thm_2Ellist_2ELZIP__LUNZIP (TMJLjGZcynxZ4JYy1455NmUXx1sSyCs9r3q)

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Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27a}$

Definition 4 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 5 *We define* c_2Ebool_2EBOUNDED *to be* $(\lambda V0v \in 2.\mathsf{c\_2Ebool\_2ET})$.

Definition 6 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 7 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \mathsf{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \mathsf{c\_2Ebool\_2EF}$

Definition 8 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{1}$$

Let $ty\_2Ellist\_2Ellist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Ellist\_2Ellist\ A0) \tag{2}$$

Let $c\_2Ellist\_2ELZIP : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Ellist\_2ELZIP\ A\_27a\ A\_27b \in ((ty\_2Ellist\_2Ellist\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b))^{(ty\_2Epair\_2Eprod\ (ty\_2Ellist\_2Ellist\ A} \tag{3}$$

Definition 9 *We define* c_2Ebool_2ELET *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0f \in (A\_27b^{A\_27a}).(\lambda V1x \in A\_27a$

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \tag{4}$$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{5}$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \tag{6}$$

Definition 10 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Definition 11 *We define* c_2Earithmetic_2EZERO *to be* c_2Enum_2E0.

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \tag{7}$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \tag{8}$$

Definition 12 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num$

Let $c\_2Earithmetic\_2E\_2B : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{9}$$

Definition 13 *We define* c_2Earithmetic_2EBIT1 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

Definition 14 *We define* c_2Earithmetic_2ENUMERAL *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x$.

Let $c\_2Earithmetic\_2E\_2D : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{10}$$

Let $ty\_2Eoption\_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Eoption\_2Eoption\ A0) \tag{11}$$

Let $c\_2Ellist\_2Ellist\_\_rep : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ellist\_2Ellist\_\_rep\ A\_27a \in (((ty\_2Eoption\_2Eoption\ A\_27a)^{ty\_2Enum\_2Enum})^{(ty\_2Ellist\_2Ellist\ A\_27a)}) \tag{12}$$

Let $ty\_2Eone\_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Eone\_2Eone \tag{13}$$

Definition 15 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in$

Let $ty\_2Esum\_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Esum\_2Esum\ A0\ A1) \tag{14}$$

Let $c\_2Esum\_2EABS\_\_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Esum\_2EABS\_\_sum\ A\_27a\ A\_27b \in ((ty\_2Esum\_2Esum\ A\_27a\ A\_27b)^{(((2^{A\_27b})^{A\_27a})^2)}) \tag{15}$$

Definition 16 *We define* c_2Esum_2EINL *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0e \in A\_27a.(ap\ (c\_2Esum\_2EABS$

Let $c\_2Eoption\_2Eoption\_\_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Eoption\_2Eoption\_\_ABS\ A\_27a \in ((ty\_2Eoption\_2Eoption\ A\_27a)^{(ty\_2Esum\_2Esum\ A\_27a\ ty\_2Eone\_2Eone)}) \tag{16}$$

Definition 17 *We define* c_2Eoption_2ESOME *to be* $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.(ap\ (c\_2Eoption\_2Eoption\_\_$

Definition 18 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

Definition 19 *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

Let $c\_2Ellist\_2Ellist\_\_abs : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ellist\_2Ellist\_\_abs\ A\_27a \in ((ty\_2Ellist\_2Ellist\ A\_27a)^{((ty\_2Eoption\_2Eoption\ A\_27a)^{ty\_2Enum\_2Enum})}) \tag{17}$$

Definition 20 *We define* c_2Ellist_2ELCONS *to be* $\lambda A\_27a : \iota.\lambda V0h \in A\_27a.\lambda V1t \in (ty\_2Ellist\_2Ellist\ A.$

Definition 21 *We define* c_2Eone_2Eone *to be* $(ap\ (\mathsf{c\_2Emin\_2E\_40}\ ty\_2Eone\_2Eone)\ (\lambda V0x \in ty\_2Eone\_2$

Definition 22 *We define* c_2Esum_2EINR *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0e \in A\_27b.(ap\ (c\_2Esum\_2EABS$

Definition 23 *We define* c_2Eoption_2ENONE *to be* $\lambda A\_27a : \iota.(ap\ (c\_2Eoption\_2Eoption\_\_ABS\ A\_27a)\ ($

Definition 24 *We define* c_2Ellist_2ELNIL *to be* $\lambda A\_27a : \iota.(ap\ (c\_2Ellist\_2Ellist\_\_abs\ A\_27a)\ (\lambda V0n \in ty$

Let $c\_2Ellist\_2ELUNZIP : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Ellist\_2ELUNZIP\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ (ty\_2Ellist\_2Ellist\ A\_27a)\ (ty\_2Ellist\_2Ellist\ A\_27b))^{(ty\_2Ellist\_2Ellist\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b))}) \tag{18}$$

Definition 25 *We define* c_2Emarker_2EAbbrev *to be* $\lambda V0x \in 2.V0x$.

