

thm_2Ellist_2ENOT__LFINITE__DROP
(TMEgm3HM8ETZVcoY61Vf2RtfNTTNgwYQswy)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a}))$

Definition 4 We define $c_2Ebool_2E_2F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p \Rightarrow q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_27E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2F$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{1}$$

Let $c_2Earithmetic_2E_2D : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2D \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})ty_2Enum_2Enum) \tag{2}$$

Definition 7 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))$

Definition 8 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A \wedge p x))$ of type $\iota \Rightarrow \iota$.

Definition 9 We define c_2Ebool_2ECOND to be $\lambda A.\lambda a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A.\lambda V2t2 \in A.\lambda V3t3 \in A.$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{3}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{4}$$

Definition 10 We define c_Enum_2E0 to be $(ap\ c_Enum_2EABS_num\ c_Enum_2EZERO_REP)$.

Definition 11 We define $c_Earithmetic_2EZERO$ to be c_Enum_2E0 .

Let $c_Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (5)$$

Let $c_Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_Enum_2ESUC_REP \in (\omega^{\omega}) \quad (6)$$

Definition 12 We define c_Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_Enum_2EABS_num$

$c_Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_Earithmetic_2E_2B \in ((ty_2Enum_2Enum)^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (7)$$

Definition 13 We define $c_Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ (ap\ c_Earithmetic$

Definition 14 We define $c_Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let $ty_2Eone_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eone_2Eone \quad (8)$$

Let $ty_2Esum_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Esum_2Esum\ A0\ A1) \quad (9)$$

Let $c_Esum_2EABS_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_Esum_2EABS_sum\ A_27a\ A_27b \in ((ty_2Esum_2Esum\ A_27a\ A_27b)^{((2^{A_27b})^{A_27a})^2}) \quad (10)$$

Definition 15 We define c_Esum_2EINL to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0e \in A_27a.(ap\ (c_Esum_2EABS$

Let $ty_2Eoption_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Eoption_2Eoption\ A0) \quad (11)$$

Let $c_Eoption_2Eoption_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_Eoption_2Eoption_ABS\ A_27a \in ((ty_2Eoption_2Eoption\ A_27a)^{(ty_2Esum_2Esum\ A_27a\ ty_2Eone_2Eone)}) \quad (12)$$

Definition 16 We define $c_Eoption_2ESOME$ to be $\lambda A_27a : \iota.\lambda V0x \in A_27a.(ap\ (c_Eoption_2Eoption$

Definition 17 We define $c_Ebool_2E_3F$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap\ V0P\ (ap\ (c_2Emin_2E_40$

Definition 18 We define c_Eone_2Eone to be $(ap\ (c_2Emin_2E_40\ ty_2Eone_2Eone)\ (\lambda V0x \in ty_2Eone_2$

Definition 19 We define c_Esum_2EINR to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0e \in A_27b. (ap\ (c_2Esum_2EABS$

Definition 20 We define $c_EOption_2ENONE$ to be $\lambda A_27a : \iota. (ap\ (c_2EOption_2EOption_ABS\ A_27a)\ ($

Definition 21 We define $c_Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in$

Definition 22 We define $c_Ellist_2ELrep_ok$ to be $\lambda A_27a : \iota. (\lambda V0a0 \in ((ty_2EOption_2EOption\ A_27a)^{ty-}$

Let $ty_2Ellist_2Ellist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. nonempty\ A0 \Rightarrow nonempty\ (ty_2Ellist_2Ellist\ A0) \quad (13)$$

Let $c_2Ellist_2Ellist_rep : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a. nonempty\ A_27a \Rightarrow c_2Ellist_2Ellist_rep\ A_27a \in \\ (((ty_2EOption_2EOption\ A_27a)^{ty_2Enum_2Enum})^{(ty_2Ellist_2Ellist\ A_27a)}) \end{aligned} \quad (14)$$

Let $c_2Ellist_2Ellist_abs : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a. nonempty\ A_27a \Rightarrow c_2Ellist_2Ellist_abs\ A_27a \in \\ ((ty_2Ellist_2Ellist\ A_27a)^{((ty_2EOption_2EOption\ A_27a)^{ty_2Enum_2Enum})}) \end{aligned} \quad (15)$$

