

thm_2Ellist_2ELUNFOLD__BISIMULATION (TMWWHsS1DLwGj5PB8p51tVWLfjzyG2UZVfg)

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Definition 1 We define `c_2Emin_2E_3D` to be $\lambda A. \lambda x \in A. \lambda y \in A. \text{inj_o } (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define `c_2Ebool_2ET` to be $(\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^2))) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x)$

Definition 3 We define `c_2Ebool_2E_21` to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^{A_27a}))))$

Definition 4 We define `c_2Ebool_2EF` to be $(\text{ap } (\text{c_2Ebool_2E_21 } 2)) (\lambda V0t \in 2.V0t)$.

Let `ty_2Eoption_2Eoption` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. \text{nonempty } A0 \Rightarrow \text{nonempty } (\text{ty_2Eoption_2Eoption } A0) \quad (1)$$

Let `ty_2Ellist_2Ellist` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. \text{nonempty } A0 \Rightarrow \text{nonempty } (\text{ty_2Ellist_2Ellist } A0) \quad (2)$$

Let `ty_2Enum_2Enum` : ι be given. Assume the following.

$$\text{nonempty } \text{ty_2Enum_2Enum} \quad (3)$$

Let `c_2Ellist_2ELNTH` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. \text{nonempty } A_27a \Rightarrow \text{c_2Ellist_2ELNTH } A_27a \in (((\text{ty_2Eoption_2Eoption } A_27a) (\text{ty_2Ellist_2Ellist } A_27a)) \text{ty_2Enum_2Enum}) \quad (4)$$

Let `c_2Enum_2EZERO__REP` : ι be given. Assume the following.

$$\text{c_2Enum_2EZERO_REP} \in \text{omega} \quad (5)$$

Let `c_2Enum_2EABS__num` : ι be given. Assume the following.

$$\text{c_2Enum_2EABS_num} \in (\text{ty_2Enum_2Enum}^{\text{omega}}) \quad (6)$$

Definition 5 We define `c_2Enum_2E0` to be $(\text{ap } \text{c_2Enum_2EABS_num } \text{c_2Enum_2EZERO_REP})$.

Definition 6 We define $c_Earithmetic_EZERO$ to be c_Enum_E0 .

Let $c_Enum_EREP_num : \iota$ be given. Assume the following.

$$c_Enum_EREP_num \in (\omega^{ty_Enum_Enum}) \quad (7)$$

Let $c_Enum_ESUC_REP : \iota$ be given. Assume the following.

$$c_Enum_ESUC_REP \in (\omega^{\omega}) \quad (8)$$

Definition 7 We define c_Enum_ESUC to be $\lambda V0m \in ty_Enum_Enum.(ap\ c_Enum_EABS_num$

Let $c_Earithmetic_E_2B : \iota$ be given. Assume the following.

$$c_Earithmetic_E_2B \in ((ty_Enum_Enum^{ty_Enum_Enum})^{ty_Enum_Enum}) \quad (9)$$

Definition 8 We define $c_Earithmetic_EBIT1$ to be $\lambda V0n \in ty_Enum_Enum.(ap\ (ap\ c_Earithmetic_E_2B$

Definition 9 We define $c_Earithmetic_ENUMERAL$ to be $\lambda V0x \in ty_Enum_Enum.V0x$.

Let $c_Earithmetic_E_2D : \iota$ be given. Assume the following.

$$c_Earithmetic_E_2D \in ((ty_Enum_Enum^{ty_Enum_Enum})^{ty_Enum_Enum}) \quad (10)$$

Let $c_Ellist_Ellist_rep : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow c_Ellist_Ellist_rep\ A_27a \in \\ (((ty_Eoption_Eoption\ A_27a)^{ty_Enum_Enum})^{(ty_Ellist_Ellist\ A_27a)}) \end{aligned} \quad (11)$$

Let $ty_Eone_Eone : \iota$ be given. Assume the following.

$$nonempty\ ty_Eone_Eone \quad (12)$$

Definition 10 We define $c_Emin_E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o\ (p\ P \Rightarrow p\ Q)$ of type ι .