Definition 26 *We define* c_2Emarker_2ECong *to be* $\lambda V0x \in 2.V0x$.

Definition 27 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap$ (c_2Emin_2E_40

Definition 28 *We define* c_2Ecombin_2EC *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.(\lambda V0f \in ((A\_27c^{A\_27b})^{A\_2}$

Let $c\_2Epair\_2EABS\_\_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})}) \tag{19}$$

Definition 29 *We define* c_2Epair_2E_2C *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2$

Definition 30 *We define* c_2Ecombin_2ES *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.(\lambda V0f \in ((A\_27c^{A\_27b})^{A\_2}$

Definition 31 *We define* c_2Ecombin_2Eo *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in (A\_27b^{A\_27c}).\lambda V1$

Let $c\_2Epair\_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2ESND\ A\_27a\ A\_27b \in (A\_27b^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \tag{20}$$

Let $c\_2Epair\_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EFST\ A\_27a\ A\_27b \in (A\_27a^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \tag{21}$$

Definition 32 *We define* c_2Epair_2EUNCURRY *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in ((A\_27c^{A\_27}$

Assume the following.

$$True \tag{22}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \tag{23}$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \tag{24}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\exists V1x \in A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \tag{25}$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge ((((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t))))))) \tag{26}$$

Assume the following.

$$(\forall V0t \in 2.(((True\vee(p\ V0t))\Leftrightarrow True)\wedge((((p\ V0t)\vee True)\Leftrightarrow True)\wedge(((False\vee(p\ V0t))\Leftrightarrow(p\ V0t))\wedge((((p\ V0t)\vee False)\Leftrightarrow(p\ V0t))\wedge(((p\ V0t)\vee(p\ V0t))\Leftrightarrow(p\ V0t))))))) \tag{27}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0x \in A\_27a.((V0x = V0x)\Leftrightarrow True)) \tag{28}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y)\Leftrightarrow(V1y = V0x)))) \tag{29}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1)\Rightarrow((p\ V1t2)\Rightarrow(p\ V2t3)))\Leftrightarrow(((p\ V0t1)\wedge(p\ V1t2))\Rightarrow(p\ V2t3)))))) \tag{30}$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x\_27 \in 2.(\forall V2y \in 2.(\forall V3y\_27 \in 2.((((p\ V0x)\Leftrightarrow(p\ V1x\_27))\wedge((p\ V1x\_27)\Rightarrow((p\ V2y)\Leftrightarrow(p\ V3y\_27))))\Rightarrow(((p\ V0x)\Rightarrow(p\ V2y))\Leftrightarrow((p\ V1x\_27)\Rightarrow(p\ V3y\_27)))))))) \tag{31}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0P \in (2^{A\_27a}).(\forall V1a \in A\_27a.((\exists V2x \in A\_27a.((V2x = V1a)\wedge(p\ (ap\ V0P\ V2x))))\Leftrightarrow(p\ (ap\ V0P\ V1a))))) \tag{32}$$

Assume the following.

