thm_2Ellist_2ELUNFOLD (TMP-Vahg8yAQYXNur6zdNcdUT1REySr3mLi4)

October 26, 2020

Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 4 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \tag{1}$$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{2}$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \tag{3}$$

Definition 5 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Definition 6 *We define* c_2Earithmetic_2EZERO *to be* c_2Enum_2E0.

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \tag{4}$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \tag{5}$$

Definition 7 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num\ ($

Let $c\_2Earithmetic\_2E\_2B : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{6}$$

Definition 8 *We define* c_2Earithmetic_2EBIT1 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic\_$

Definition 9 *We define* c_2Earithmetic_2ENUMERAL *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x$.

Let $c\_2Earithmetic\_2E\_2D : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{7}$$

Let $ty\_2Eone\_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Eone\_2Eone \tag{8}$$

Definition 10 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 11 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in$

Let $ty\_2Esum\_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Esum\_2Esum\ A0\ A1) \tag{9}$$

Let $c\_2Esum\_2EABS\_\_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Esum\_2EABS\_\_sum\ A\_27a\ A\_27b \in ((ty\_2Esum\_2Esum\ A\_27a\ A\_27b)^{(((2^{A\_27b})^{A\_27a})^{2})}) \tag{10}$$

Definition 12 *We define* c_2Esum_2EINL *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0e \in A\_27a.(ap\ (c\_2Esum\_2EABS$

Let $ty\_2Eoption\_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Eoption\_2Eoption\ A0) \tag{11}$$

Let $c\_2Eoption\_2Eoption\_\_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Eoption\_2Eoption\_\_ABS\ A\_27a \in ((ty\_2Eoption\_2Eoption\ A\_27a)^{(ty\_2Esum\_2Esum\ A\_27a\ ty\_2Eone\_2Eone)}) \tag{12}$$

Definition 13 *We define* c_2Eoption_2ESOME *to be* $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.(ap\ (c\_2Eoption\_2Eoption\_\_$

Definition 14 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^{A}.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge$ *of type* $\iota \Rightarrow \iota$.

Definition 15 *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

Definition 16 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a:\iota.(\lambda V0P\in(2^{A\_27a}).(ap\ V0P\ (ap$ (c_2Emin_2E_40

Definition 17 *We define* c_2Eone_2Eone *to be* $(ap$ (c_2Emin_2E_40 $ty\_2Eone\_2Eone)\ (\lambda V0x\in ty\_2Eone\_2$

Definition 18 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t\in 2.(ap\ (ap$ c_2Emin_2E_3D_3D_3E $V0t)$ c_2Ebool_2E

Definition 19 *We define* c_2Esum_2EINR *to be* $\lambda A\_27a:\iota.\lambda A\_27b:\iota.\lambda V0e\in A\_27b.(ap\ (c\_2Esum\_2EABS$

Definition 20 *We define* c_2Eoption_2ENONE *to be* $\lambda A\_27a:\iota.(ap\ (c\_2Eoption\_2Eoption\_\_ABS\ A\_27a)\ ($

Definition 21 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1\in 2.(\lambda V1t2\in 2.(ap$ (c_2Ebool_2E_21 2) $(\lambda V2t\in$

Definition 22 *We define* c_2Ellist_2Elrep__ok *to be* $\lambda A\_27a:\iota.(\lambda V0a0\in((ty\_2Eoption\_2Eoption\ A\_27a)^{ty\text{-}}$

Let $ty\_2Ellist\_2Ellist:\iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0\Rightarrow nonempty\ (ty\_2Ellist\_2Ellist\ A0) \tag{13}$$

Let $c\_2Ellist\_2Ellist\_\_rep:\iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow c\_2Ellist\_2Ellist\_\_rep\ A\_27a\in (((ty\_2Eoption\_2Eoption\ A\_27a)^{ty\_2Enum\_2Enum})^{(ty\_2Ellist\_2Ellist\ A\_27a)}) \tag{14}$$

Let $c\_2Ellist\_2Ellist\_\_abs:\iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow c\_2Ellist\_2Ellist\_\_abs\ A\_27a\in ((ty\_2Ellist\_2Ellist\ A\_27a)^{((ty\_2Eoption\_2Eoption\ A\_27a)^{ty\_2Enum\_2Enum})}) \tag{15}$$