Definition 11 We define $c_Ebool_E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_Ebool_E_21\ 2)\ (\lambda V2t \in 2.$

Let $ty_Esum_Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_Esum_Esum\ A0\ A1) \quad (13)$$

Let $c_Esum_EABS_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_Esum_EABS_sum\ A_27a\ A_27b \in ((ty_Esum_Esum\ A_27a\ A_27b)^{((2^{A_27b})^{A_27a})^2}) \quad (14)$$

Definition 12 We define c_2Esum_2EINL to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0e \in A_27a. (ap (c_2Esum_2EABS$

Let $c_2Eoption_2Eoption_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow c_2Eoption_2Eoption_ABS A_27a \in ((ty_2Eoption_2Eoption A_27a)^{(ty_2Esum_2Esum A_27a ty_2Eone_2Eone)}) \quad (15)$$

Definition 13 We define $c_2Eoption_2ESOME$ to be $\lambda A_27a : \iota. \lambda V0x \in A_27a. (ap (c_2Eoption_2Eoption_ABS$

Definition 14 We define $c_2Emin_2E_40$ to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p (ap P x)) \text{ then } (the (\lambda x. x \in A \wedge P x))$ of type $\iota \Rightarrow \iota$.

Definition 15 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A_27a. (\lambda V2t2 \in A_27a. (ap$

Let $c_2Ellist_2Ellist_abs : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow c_2Ellist_2Ellist_abs A_27a \in ((ty_2Ellist_2Ellist A_27a)^{(ty_2Eoption_2Eoption A_27a)^{ty_2Enum_2Enum}}) \quad (16)$$

Definition 16 We define $c_2Ellist_2ELCONS$ to be $\lambda A_27a : \iota. \lambda V0h \in A_27a. \lambda V1t \in (ty_2Ellist_2Ellist A_27a)$

Definition 17 We define c_2Eone_2Eone to be $(ap (c_2Emin_2E_40 ty_2Eone_2Eone)) (\lambda V0x \in ty_2Eone_2Eone)$

Definition 18 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2. (ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_7E$

Definition 19 We define c_2Esum_2EINR to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0e \in A_27b. (ap (c_2Esum_2EABS$

Definition 20 We define $c_2Eoption_2ENONE$ to be $\lambda A_27a : \iota. (ap (c_2Eoption_2Eoption_ABS A_27a))$

Definition 21 We define $c_2Ellist_2ELNIL$ to be $\lambda A_27a : \iota. (ap (c_2Ellist_2Ellist_abs A_27a)) (\lambda V0n \in ty_2Ellist_2Ellist A_27a)$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. nonempty A0 \Rightarrow \forall A1. nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \quad (17)$$

Definition 22 We define $c_2Ecombin_2EK$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. (\lambda V0x \in A_27a. (\lambda V1y \in A_27b. V0x)$

Definition 23 We define $c_2Ecombin_2Eo$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda A_27c : \iota. \lambda V0f \in (A_27b^{A_27c}). \lambda V1g \in (A_27c^{A_27b})$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow \forall A_27b. nonempty A_27b \Rightarrow c_2Epair_2ESND A_27a A_27b \in (A_27b^{(ty_2Epair_2Eprod A_27a A_27b)}) \quad (18)$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow \forall A_27b. nonempty A_27b \Rightarrow c_2Epair_2EFST A_27a A_27b \in (A_27a^{(ty_2Epair_2Eprod A_27a A_27b)}) \quad (19)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2. ((\exists V1x \in A_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \quad (28)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \end{aligned} \quad (29)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \vee (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \vee True) \Leftrightarrow True) \wedge \\ & (((False \vee (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \vee False) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \vee \\ & (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \end{aligned} \quad (30)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((\\ & (p\ V0t) \Rightarrow False) \Leftrightarrow \neg(p\ V0t)))))) \end{aligned} \quad (31)$$

Assume the following.

$$\begin{aligned} & ((\forall V0t \in 2. ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge \\ & ((\neg False) \Leftrightarrow True)))) \end{aligned} \quad (32)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (33)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (34)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg(\\ & p\ V0t)))))) \end{aligned} \quad (35)$$

Assume the following.