Definition 23 We define c_Ellist_2ELHD to be $\lambda A_27a : \iota. \lambda V0ll \in (ty_2Ellist_2Ellist\ A_27a). (ap\ (ap\ (c_2$

Definition 24 We define $c_Ellist_2ELCONS$ to be $\lambda A_27a : \iota. \lambda V0h \in A_27a. \lambda V1t \in (ty_2Ellist_2Ellist\ A$

Definition 25 We define c_Ellist_2ELNIL to be $\lambda A_27a : \iota. (ap\ (c_2Ellist_2Ellist_abs\ A_27a)\ (\lambda V0n \in ty$

Definition 26 We define $c_Ellist_2ELFINITE$ to be $\lambda A_27a : \iota. (\lambda V0a0 \in (ty_2Ellist_2Ellist\ A_27a). (ap\ (c$

Let $c_2EOption_2EOption_CASE : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a. nonempty\ A_27a \Rightarrow \forall A_27b. nonempty\ A_27b \Rightarrow c_2EOption_2EOption_CASE \\ A_27a\ A_27b \in (((A_27b^{(A_27b^{A_27a})})^{A_27b})^{(ty_2EOption_2EOption\ A_27a)}) \end{aligned} \quad (16)$$

Definition 27 We define c_Ellist_2ELTL to be $\lambda A_27a : \iota. \lambda V0ll \in (ty_2Ellist_2Ellist\ A_27a). (ap\ (ap\ (ap$

Let $c_2Ellist_2ELDROP : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a. nonempty\ A_27a \Rightarrow c_2Ellist_2ELDROP\ A_27a \in (((ty_2EOption_2EOption \\ (ty_2Ellist_2Ellist\ A_27a))^{(ty_2Ellist_2Ellist\ A_27a)})^{ty_2Enum_2Enum}) \end{aligned} \quad (17)$$

Let $c_2EOption_2EOPTION_MAP : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a. nonempty\ A_27a \Rightarrow \forall A_27b. nonempty\ A_27b \Rightarrow c_2EOption_2EOPTION_MAP \\ A_27a\ A_27b \in (((ty_2EOption_2EOption\ A_27b)^{(ty_2EOption_2EOption\ A_27a)})^{(A_27b^{A_27a})}) \end{aligned} \quad (18)$$

Let $c_2Eoption_2EOPTION_JOIN : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Eoption_2EOPTION_JOIN\ A_27a \in ((ty_2Eoption_2Eoption\ A_27a)^{(ty_2Eoption_2Eoption\ (ty_2Eoption_2Eoption\ A_27a))}) \quad (19)$$

Assume the following.

$$True \quad (20)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (21)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \quad (22)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p\ V0t) \Leftrightarrow (p\ V0t)))) \quad (23)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((p\ V0t) \Rightarrow False) \Leftrightarrow \neg(p\ V0t)))))) \quad (24)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t)) \wedge ((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (25)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (26)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (27)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg(p\ V0t)))))) \quad (28)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \quad (29)$$

Assume the following.

$$2.(((\forall V0x \in 2.(\forall V1x.27 \in 2.(\forall V2y \in 2.(\forall V3y.27 \in 2.(((p V0x) \Leftrightarrow (p V1x.27)) \wedge ((p V1x.27) \Rightarrow ((p V2y) \Leftrightarrow (p V3y.27)))))) \Rightarrow ((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x.27) \Rightarrow (p V3y.27)))))) \Rightarrow (30)$$

Assume the following.

$$2.(((\forall V0x \in 2.(\forall V1y \in 2.(\forall V2z \in 2.(\forall V3w \in 2.(((p V0x) \Rightarrow (p V1y)) \wedge ((p V2z) \Rightarrow (p V3w))) \Rightarrow (((p V0x) \wedge (p V2z)) \Rightarrow ((p V1y) \wedge (p V3w)))))) \Rightarrow (31)$$

Assume the following.