Definition 23 *We define* c_2Ellist_2ELCONS *to be* $\lambda A\_27a:\iota.\lambda V0h\in A\_27a.\lambda V1t\in(ty\_2Ellist\_2Ellist\ A$

Definition 24 *We define* c_2Ellist_2ELNIL *to be* $\lambda A\_27a:\iota.(ap\ (c\_2Ellist\_2Ellist\_\_abs\ A\_27a)\ (\lambda V0n\in ty$

Definition 25 *We define* c_2Ecombin_2EK *to be* $\lambda A\_27a:\iota.\lambda A\_27b:\iota.(\lambda V0x\in A\_27a.(\lambda V1y\in A\_27b.V0x)$

Definition 26 *We define* c_2Ecombin_2Eo *to be* $\lambda A\_27a:\iota.\lambda A\_27b:\iota.\lambda A\_27c:\iota.\lambda V0f\in(A\_27b^{A\_27c}).\lambda V1$

Let $c\_2Earithmetic\_2EFUNPOW:\iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow c\_2Earithmetic\_2EFUNPOW\ A\_27a\in (((A\_27a^{A\_27a})^{ty\_2Enum\_2Enum})^{(A\_27a^{A\_27a})}) \tag{16}$$

Let $ty\_2Epair\_2Eprod:\iota\Rightarrow\iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0\Rightarrow\forall A1.nonempty\ A1\Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{17}$$

Let $c\_2Epair\_2ESND:\iota\Rightarrow\iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow\forall A\_27b.nonempty\ A\_27b\Rightarrow c\_2Epair\_2ESND\ A\_27a\ A\_27b\in(A\_27b^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \tag{18}$$

Let $c\_2Epair\_2EFST:\iota\Rightarrow\iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow\forall A\_27b.nonempty\ A\_27b\Rightarrow c\_2Epair\_2EFST\ A\_27a\ A\_27b\in(A\_27a^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \tag{19}$$

Definition 27 *We define* c_2Epair_2EUNCURRY *to be* $\lambda A\_27a:\iota.\lambda A\_27b:\iota.\lambda A\_27c:\iota.\lambda V0f\in((A\_27c^{A\_27}$

Let $c\_2Eoption\_2EOPTION\_\_BIND:\iota\Rightarrow\iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow\forall A\_27b.nonempty\ A\_27b\Rightarrow c\_2Eoption\_2EOPTION\_\_BIND\ A\_27a\ A\_27b\in(((ty\_2Eoption\_2Eoption\ A\_27a)^{((ty\_2Eoption\_2Eoption\ A\_27a)^{A\_27b})})^{(ty\_2Eoption\_2Eoption\ A\_27} \tag{20}$$

Let $c\_2Eoption\_2EOPTION\_\_MAP:\iota\Rightarrow\iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow\forall A\_27b.nonempty\ A\_27b\Rightarrow c\_2Eoption\_2EOPTION\_\_MAP\ A\_27a\ A\_27b\in(((ty\_2Eoption\_2Eoption\ A\_27b)^{(ty\_2Eoption\_2Eoption\ A\_27a)})^{(A\_27b^{A\_27a})}) \tag{21}$$

Definition 28 *We define* c_2Ellist_2ELUNFOLD *to be* $\lambda A\_27a:\iota.\lambda A\_27b:\iota.\lambda V0f\in((ty\_2Eoption\_2Eoptio$

Let $c\_2Eoption\_2Eoption\_\_CASE:\iota\Rightarrow\iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow\forall A\_27b.nonempty\ A\_27b\Rightarrow c\_2Eoption\_2Eoption\_\_CASE\ A\_27a\ A\_27b\in(((A\_27b^{(A\_27b^{A\_27a})})^{A\_27b})^{(ty\_2Eoption\_2Eoption\ A\_27a)}) \tag{22}$$