$$\begin{aligned} & (\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p\ V0t1) \Rightarrow \\ & ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \end{aligned} \quad (36)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in 2. (\forall V1x_27 \in 2. (\forall V2y \in 2. (\forall V3y_27 \in \\ & 2. (((p\ V0x) \Leftrightarrow (p\ V1x_27)) \wedge ((p\ V1x_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y_27)))) \Rightarrow \\ & (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x_27) \Rightarrow (p\ V3y_27)))))) \end{aligned} \quad (37)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}).(\forall V1a \in A_27a.((\exists V2x \in A_27a.((V2x = V1a) \wedge (p (ap\ V0P\ V2x)))) \Leftrightarrow (p (ap\ V0P\ V1a)))))) \quad (38)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}).(\forall V1Q \in 2.(((p\ V1Q) \Rightarrow (\forall V2x \in A_27a.(p (ap\ V0P\ V2x)))) \Leftrightarrow (\forall V3x \in A_27a.((p\ V1Q) \Rightarrow (p (ap\ V0P\ V3x)))))) \wedge (((\forall V4x \in A_27a.(p (ap\ V0P\ V4x)) \wedge (p\ V1Q)) \Leftrightarrow (\forall V5x \in A_27a.((p (ap\ V0P\ V5x)) \wedge (p\ V1Q)))) \wedge (((p\ V1Q) \wedge (\forall V6x \in A_27a.(p (ap\ V0P\ V6x)))) \Leftrightarrow (\forall V7x \in A_27a.((p\ V1Q) \wedge (p (ap\ V0P\ V7x)))))))))) \quad (39)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0h \in A_27a.(\forall V1t \in (ty_2Ellist_2Ellist\ A_27a).((\neg((ap (ap (c_2Ellist_2ELCONS\ A_27a)\ V0h)\ V1t) = (c_2Ellist_2ELNIL\ A_27a))) \wedge (\neg((c_2Ellist_2ELNIL\ A_27a) = (ap (ap (c_2Ellist_2ELCONS\ A_27a)\ V0h)\ V1t)))))) \quad (40)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0h1 \in A_27a.(\forall V1t1 \in (ty_2Ellist_2Ellist\ A_27a).(\forall V2h2 \in A_27a.(\forall V3t2 \in (ty_2Ellist_2Ellist\ A_27a).((ap (ap (c_2Ellist_2ELCONS\ A_27a)\ V0h1)\ V1t1) = (ap (ap (c_2Ellist_2ELCONS\ A_27a)\ V2h2)\ V3t2)) \Leftrightarrow ((V0h1 = V2h2) \wedge (V1t1 = V3t2)))))) \quad (41)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c.nonempty\ A_27c \Rightarrow ((\forall V0n \in ty_2Enum_2Enum.((ap (ap (c_2Ellist_2ELNTH\ A_27a)\ V0n)\ (c_2Ellist_2ELNIL\ A_27a)) = (c_2Eoption_2ENONE\ A_27a))) \wedge ((\forall V1h \in A_27b.(\forall V2t \in (ty_2Ellist_2Ellist\ A_27b).((ap (ap (c_2Ellist_2ELNTH\ A_27b)\ c_2Enum_2E0)\ (ap (ap (c_2Ellist_2ELCONS\ A_27b)\ V1h)\ V2t)) = (ap (c_2Eoption_2ESOME\ A_27b)\ V1h)))) \wedge (\forall V3n \in ty_2Enum_2Enum.(\forall V4h \in A_27c.(\forall V5t \in (ty_2Ellist_2Ellist\ A_27c).((ap (ap (c_2Ellist_2ELNTH\ A_27c)\ (ap\ c_2Enum_2ESUC\ V3n))\ (ap (ap (c_2Ellist_2ELCONS\ A_27c)\ V4h)\ V5t)) = (ap (ap (c_2Ellist_2ELNTH\ A_27c)\ V3n)\ V5t)))))) \quad (42)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0ll1 \in (ty_2Ellist_2Ellist\ A_27a).(\forall V1ll2 \in (ty_2Ellist_2Ellist\ A_27a).((V0ll1 = V1ll2) \Leftrightarrow (\forall V2n \in ty_2Enum_2Enum.((ap (ap (c_2Ellist_2ELNTH\ A_27a)\ V2n)\ V0ll1) = (ap (ap (c_2Ellist_2ELNTH\ A_27a)\ V2n)\ V1ll2)))))) \quad (43)$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0f \in ((ty_2Eoption_2Eoption\ (ty_2Epair_2Eprod\ A_27a \\
& \quad A_27b))^{A_27a}).(\forall V1x \in A_27a.((ap\ (ap\ (c_2Elist_2ELUNFOLD \\
& \quad A_27b\ A_27a)\ V0f)\ V1x) = (ap\ (ap\ (ap\ (c_2Eoption_2Eoption_CASE \\
& \quad (ty_2Epair_2Eprod\ A_27a\ A_27b)\ (ty_2Elist_2Elist\ A_27b))\ (\\
