_2ENONE \\
& (ty\_2Ellist\_2Ellist\ A\_27a))) \wedge (\forall V0h \in A\_27b. (\forall V1t \in \\
& (ty\_2Ellist\_2Ellist\ A\_27b). ((ap\ (c\_2Ellist\_2ELTL\ A\_27b)\ (ap \\
& (ap\ (c\_2Ellist\_2ELCONS\ A\_27b)\ V0h)\ V1t) = (ap\ (c\_2Eoption\_2ESOME \\
& (ty\_2Ellist\_2Ellist\ A\_27b))\ V1t))))))
\end{aligned}$$