$$2.(((\forall V0x \in 2.(\forall V1y \in 2.(\forall V2z \in 2.(\forall V3w \in 2.(((p V0x) \Rightarrow (p V1y)) \wedge ((p V2z) \Rightarrow (p V3w))) \Rightarrow (((p V0x) \vee (p V2z)) \Rightarrow ((p V1y) \vee (p V3w)))))) \Rightarrow (32)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).(\forall V1Q \in (2^{A.27a}).((\forall V2x \in A.27a.((p (ap V0P V2x)) \Rightarrow (p (ap V1Q V2x)))) \Rightarrow ((\exists V3x \in A.27a.(p (ap V0P V3x))) \Rightarrow (\exists V4x \in A.27a.(p (ap V1Q V4x)))))) \Rightarrow (33)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0a \in A.27a.(\exists V1x \in A.27a.(V1x = V0a))) \Rightarrow (34)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow ((\forall V0a \in (ty_2Ellist_2Ellist A.27a).((ap (c_2Ellist_2Ellist_abs A.27a) (ap (c_2Ellist_2Ellist_rep A.27a) V0a)) = V0a)) \wedge (\forall V1r \in ((ty_2Eoption_2Eoption A.27a)^{ty_2Enum_2Enum}).((p (ap (c_2Ellist_2Elrep_ok A.27a) V1r)) \Leftrightarrow ((ap (c_2Ellist_2Ellist_rep A.27a) (ap (c_2Ellist_2Ellist_abs A.27a) V1r)) = V1r)))) \Rightarrow (35)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0h \in A.27a.(\forall V1t \in (ty_2Ellist_2Ellist A.27a).(((ap (c_2Ellist_2ELHD A.27a) (ap (ap (c_2Ellist_2ELCONS A.27a) V0h) V1t)) = (ap (c_2Eoption_2ESOME A.27a) V0h)) \wedge ((ap (c_2Ellist_2ELTL A.27a) (ap (ap (c_2Ellist_2ELCONS A.27a) V0h) V1t)) = (ap (c_2Eoption_2ESOME (ty_2Ellist_2Ellist A.27a) V1t)))))) \Rightarrow (36)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0l \in (ty_2Ellist_2Ellist \\ A_27a).((V0l = (c_2Ellist_2ELNIL\ A_27a)) \vee (\exists V1h \in A_27a. \\ (\exists V2t \in (ty_2Ellist_2Ellist\ A_27a).(V0l = (ap\ (ap\ (c_2Ellist_2ELCONS \\ A_27a)\ V1h)\ V2t))))))) \end{aligned} \quad (37)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ ((p\ (ap\ (c_2Ellist_2ELFINITE\ A_27a)\ (c_2Ellist_2ELNIL\ A_27a))) \Leftrightarrow \\ True) \wedge (\forall V0h \in A_27b.(\forall V1t \in (ty_2Ellist_2Ellist \\ A_27b).((p\ (ap\ (c_2Ellist_2ELFINITE\ A_27b)\ (ap\ (ap\ (c_2Ellist_2ELCONS \\ A_27b)\ V0h)\ V1t))) \Leftrightarrow (p\ (ap\ (c_2Ellist_2ELFINITE\ A_27b)\ V1t)))))) \end{aligned} \quad (38)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow ((\forall V0ll \in (ty_2Ellist_2Ellist \\ A_27a).((ap\ (ap\ (c_2Ellist_2ELDROP\ A_27a)\ c_2Enum_2E0)\ V0ll) = \\ (ap\ (c_2Eoption_2ESOME\ (ty_2Ellist_2Ellist\ A_27a)\ V0ll))) \wedge \\ (\forall V1n \in ty_2Enum_2Enum.(\forall V2ll \in (ty_2Ellist_2Ellist \\ A_27a).((ap\ (ap\ (c_2Ellist_2ELDROP\ A_27a)\ (ap\ c_2Enum_2ESUC\ V1n)) \\ V2ll) = (ap\ (c_2Eoption_2EOPTION_JOIN\ (ty_2Ellist_2Ellist\ A_27a) \\ (ap\ (ap\ (c_2Eoption_2EOPTION_MAP\ (ty_2Ellist_2Ellist\ A_27a) \\ (ty_2Eoption_2Eoption\ (ty_2Ellist_2Ellist\ A_27a))\ (ap\ (c_2Ellist_2ELDROP \\ A_27a)\ V1n))\ (ap\ (c_2Ellist_2ELTL\ A_27a)\ V2ll))))))) \end{aligned} \quad (39)$$