& \quad ap\ V0f\ V1x))\ (c_2Elist_2ELNIL\ A_27b))\ (\lambda V2v \in (ty_2Epair_2Eprod \\
& \quad A_27a\ A_27b).(\lambda V3v1 \in A_27a.(\lambda V4v2 \in A_27b.(ap \\
& \quad (ap\ (c_2Elist_2ELCONS\ A_27b)\ V4v2)\ (ap\ (ap\ (c_2Elist_2ELUNFOLD \\
& \quad A_27b\ A_27a)\ V0f)\ V3v1))))))))) \\
& \hspace{15em} (44)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0P \in (2^{ty_2Enum_2Enum}).(((p\ (ap\ V0P\ c_2Enum_2E0)) \wedge \\
& (\forall V1n \in ty_2Enum_2Enum.((p\ (ap\ V0P\ V1n)) \Rightarrow (p\ (ap\ V0P\ (ap\ c_2Enum_2ESUC \\
& \quad V1n)))))) \Rightarrow (\forall V2n \in ty_2Enum_2Enum.(p\ (ap\ V0P\ V2n)))))) \\
& \hspace{15em} (45)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0opt \in (ty_2Eoption_2Eoption \\
& \quad A_27a).((V0opt = (c_2Eoption_2ENONE\ A_27a)) \vee (\exists V1x \in A_27a. \\
& \quad (V0opt = (ap\ (c_2Eoption_2ESOME\ A_27a)\ V1x)))))) \\
& \hspace{15em} (46)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad (\forall V0v \in A_27b.(\forall V1f \in (A_27b^{A_27a}).((ap\ (ap\ (ap\ (c_2Eoption_2Eoption_CASE \\
& \quad A_27a\ A_27b)\ (c_2Eoption_2ENONE\ A_27a))\ V0v)\ V1f) = V0v))) \wedge (\forall V2x \in \\
& \quad A_27a.(\forall V3v \in A_27b.(\forall V4f \in (A_27b^{A_27a}).((ap\ (ap \\
& \quad (ap\ (c_2Eoption_2Eoption_CASE\ A_27a\ A_27b)\ (ap\ (c_2Eoption_2ESOME \\
& \quad A_27a)\ V2x))\ V3v)\ V4f) = (ap\ V4f\ V2x)))))) \\
& \hspace{15em} (47)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in \\
& \quad A_27a.(((ap\ (c_2Eoption_2ESOME\ A_27a)\ V0x) = (ap\ (c_2Eoption_2ESOME \\
& \quad A_27a)\ V1y)) \Leftrightarrow (V0x = V1y)))) \\
& \hspace{15em} (48)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.(\neg((c_2Eoption_2ENONE \\
& \quad A_27a) = (ap\ (c_2Eoption_2ESOME\ A_27a)\ V0x)))) \\
& \hspace{15em} (49)
\end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \quad \forall V0x \in A_27a. (\forall V1y \in A_27b. (\forall V2a \in A_27a. (\forall V3b \in \\ & \quad A_27b. (((ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27b)\ V0x)\ V1y) = (ap\ (ap \\ & \quad (c_2Epair_2E_2C\ A_27a\ A_27b)\ V2a)\ V3b)) \Leftrightarrow ((V0x = V2a) \wedge (V1y = V3b)))))) \\ & \end{aligned} \tag{50}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \quad \forall V0x \in (ty_2Epair_2Eprod\ A_27a\ A_27b). (\exists V1q \in A_27a. \\ & \quad (\exists V2r \in A_27b. (V0x = (ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27b)\ \\ & \quad V1q)\ V2r)))))) \\ & \end{aligned} \tag{51}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \quad \forall V0x \in A_27a. (\forall V1y \in A_27b. ((ap\ (c_2Epair_2EFST\ A_27a \\ & \quad A_27b)\ (ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27b)\ V0x)\ V1y)) = V0x))) \\ & \end{aligned} \tag{52}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \quad \forall V0x \in A_27a. (\forall V1y \in A_27b. ((ap\ (c_2Epair_2ESND\ A_27a \\ & \quad A_27b)\ (ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27b)\ V0x)\ V1y)) = V1y))) \\ & \end{aligned} \tag{53}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\ & \quad nonempty\ A_27c \Rightarrow (\forall V0x \in A_27b. (\forall V1y \in A_27c. (\forall V2f \in \\ & \quad ((A_27a^{A_27c})^{A_27b}). ((ap\ (ap\ (c_2Epair_2Epair_CASE\ A_27a\ A_27b \\ & \quad A_27c)\ (ap\ (ap\ (c_2Epair_2E_2C\ A_27b\ A_27c)\ V0x)\ V1y))\ V2f) = (ap \\ & \quad (ap\ V2f\ V0x)\ V1y)))))) \\ & \end{aligned} \tag{54}$$