Definition 29 *We define* c_2Epair_2Epair__CASE *to be* $\lambda A\_27a:\iota.\lambda A\_27b:\iota.\lambda A\_27c:\iota.\lambda V0p\in(ty\_2Epair\_$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow((\forall V0f\in(A\_27a^{A\_27a}).(\forall V1x\in A\_27a.((ap\ (ap\ (ap\ (c\_2Earithmetic\_2EFUNPOW\ A\_27a)\ V0f)\ \mathsf{c\_2Enum\_2E0})\ V1x)=V1x)))\wedge(\forall V2f\in(A\_27a^{A\_27a}).(\forall V3n\in ty\_2Enum\_2Enum.(\forall V4x\in A\_27a.((ap\ (ap\ (ap\ (c\_2Earithmetic\_2EFUNPOW\ A\_27a)\ V2f)\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V3n))\ V4x)=(ap\ (ap\ (ap\ (c\_2Earithmetic\_2EFUNPOW\ A\_27a)\ V2f)\ V3n)\ (ap\ V2f\ V4x)))))))) \tag{23}$$

Assume the following.

$$(\forall V0m\in ty\_2Enum\_2Enum.((V0m=\mathsf{c\_2Enum\_2E0})\vee(\exists V1n\in ty\_2Enum\_2Enum.(V0m=(ap\ \mathsf{c\_2Enum\_2ESUC}\ V1n))))) \tag{24}$$

Assume the following.

$$(\forall V0m\in ty\_2Enum\_2Enum.(((ap\ (ap\ c\_2Earithmetic\_2E\_2D\ \mathsf{c\_2Enum\_2E0})\ V0m)=\mathsf{c\_2Enum\_2E0})\wedge((ap\ (ap\ c\_2Earithmetic\_2E\_2D\ V0m)\ \mathsf{c\_2Enum\_2E0})=V0m))) \tag{25}$$

Assume the following.

$$(\forall V0m\in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2D\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V0m))\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO})))=V0m)) \tag{26}$$

Assume the following.