Assume the following.

$$\begin{aligned} (\forall V0P \in (2^{ty_2Enum_2Enum}).(((p\ (ap\ V0P\ c_2Enum_2E0)) \wedge \\ (\forall V1n \in ty_2Enum_2Enum.((p\ (ap\ V0P\ V1n)) \Rightarrow (p\ (ap\ V0P\ (ap\ c_2Enum_2ESUC \\ V1n)))))) \Rightarrow (\forall V2n \in ty_2Enum_2Enum.(p\ (ap\ V0P\ V2n)))))) \end{aligned} \quad (40)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ (\forall V0v \in A_27b.(\forall V1f \in (A_27b^{A_27a}).((ap\ (ap\ (ap\ (c_2Eoption_2Eoption_CASE \\ A_27a\ A_27b)\ (c_2Eoption_2ENONE\ A_27a)\ V0v)\ V1f) = V0v))) \wedge (\forall V2x \in \\ A_27a.(\forall V3v \in A_27b.(\forall V4f \in (A_27b^{A_27a}).((ap\ (ap \\ (ap\ (c_2Eoption_2Eoption_CASE\ A_27a\ A_27b)\ (ap\ (c_2Eoption_2ESOME \\ A_27a)\ V2x))\ V3v)\ V4f) = (ap\ V4f\ V2x)))))) \end{aligned} \quad (41)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in \\ A_27a.(((ap\ (c_2Eoption_2ESOME\ A_27a)\ V0x) = (ap\ (c_2Eoption_2ESOME \\ A_27a)\ V1y)) \Leftrightarrow (V0x = V1y)))) \end{aligned} \quad (42)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & (\forall V0f \in (A_27b^{A_27a}).(\forall V1x \in A_27a.((ap\ (ap\ (c_2Eoption_2EOPTION_MAP \\ & A_27a\ A_27b)\ V0f)\ (ap\ (c_2Eoption_2ESOME\ A_27a)\ V1x)) = (ap\ (c_2Eoption_2ESOME \\ & A_27b)\ (ap\ V0f\ V1x)))))) \wedge (\forall V2f \in (A_27b^{A_27a}).((ap\ (ap\ (c_2Eoption_2EOPTION_MAP \\ & A_27a\ A_27b)\ V2f)\ (c_2Eoption_2ENONE\ A_27a)) = (c_2Eoption_2ENONE \\ & A_27b)))))) \end{aligned} \tag{43}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (((ap\ (c_2Eoption_2EOPTION_JOIN \\ & A_27a)\ (c_2Eoption_2ENONE\ (ty_2Eoption_2Eoption\ A_27a))) = (\\ & c_2Eoption_2ENONE\ A_27a)) \wedge (\forall V0x \in (ty_2Eoption_2Eoption \\ & A_27a).((ap\ (c_2Eoption_2EOPTION_JOIN\ A_27a)\ (ap\ (c_2Eoption_2ESOME \\ & (ty_2Eoption_2Eoption\ A_27a))\ V0x)) = V0x))) \end{aligned} \tag{44}$$

Theorem 1

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0ll \in (ty_2Ellist_2Ellist \\ & A_27a).((\neg(p\ (ap\ (c_2Ellist_2ELFINITE\ A_27a)\ V0ll))) \Rightarrow (\forall V1n \in \\ & ty_2Enum_2Enum.(\exists V2y \in (ty_2Ellist_2Ellist\ A_27a).((\\ & ap\ (ap\ (c_2Ellist_2ELDROP\ A_27a)\ V1n)\ V0ll) = (ap\ (c_2Eoption_2ESOME \\ & (ty_2Ellist_2Ellist\ A_27a)\ V2y))))))) \end{aligned}$$