Theorem 1

$$\begin{aligned}
& \forall A_27a.\text{nonempty } A_27a \Rightarrow \forall A_27b.\text{nonempty } A_27b \Rightarrow \forall A_27c. \\
& \text{nonempty } A_27c \Rightarrow (\forall V0f1 \in ((\text{ty_2Eoption_2Eoption } (\text{ty_2Epair_2Eprod} \\
& \quad A_27a \ A_27b))^{A_27a}).(\forall V1f2 \in ((\text{ty_2Eoption_2Eoption } (\\
& \quad \text{ty_2Epair_2Eprod } A_27c \ A_27b))^{A_27c}).(\forall V2x1 \in A_27a.(\\
& \quad \forall V3x2 \in A_27c.(((\text{ap } (\text{ap } (\text{c_2Ellist_2ELUNFOLD } A_27b \ A_27a) \\
V0f1) \ V2x1) = (\text{ap } (\text{ap } (\text{c_2Ellist_2ELUNFOLD } A_27b \ A_27c) \ V1f2) \ V3x2)) \Leftrightarrow \\
& \quad (\exists V4R \in ((2^{A_27c})^{A_27a}).((p \ (\text{ap } (\text{ap } \ V4R \ V2x1) \ V3x2)) \wedge (\forall V5y1 \in \\
& \quad A_27a.(\forall V6y2 \in A_27c.((p \ (\text{ap } (\text{ap } \ V4R \ V5y1) \ V6y2)) \Rightarrow (((\text{ap} \\
V0f1 \ V5y1) = (\text{c_2Eoption_2ENONE } (\text{ty_2Epair_2Eprod } A_27a \ A_27b))) \wedge \\
& \quad ((\text{ap } \ V1f2 \ V6y2) = (\text{c_2Eoption_2ENONE } (\text{ty_2Epair_2Eprod } A_27c \ A_27b)))))) \vee \\
& \quad (\exists V7h \in A_27b.(\exists V8t1 \in A_27a.(\exists V9t2 \in A_27c. \\
& \quad (((\text{ap } \ V0f1 \ V5y1) = (\text{ap } (\text{c_2Eoption_2ESOME } (\text{ty_2Epair_2Eprod } A_27a \\
& \quad A_27b)) \ (\text{ap } (\text{ap } (\text{c_2Epair_2E_2C } A_27a \ A_27b) \ V8t1) \ V7h))) \wedge ((\text{ap} \\
V1f2 \ V6y2) = (\text{ap } (\text{c_2Eoption_2ESOME } (\text{ty_2Epair_2Eprod } A_27c \ A_27b)) \\
& \quad (\text{ap } (\text{ap } (\text{c_2Epair_2E_2C } A_27c \ A_27b) \ V9t2) \ V7h))) \wedge (p \ (\text{ap } (\text{ap } \ V4R \\
& \quad \ V8t1) \ V9t2))))))))))))))
\end{aligned}$$