$$(\forall V0v \in 2.((p\ (ap\ \mathsf{c\_2Ebool\_2EBOUNDED}\ V0v))\Leftrightarrow True)) \tag{33}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow\forall A\_27b.nonempty\ A\_27b\Rightarrow\forall A\_27c.nonempty\ A\_27c\Rightarrow(\forall V0f \in (A\_27b^{A\_27a}).(\forall V1g \in (A\_27a^{A\_27c}).(\forall V2x \in A\_27c.((ap\ (ap\ (ap\ (\mathsf{c\_2Ecombin\_2Eo}\ A\_27c\ A\_27b\ A\_27a)\ V0f)\ V1g)\ V2x) = (ap\ V0f\ (ap\ V1g\ V2x)))))) \tag{34}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow\forall A\_27b.nonempty\ A\_27b\Rightarrow\forall A\_27c.nonempty\ A\_27c\Rightarrow(\forall V0f \in (A\_27b^{A\_27c}).(\forall V1g \in (A\_27c^{A\_27a}).((ap\ (ap\ (\mathsf{c\_2Ecombin\_2Eo}\ A\_27a\ A\_27b\ A\_27c)\ V0f)\ (\lambda V2x \in A\_27a.(ap\ V1g\ V2x))) = (\lambda V3x \in A\_27a.(ap\ V0f\ (ap\ V1g\ V3x)))))) \tag{35}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c.\ nonempty\ A\_27c \Rightarrow (\forall V0f \in ((A\_27b^{A\_27c})^{A\_27a}).(\forall V1g \in (A\_27c^{A\_27a}).((ap\ (ap\ (\mathsf{c\_2Ecombin\_2ES}\ A\_27a\ A\_27c\ A\_27b)\ V0f)\ (\lambda V2x \in A\_27a.(ap\ V1g\ V2x))) = (\lambda V3x \in A\_27a.(ap\ (ap\ V0f\ V3x)\ (ap\ V1g\ V3x)))))) \tag{36}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c.\ nonempty\ A\_27c \Rightarrow (\forall V0f \in ((A\_27c^{A\_27b})^{A\_27a}).(\forall V1x \in A\_27b.(\forall V2y \in A\_27a.((ap\ (ap\ (ap\ (\mathsf{c\_2Ecombin\_2EC}\ A\_27a\ A\_27b\ A\_27c)\ V0f)\ V1x)\ V2y) = (ap\ (ap\ V0f\ V2y)\ V1x))))) \tag{37}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c.\ nonempty\ A\_27c \Rightarrow (\forall V0f \in ((A\_27b^{A\_27c})^{A\_27a}).(\forall V1y \in A\_27c.((ap\ (ap\ (\mathsf{c\_2Ecombin\_2EC}\ A\_27a\ A\_27c\ A\_27b)\ (\lambda V2x \in A\_27a.\ (ap\ V0f\ V2x)))\ V1y) = (\lambda V3x \in A\_27a.(ap\ (ap\ V0f\ V3x)\ V1y))))) \tag{38}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c.\ nonempty\ A\_27c \Rightarrow (\forall V0P \in (A\_27a^{A\_27b}).(\forall V1f \in (A\_27b^{A\_27c}).\ (\forall V2v \in A\_27c.((ap\ V0P\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ELET}\ A\_27c\ A\_27b)\ V1f)\ V2v)) = (ap\ (ap\ (\mathsf{c\_2Ebool\_2ELET}\ A\_27c\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Ecombin\_2Eo}\ A\_27c\ A\_27a\ A\_27b)\ V0P)\ V1f))\ V2v))))) \tag{39}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c.\ nonempty\ A\_27c \Rightarrow (\forall V0f \in ((A\_27a^{A\_27c})^{A\_27b}).(\forall V1v \in A\_27b.(\forall V2x \in A\_27c.((ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ELET}\ A\_27b\ (A\_27a^{A\_27c}))\ V0f)\ V1v)\ V2x) = (ap\ (ap\ (\mathsf{c\_2Ebool\_2ELET}\ A\_27b\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Ecombin\_2EC}\ A\_27b\ A\_27c\ A\_27a)\ V0f)\ V2x))\ V1v))))) \tag{40}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0f \in (2^{A\_27a}).(\forall V1v \in A\_27a.