$$True \tag{27}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \tag{28}$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \tag{29}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \tag{30}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{31}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{32}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0f \in (A\_27b^{A\_27a}).(\forall V1g \in (A\_27b^{A\_27a}).((V0f = V1g) \Leftrightarrow (\forall V2x \in A\_27a.((ap\ V0f\ V2x) = (ap\ V1g\ V2x)))))) \tag{33}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg (p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg (p\ V0t))))))) \tag{34}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t1 \in A\_27a.(\forall V1t2 \in A\_27a.(((ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ \mathsf{c\_2Ebool\_2ET})\ V0t1)\ V1t2) = V0t1) \wedge ((ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ \mathsf{c\_2Ebool\_2EF})\ V0t1)\ V1t2) = V1t2)))) \tag{35}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \tag{36}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in 2.(\forall V2x \in A\_27a.(\forall V3x\_27 \in A\_27a.(\forall V4y \in A\_27a.(\forall V5y\_27 \in A\_27a.((((p\ V0P) \Leftrightarrow (p\ V1Q)) \wedge (((p\ V1Q) \Rightarrow (V2x = V3x\_27)) \wedge ((\neg(p\ V1Q)) \Rightarrow (V4y = V5y\_27)))) \Rightarrow ((ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ V0P)\ V2x)\ V4y) = (ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ V1Q)\ V3x\_27)\ V5y\_27))))))))) \tag{37}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c.nonempty\ A\_27c \Rightarrow (\forall V0f \in (A\_27b^{A\_27a}).(\forall V1g \in (A\_27a^{A\_27c}).(\forall V2x \in A\_27c.((ap\ (ap\ (ap\ (\mathsf{c\_2Ecombin\_2Eo}\ A\_27c\ A\_27b\ A\_27a)\ V0f)\ V1g)\ V2x) = (ap\ V0f\ (ap\ V1g\ V2x)))))) \tag{38}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27b.((ap\ (ap\ (\mathsf{c\_2Ecombin\_2EK}\ A\_27a\ A\_27b)\ V0x)\ V1y) = V0x))) \tag{39}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow ((\forall V0a \in (ty\_2Ellist\_2Ellist\ A\_27a).((ap\ (c\_2Ellist\_2Ellist\_\_abs\ A\_27a)\ (ap\ (c\_2Ellist\_2Ellist\_\_rep\ A\_27a)\ V0a)) = V0a)) \wedge (\forall V1r \in ((ty\_2Eoption\_2Eoption\ A\_27a)^{ty\_2Enum\_2Enum}).((p\ (ap\ (\mathsf{c\_2Ellist\_2Elrep\_\_ok}\ A\_27a)\ V1r)) \Leftrightarrow ((ap\ (c\_2Ellist\_2Ellist\_\_rep\ A\_27a)\ (ap\ (c\_2Ellist\_2Ellist\_\_abs\ A\_27a)\ V1r)) = V1r)))) \tag{40}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0f \in (A\_27a^{A\_27b}).(\forall V1g \in ((ty\_2Eoption\_2Eoption\ A\_27b)^{ty\_2Enum\_2Enum}).((p\ (ap\ (\mathsf{c\_2Ellist\_2Elrep\_\_ok}\ A\_27a)\ (\lambda V2n \in ty\_2Enum\_2Enum.(ap\ (ap\ (c\_2Eoption\_2EOPTION\_\_MAP\ A\_27b\ A\_27a)\ V0f)\ (ap\ V1g\ V2n))))) \Leftrightarrow (p\ (ap\ (\mathsf{c\_2Ellist\_2Elrep\_\_ok}\ A\_27b)\ V1g))))) \tag{41}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0g \in ((ty\_2Eoption\_2Eoption\ A\_27a)^{A\_27a}).(\forall V1fz \in (ty\_2Eoption\_2Eoption\ A\_27a).(p\ (ap\ (\mathsf{c\_2Ellist\_2Elrep\_\_ok}\ A\_27a)\ (\lambda V2n \in ty\_2Enum\_2Enum.(ap\ (ap\ (ap\ (c\_2Earithmetic\_2EFUNPOW\ (ty\_2Eoption\_2Eoption\ A\_27a))\ (\lambda V3m \in (ty\_2Eoption\_2Eoption\ A\_27a).(ap\ (ap\ (c\_2Eoption\_2EOPTION\_\_BIND\ A\_27a\ A\_27a)\ V3m)\ V0g)))\ V2n)\ V1fz)))))) \tag{42}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0h \in A\_27a.(\forall V1t \in (ty\_2Ellist\_2Ellist\ A\_27a).((ap\ (c\_2Ellist\_2Ellist\_\_rep\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Ellist\_2ELCONS}\ A\_27a)\ V0h)\ V1t)) = (\lambda V2n \in ty\_2Enum\_2Enum.(ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ (ty\_2Eoption\_2Eoption\ A\_27a))\ (ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ ty\_2Enum\_2Enum)\ V2n)\ \mathsf{c\_2Enum\_2E0}))\ (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V0h))\ (ap\ (ap\ (c\_2Ellist\_2Ellist\_\_rep\ A\_27a)\ V1t)\ (ap\ (ap\ c\_2Earithmetic\_2E\_2D\ V2n)\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO})))))))))) \tag{43}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow ((ap\ (c\_2Ellist\_2Ellist\_\_rep\ A\_27a)\ (\mathsf{c\_2Ellist\_2ELNIL}\ A\_27a)) = (\lambda V0n \in ty\_2Enum\_2Enum.(\mathsf{c\_2Eoption\_2ENONE}\ A\_27a))) \tag{44}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\neg((ap\ \mathsf{c\_2Enum\_2ESUC}\ V0n) = \mathsf{c\_2Enum\_2E0}))) \tag{45}$$

Assume the following.

$$(\forall V0P \in (2^{ty\_2Enum\_2Enum}).(((p\ (ap\ V0P\ \mathsf{c\_2Enum\_2E0})) \land (\forall V1n \in ty\_2Enum\_2Enum.((p\ (ap\ V0P\ V1n)) \Rightarrow (p\ (ap\ V0P\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V1n)))))) \Rightarrow (\forall V2n \in ty\_2Enum\_2Enum.(p\ (ap\ V0P\ V2n))))) \tag{46}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0opt \in (ty\_2Eoption\_2Eoption\ A\_27a).((V0opt = (\mathsf{c\_2Eoption\_2ENONE}\ A\_27a)) \lor (\exists V1x \in A\_27a.(V0opt = (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V1x))))) \tag{47}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ((\forall V0v \in A\_27b.(\forall V1f \in (A\_27b^{A\_27a}).((ap\ (ap\ (ap\ (c\_2Eoption\_2Eoption\_\_CASE\ A\_27a\ A\_27b)\ (\mathsf{c\_2Eoption\_2ENONE}\ A\_27a))\ V0v)\ V1f) = V0v))) \land (\forall V2x \in A\_27a.(\forall V3v \in A\_27b.(\forall V4f \in (A\_27b^{A\_27a}).((ap\ (ap\ (ap\ (c\_2Eoption\_2Eoption\_\_CASE\ A\_27a\ A\_27b)\ (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V2x))\ V3v)\ V4f) = (ap\ V4f\ V2x)))))) \tag{48}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.(((ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V0x) = (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V1y)) \Leftrightarrow (V0x = V1y)))) \tag{49}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ((\forall V0f \in (A\_27b^{A\_27a}).(\forall V1x \in A\_27a.((ap\ (ap\ (c\_2Eoption\_2EOPTION\_\_MAP\ A\_27a\ A\_27b)\ V0f)\ (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V1x)) = (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27b)\ (ap\ V0f\ V1x)))))\wedge(\forall V2f \in (A\_27b^{A\_27a}).((ap\ (ap\ (c\_2Eoption\_2EOPTION\_\_MAP\ A\_27a\ A\_27b)\ V2f)\ (\mathsf{c\_2Eoption\_2ENONE}\ A\_27a)) = (\mathsf{c\_2Eoption\_2ENONE}\ A\_27b)))) \tag{50}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ((\forall V0f \in ((ty\_2Eoption\_2Eoption\ A\_27a)^{A\_27b}).((ap\ (ap\ (c\_2Eoption\_2EOPTION\_\_BIND\ A\_27a\ A\_27b)\ (\mathsf{c\_2Eoption\_2ENONE}\ A\_27b))\ V0f) = (\mathsf{c\_2Eoption\_2ENONE}\ A\_27a)))\wedge(\forall V1x \in A\_27b.(\forall V2f \in ((ty\_2Eoption\_2Eoption\ A\_27a)^{A\_27b}).((ap\ (ap\ (c\_2Eoption\_2EOPTION\_\_BIND\ A\_27a\ A\_27b)\ (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27b)\ V1x))\ V2f) = (ap\ V2f\ V1x))))) \tag{51}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c. nonempty\ A\_27c \Rightarrow (\forall V0f \in ((A\_27c^{A\_27b})^{A\_27a}).(\forall V1v \in (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b).((ap\ (ap\ (\mathsf{c\_2Epair\_2EUNCURRY}\ A\_27a\ A\_27b\ A\_27c)\ V0f)\ V1v) = (ap\ (ap\ V0f\ (ap\ (c\_2Epair\_2EFST\ A\_27a\ A\_27b)\ V1v))\ (ap\ (c\_2Epair\_2ESND\ A\_27a\ A\_27b)\ V1v))))) \tag{52}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0f \in ((ty\_2Eoption\_2Eoption\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b))^{A\_27a}).(\forall V1x \in A\_27a.((ap\ (ap\ (\mathsf{c\_2Ellist\_2ELUNFOLD}\ A\_27b\ A\_27a)\ V0f)\ V1x) = (ap\ (ap\ (ap\ (c\_2Eoption\_2Eoption\_\_CASE\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)\ (ty\_2Ellist\_2Ellist\ A\_27b))\ (ap\ V0f\ V1x))\ (\mathsf{c\_2Ellist\_2ELNIL}\ A\_27b))\ (\lambda V2v \in (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b).(ap\ (ap\ (\mathsf{c\_2Epair\_2Epair\_\_CASE}\ (ty\_2Ellist\_2Ellist\ A\_27b)\ A\_27a\ A\_27b)\ V2v)\ (\lambda V3v1 \in A\_27a.(\lambda V4v2 \in A\_27b.(ap\ (ap\ (\mathsf{c\_2Ellist\_2ELCONS}\ A\_27b)\ V4v2)\ (ap\ (ap\ (\mathsf{c\_2Ellist\_2ELUNFOLD}\ A\_27b\ A\_27a)\ V0f)\ V3v1)))))))))))$$