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ELET}\ A\_27a\ 2)\ V0f)\ V1v)) \Leftrightarrow (p\ (ap\ (\mathsf{c\_2Ebool\_2E\_21}\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Ecombin\_2ES}\ A\_27a\ 2\ 2)\ (ap\ (ap\ (\mathsf{c\_2Ecombin\_2Eo}\ A\_27a\ (2^{2})\ 2)\ \mathsf{c\_2Emin\_2E\_3D\_3D\_3E})\ (ap\ (ap\ (\mathsf{c\_2Ecombin\_2Eo}\ A\_27a\ 2\ 2)\ \mathsf{c\_2Emarker\_2EAbbrev})\ (ap\ (ap\ (\mathsf{c\_2Ecombin\_2EC}\ A\_27a\ A\_27a\ 2)\ (\mathsf{c\_2Emin\_2E\_3D}\ A\_27a))\ V1v))))\ V0f)))))) \tag{41}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0l \in (ty\_2Ellist\_2Ellist\ A\_27a).((V0l = (\mathsf{c\_2Ellist\_2ELNIL}\ A\_27a)) \vee (\exists V1h \in A\_27a.(\exists V2t \in (ty\_2Ellist\_2Ellist\ A\_27a).(V0l = (ap\ (ap\ (\mathsf{c\_2Ellist\_2ELCONS}\ A\_27a)\ V1h)\ V2t)))))) \tag{42}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0h \in A\_27a.(\forall V1t \in (ty\_2Ellist\_2Ellist\ A\_27a).((\neg((ap\ (ap\ (\mathsf{c\_2Ellist\_2ELCONS}\ A\_27a)\ V0h)\ V1t) = (\mathsf{c\_2Ellist\_2ELNIL}\ A\_27a))) \wedge (\neg((\mathsf{c\_2Ellist\_2ELNIL}\ A\_27a) = (ap\ (ap\ (\mathsf{c\_2Ellist\_2ELCONS}\ A\_27a)\ V0h)\ V1t)))))) \tag{43}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0h1 \in A\_27a.(\forall V1t1 \in (ty\_2Ellist\_2Ellist\ A\_27a).(\forall V2h2 \in A\_27a.(\forall V3t2 \in (ty\_2Ellist\_2Ellist\ A\_27a).(((ap\ (ap\ (\mathsf{c\_2Ellist\_2ELCONS}\ A\_27a)\ V0h1)\ V1t1) = (ap\ (ap\ (\mathsf{c\_2Ellist\_2ELCONS}\ A\_27a)\ V2h2)\ V3t2)) \Leftrightarrow ((V0h1 = V2h2) \wedge (V1t1 = V3t2))))))) \tag{44}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0ll1 \in (ty\_2Ellist\_2Ellist\ A\_27a).(\forall V1ll2 \in (ty\_2Ellist\_2Ellist\ A\_27a).((V0ll1 = V1ll2) \Leftrightarrow (\exists V2R \in ((2^{(ty\_2Ellist\_2Ellist\ A\_27a)})^{(ty\_2Ellist\_2Ellist\ A\_27a)}).((p\ (ap\ (ap\ V2R\ V0ll1)\ V1ll2)) \wedge (\forall V3ll3 \in (ty\_2Ellist\_2Ellist\ A\_27a).(\forall V4ll4 \in (ty\_2Ellist\_2Ellist\ A\_27a).((p\ (ap\ (ap\ V2R\ V3ll3)\ V4ll4)) \Rightarrow ((V3ll3 = V4ll4) \vee (\exists V5h \in A\_27a.(\exists V6t1 \in (ty\_2Ellist\_2Ellist\ A\_27a).(\exists V7t2 \in (ty\_2Ellist\_2Ellist\ A\_27a).((V3ll3 = (ap\ (ap\ (\mathsf{c\_2Ellist\_2ELCONS}\ A\_27a)\ V5h)\ V6t1)) \wedge ((V4ll4 = (ap\ (ap\ (\mathsf{c\_2Ellist\_2ELCONS}\ A\_27a)\ V5h)\ V7t2)) \wedge (p\ (ap\ (ap\ V2R\ V6t1)\ V7t2)))))))))))))))) \tag{45}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ((\forall V0l1 \in (ty\_2Ellist\_2Ellist\ A\_27a).((ap\ (c\_2Ellist\_2ELZIP\ A\_27a\ A\_27b)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (ty\_2Ellist\_2Ellist\ A\_27a)\ (ty\_2Ellist\_2Ellist\ A\_27b))\ V0l1)\ (\mathsf{c\_2Ellist\_2ELNIL}\ A\_27b))) = (\mathsf{c\_2Ellist\_2ELNIL}\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b))))\wedge((\forall V1l2 \in (ty\_2Ellist\_2Ellist\ A\_27b).((ap\ (c\_2Ellist\_2ELZIP\ A\_27a\ A\_27b)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (ty\_2Ellist\_2Ellist\ A\_27a)\ (ty\_2Ellist\_2Ellist\ A\_27b))\ (\mathsf{c\_2Ellist\_2ELNIL}\ A\_27a))\ V1l2)) = (\mathsf{c\_2Ellist\_2ELNIL}\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b))))\wedge(\forall V2h1 \in A\_27a.(\forall V3h2 \in A\_27b.(\forall V4t1 \in (ty\_2Ellist\_2Ellist\ A\_27a).(\forall V5t2 \in (ty\_2Ellist\_2Ellist\ A\_27b).((ap\ (c\_2Ellist\_2ELZIP\ A\_27a\ A\_27b)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (ty\_2Ellist\_2Ellist\ A\_27a)\ (ty\_2Ellist\_2Ellist\ A\_27b))\ (ap\ (ap\ (\mathsf{c\_2Ellist\_2ELCONS}\ A\_27a)\ V2h1)\ V4t1))\ (ap\ (ap\ (\mathsf{c\_2Ellist\_2ELCONS}\ A\_27b)\ V3h2)\ V5t2))) = (ap\ (ap\ (\mathsf{c\_2Ellist\_2ELCONS}\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b))\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27b)\ V2h1)\ V3h2))\ (ap\ (c\_2Ellist\_2ELZIP\ A\_27a\ A\_27b)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (ty\_2Ellist\_2Ellist\ A\_27a)\ (ty\_2Ellist\_2Ellist\ A\_27b))\ V4t1)\ V5t2))))))))))) \tag{46}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (((ap\ (c\_2Ellist\_2ELUNZIP\ A\_27a\ A\_27b)\ (\mathsf{c\_2Ellist\_2ELNIL}\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b))) = (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (ty\_2Ellist\_2Ellist\ A\_27a)\ (ty\_2Ellist\_2Ellist\ A\_27b))\ (\mathsf{c\_2Ellist\_2ELNIL}\ A\_27a))\ (\mathsf{c\_2Ellist\_2ELNIL}\ A\_27b)))\wedge(\forall V0x \in A\_27a.(\forall V1y \in A\_27b.(\forall V2t \in (ty\_2Ellist\_2Ellist\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)).((ap\ (c\_2Ellist\_2ELUNZIP\ A\_27a\ A\_27b)\ (ap\ (ap\ (\mathsf{c\_2Ellist\_2ELCONS}\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b))\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27b)\ V0x)\ V1y))\ V2t)) = (ap\ (ap\ (\mathsf{c\_2Ebool\_2ELET}\ (ty\_2Epair\_2Eprod\ (ty\_2Ellist\_2Ellist\ A\_27a)\ (ty\_2Ellist\_2Ellist\ A\_27b))\ (ty\_2Epair\_2Eprod\ (ty\_2Ellist\_2Ellist\ A\_27a)\ (ty\_2Ellist\_2Ellist\ A\_27b)))\ (ap\ (\mathsf{c\_2Epair\_2EUNCURRY}\ (ty\_2Ellist\_2Ellist\ A\_27a)\ (ty\_2Ellist\_2Ellist\ A\_27b)\ (ty\_2Epair\_2Eprod\ (ty\_2Ellist\_2Ellist\ A\_27a)\ (ty\_2Ellist\_2Ellist\ A\_27b)))\ (\lambda V3ll1 \in (ty\_2Ellist\_2Ellist\ A\_27a).(\lambda V4ll2 \in (ty\_2Ellist\_2Ellist\ A\_27b).(ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (ty\_2Ellist\_2Ellist\ A\_27a)\ (ty\_2Ellist\_2Ellist\ A\_27b))\ (ap\ (ap\ (\mathsf{c\_2Ellist\_2ELCONS}\ A\_27a)\ V0x)\ V3ll1))\ (ap\ (ap\ (\mathsf{c\_2Ellist\_2ELCONS}\ A\_27b)\ V1y)\ V4ll2))))))\ (ap\ (c\_2Ellist\_2ELUNZIP\ A\_27a\ A\_27b)\ V2t))))))) \tag{47}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow\forall A\_27b.nonempty\ A\_27b\Rightarrow(\forall V0x\in A\_27a.(\forall V1y\in A\_27b.(\forall V2a\in A\_27a.(\forall V3b\in A\_27b.(((ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27b)\ V0x)\ V1y) = (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27b)\ V2a)\ V3b))\Leftrightarrow((V0x = V2a)\wedge(V1y = V3b)))))))\tag{48}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow\forall A\_27b.nonempty\ A\_27b\Rightarrow(\forall V0x\in(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b).(\exists V1q\in A\_27a. (\exists V2r\in A\_27b.(V0x = (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27b)\ V1q)\ V2r)))))\tag{49}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow\forall A\_27b.nonempty\ A\_27b\Rightarrow\forall A\_27c. nonempty\ A\_27c\Rightarrow\forall A\_27d.nonempty\ A\_27d\Rightarrow(\forall V0f\in(A\_27c^{A\_27d}). (\forall V1g\in((A\_27d^{A\_27b})^{A\_27a}).((ap\ (ap\ (\mathsf{c\_2Ecombin\_2Eo}\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)\ A\_27c\ A\_27d)\ V0f)\ (ap\ (\mathsf{c\_2Epair\_2EUNCURRY}\ A\_27a\ A\_27b\ A\_27d)\ V1g)) = (ap\ (\mathsf{c\_2Epair\_2EUNCURRY}\ A\_27a\ A\_27b\ A\_27c) (ap\ (ap\ (\mathsf{c\_2Ecombin\_2Eo}\ A\_27a\ (A\_27c^{A\_27b})\ (A\_27d^{A\_27b}))\ (ap\ (\mathsf{c\_2Ecombin\_2Eo}\ A\_27b\ A\_27c\ A\_27d)\ V0f))\ V1g)))))\tag{50}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow\forall A\_27b.nonempty\ A\_27b\Rightarrow\forall A\_27c. nonempty\ A\_27c\Rightarrow\forall A\_27d.nonempty\ A\_27d\Rightarrow(\forall V0f\in(((A\_27c^{A\_27d})^{A\_27b})^{A\_27a}).(\forall V1x\in A\_27d.((ap\ (ap\ (\mathsf{c\_2Ecombin\_2EC} (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)\ A\_27d\ A\_27c)\ (ap\ (\mathsf{c\_2Epair\_2EUNCURRY} A\_27a\ A\_27b\ (A\_27c^{A\_27d}))\ V0f))\ V1x) = (ap\ (\mathsf{c\_2Epair\_2EUNCURRY} A\_27a\ A\_27b\ A\_27c)\ (ap\ (ap\ (\mathsf{c\_2Ecombin\_2EC}\ A\_27a\ A\_27d\ (A\_27c^{A\_27b})) (ap\ (ap\ (\mathsf{c\_2Ecombin\_2Eo}\ A\_27a\ ((A\_27c^{A\_27b})^{A\_27d})\ ((A\_27c^{A\_27d})^{A\_27b})) (\mathsf{c\_2Ecombin\_2EC}\ A\_27b\ A\_27d\ A\_27c))\ V0f))\ V1x)))))\tag{51}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c.\ nonempty\ A\_27c \Rightarrow \forall A\_27d.nonempty\ A\_27d \Rightarrow (\forall V0f \in ((A\_27c^{A\_27d})^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}).(\forall V1g \in ((A\_27d^{A\_27b})^{A\_27a}).((ap\ (ap\ (\mathsf{c\_2Ecombin\_2ES}\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)\ A\_27d\ A\_27c)\ V0f)\ (ap\ (\mathsf{c\_2Epair\_2EUNCURRY}\ A\_27a\ A\_27b\ A\_27d)\ V1g)) = (ap\ (\mathsf{c\_2Epair\_2EUNCURRY}\ A\_27a\ A\_27b\ A\_27c)\ (ap\ (ap\ (\mathsf{c\_2Ecombin\_2ES}\ A\_27a\ (A\_27d^{A\_27b})\ (A\_27c^{A\_27b}))\ (ap\ (ap\ (\mathsf{c\_2Ecombin\_2Eo}\ A\_27a\ ((A\_27c^{A\_27b})^{(A\_27d^{A\_27b})})\ ((A\_27c^{A\_27d})^{A\_27b}))\ (\mathsf{c\_2Ecombin\_2ES}\ A\_27b\ A\_27d\ A\_27c))\ (ap\ (ap\ (\mathsf{c\_2Ecombin\_2Eo}\ A\_27a\ ((A\_27c^{A\_27d})^{A\_27b})\ ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{A\_27b}))\ (ap\ (\mathsf{c\_2Ecombin\_2Eo}\ A\_27b\ (A\_27c^{A\_27d})\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b))\ V0f))\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27b))))\ V1g))))) \tag{52}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0f \in ((2^{A\_27b})^{A\_27a}).((p\ (ap\ (\mathsf{c\_2Ebool\_2E\_21}\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b))\ (ap\ (\mathsf{c\_2Epair\_2EUNCURRY}\ A\_27a\ A\_27b\ 2)\ V0f))) \Leftrightarrow (p\ (ap\ (\mathsf{c\_2Ebool\_2E\_21}\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Ecombin\_2Eo}\ A\_27a\ 2\ (2^{A\_27b}))\ (\mathsf{c\_2Ebool\_2E\_21}\ A\_27b))\ V0f))))) \tag{53}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0ll \in (ty\_2Ellist\_2Ellist\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)).((ap\ (c\_2Ellist\_2ELZIP\ A\_27a\ A\_27b)\ (ap\ (c\_2Ellist\_2ELUNZIP\ A\_27a\ A\_27b)\ V0ll)) = V0ll))$$